Indian National Physics Olympiad - 2015

Roll Number ☐☐☐☐ - ☐☐☐☐ - ☐☐☐☐

Date : 1st February 2015

Time: **9.00 to 12.00 (3 hours)**

Maximum Marks 100

Please fill in all the data below correctly. The contact details provided here would be used for all further correspondence.
Only BASIC / NORMAL calculator is allowed.

Full Name(BLOCK letters) Ms. / Mr. ____

____

Sex: M / F T-shirt size: S/M/L/XL/XXL Date of Birth (dd/mm/yyyy) ____________________

Name of the school / junior college: ____

____

Class: XI / XII Board: ICSE / CBSE / State Board / Other

Address for correspondence (include city and PIN code: ____

____

____PIN code ☐☐☐☐☐☐

Telephone(with area code):____________________________Mobile:____________________________

Email address:____

I **permit/do not permit** (strike out one) IAPT_APhO Cell to reveal my academic performance and personal details to a third party.

Besides the International Physics Olympiad, do you also want to be considered for the Asian Physics Olympiad? APhO-2015 shall be held in Zhejiang University, China from 3rd May 2015 to 11th May 2015. The pre-departure training shall be held in Delhi. Your presence in Delhi-cum-China will be required from April 27th 2015 to May 12th 2015. In principle you can participate in both Olympiads.

Yes/No.

I have read the procedural rules for INPhO and agree to abide by them. Signature

(Do not write below this line)

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MARKS

Question:	1	2	3	4	5	6	**Total**
Marks:	20	20	20	10	20	10	100
Score:							

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Instructions:

1. Write your Roll Number on every page of this booklet.
2. Fill out the attached performance card. **Do not detach it from this booklet**.
3. Booklet consists of 25 pages and 6 questions and 3 extra graph papers.
4. Questions consist of sub-questions. Write your **detailed answer** in the blank space provided below the sub-question and final answer to the sub-question in the **smaller box** which follows the blank space.
5. You may use the empty pages if you need more space. You may also use these sheets for rough work.
6. Only **normal, basic or non-scientific calculator** is allowed. Calculator containing any trigonometric, log or plotting function will lead to your disqualification from the exam.
7. A mobile phone **cannot** be used as a calculator.
8. Mobiles, pagers, smart watches, slide rules, log tables etc. are **not** allowed.
9. **This entire booklet must be returned.**

Table of Information:

Speed of light in vacuum	$c = 3.00 \times 10^{8} ms^{-1}$
Planck's constant	$h = 6.63 \times 10^{-34} Js$
Universal constant of Gravitation	$G = 6.67 \times 10^{-11} Nm^{2}Kg^{-2}$
Magnitude of the electron charge	$e = 1.60 \times 10^{-19} C$
Mass of the electron	$m_e = 9.11 \times 10^{-11} kg = 0.51\, MeVc^{-2}$
Stefan - Boltzmann constant	$\sigma = 5.67 \times 10^{-8} Wm^{-2}K^{-4}$
Value of $\frac{1}{4\pi\epsilon_0}$	$= 9.00 \times 10^{9} Nm^{2}C^{-2}$
Permeability constant	$\mu_0 = 4\pi \times 10^{7} Hm^{-1}$
Universal Gas constant	$R = 8.31\, JK^{-1}mole^{-1}$
Molar mass of air	$= 29.0\, kgkmol^{-1}$
Rest mass of the proton	$m_p = 1.67 \times 10^{-27} kg$
1 atomic mass unit	$1u = 1.661 \times 10^{-27} kg$
Boltzmann constant	$k = 1.38 \times 10^{-22} JK^{-1}$

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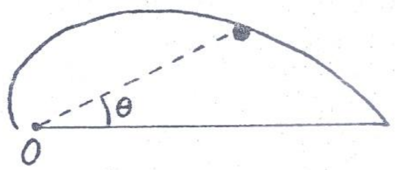
Q1. A particle of mass m slides along a frictionless track, fixed in the horizontal plane, of the form $r = r_0 \exp(-a\theta)$ where a is a positive constant . Its initial speed at $t = 0$ is v_0 when $\theta = 0$. Recall that in polar coordinates $\vec{\boldsymbol{r}} = r\,\hat{\boldsymbol{r}}$, $\frac{d\vec{r}}{dt} = \vec{\boldsymbol{v}} = \frac{dr}{dt}\hat{\boldsymbol{r}} + r\frac{d\theta}{dt}\,\hat{\theta}$, $\hat{\boldsymbol{r}}$ and $\hat{\boldsymbol{\theta}}$ are unit vectors in the radial direction and a direction perpendicular to $\hat{\boldsymbol{r}}$ respectively.



i) Find the θ dependence of both the speed and the angle α that the velocity vector makes with the radial line connecting origin O with the particle. (2 + 2)

$v(\vartheta)$ = constant = v_0 and independent of θ $\alpha(\vartheta)$ = constant = $\tan^{-1} 1/a$ and independent of θ

ii) Obtain an expression for the rate of change of angular momentum $\frac{dL}{dt}$ about O in terms of α and other related quantities. (4)

dL/dt $= -mv_0^2 \sin\alpha \cos\alpha$

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iii) Obtain an expression for the normal force N exerted by the track on the particle in terms of θ , α and other related quantities. (2)

N(θ) = $\frac{mv_0^2}{r_{0\sqrt{1+a^2}}}$ $e^{a\theta}$

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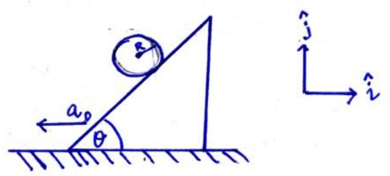
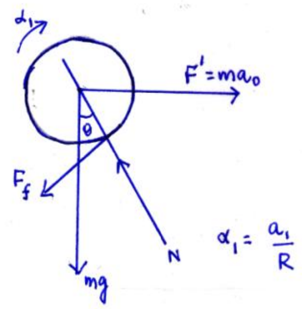
Q 1 b) A cylinder of mass M and radius R is placed on an inclined plane with angle of inclination θ. The inclined plane has acceleration a_0 with respect to an inertial frame as shown in the figure.



Assuming the cylinder rolls without slipping,

(i) Draw the free body diagram of the cylinder in the frame of the inclined plane. (2)



(ii) Find the acceleration $\vec{a}$ of the center of mass of the cylinder with respect to an inertial frame. (8)

$\vec{a} = (a_1 \cos\theta - a_0)\hat{\imath} + a_1 \sin\theta\, \hat{\jmath}$ where $a_1 = \frac{2}{3}(a_0 \cos\theta - g \sin\theta)$

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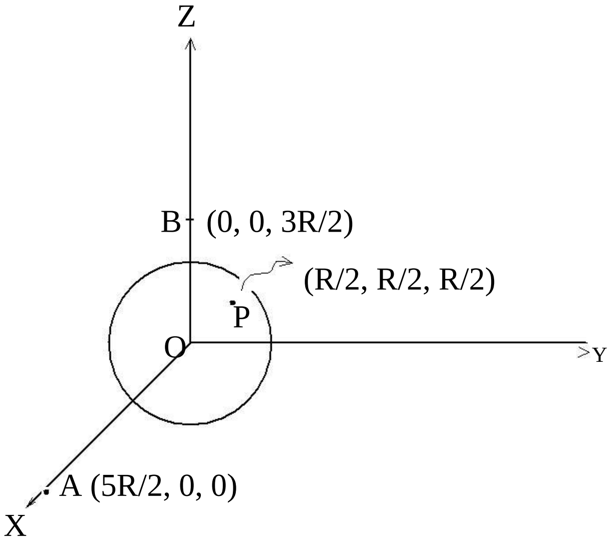
Q 2a) An uncharged solid spherical conductor of radius R has its centre at the origin of a cartesian frame OXY Z. Two point charges each of +Q are placed at fixed positions at (5R/2, 0, 0) and (0, 0, 3R/2). There will be induced charges on the sphere due to these charges.



(i) Find the electric field $\vec{E}$ at point P (R/2, R/2, R/2) due to the induced charges. (4)

$$\boldsymbol{E} = -\left(\frac{Q}{4\pi\varepsilon_0 R^2}\right)\frac{2\sqrt{2}}{27\sqrt{3}}\left(\left(-2\sqrt{3}+\frac{9}{2}\right)\boldsymbol{i}+\left(\frac{\sqrt{3}}{2}+\frac{9}{2}\right)\boldsymbol{j}+\left(\frac{\sqrt{3}}{2}-9\right)\boldsymbol{k}\right)$$

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(ii) What is the electric potential V at point P due to the induced charges? (4)

$V = Q/(4\pi\varepsilon_0 R)(16/15 - (2/9)^{1/2} - (2/3)^{1/2})$

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Q2b) (i) Consider two point charges Q and Q1 at the points (0,0,d) and (0,0,b) with d > b. Express Q1 and b in terms of Q, d and R such that the equipotential surface with 0 potential is a sphere of radius R for d>R. (2 + 2)

Q1= -QR/d

$b = R^2/d$

(ii) Consider a point charge Q at (0,0,d) near a grounded spherical conductor of radius R < d. What is the electric field ***E*** on the charge Q due to the induced charges on the sphere? (2)

$\boldsymbol{E} = 1/4\pi\varepsilon_0 \ (RdQ/(d^2-R^2)^2)(-\mathbf{z})$

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2c)(i) Consider a point charge Q at (0,0,d) near a spherical conductor of radius $R < d$ charged to a potential V .Find the total charge q on the sphere. (1)

$q = 4\pi\varepsilon_0 VR - QR/d$

(ii)Find the force ***F*** on the charge Q. (1)

$\boldsymbol{F} = Q/4\pi\varepsilon_0((q+QR/d)/d^2 - QR/d(d-R^2/d)^2)\mathbf{z}$

(iii) If $d>>R$ find an expression for the force ***F*** on Q and also comment on its nature if $qQ > 0$ (1)

$\boldsymbol{F} = qQ/4\pi\varepsilon_0 1/d^2 \mathbf{z}$

Nature of force: repulsive

(iv) If d=R + δ with δ→0 find an expression for the force ***F*** on Q and also comment on its nature if qQ>0. (1)

$\boldsymbol{F} = Q^2/4\pi\varepsilon_0 1/4\ \delta^2\ \mathbf{(-z)}$

Nature of force: attractive

(v) Briefly explain the observation on the nature of the forces in parts (iii) and (iv) (2)

When d>>R, the electric field at d looks like the Electric field due to a charge q. Thus the force is repulsive.

When d=R+δ with δ --→ 0, the field at d due to the negative induced charge is larger than the uniform field due to the conductor being charged to potential V and hence the attraction.

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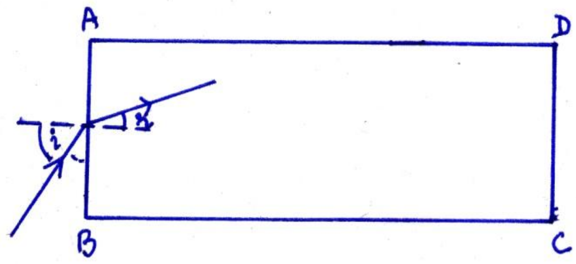
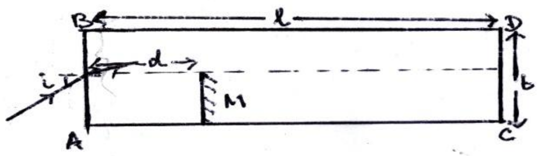
Q3 a). Light falls on the surface AB of a rectangular slab from air. Determine the smallest refractive index n that the material of the slab can have so that all incident light emerges from the opposite face CD. (2)



n $=2^{1/2}$

3 b) (i) Consider a slab of transparent material of thickness b, length l and refractive index n = $2^{1/2}$,surrounded by air. There is a mirror M of height b/2 inside the slab at a distance d from the face AB. The mirror is placed parallel to face AB, its lower end touching the face BC and the reflecting side towards AB. A narrow beam of monochromatic rays of light is incident at the center P of the face AB. The incident beam has an angle of incidence i lying between -π/2 and π/2. Ignoring diffraction effects find the values of i such that the rays emerge out of the opposite face CD. (6)



For $0 \le i \le \frac{\pi}{2}$

Sin^{-1}(n Sin(tan^{-1}((2m)b/d)) < i < Sin^{-1}(n Sin(tan^{-1}((2m+1)b/d)) for m= 0, 1,2,3 etc.

For $-\frac{\pi}{2} \le i \le 0$

Sin^{-1}(n Sin(tan^{-1}((2m-1)b/d)) < i < Sin^{-1}(n Sin(tan^{-1}((2m)b/d)) for m= 1,2,3 etc.

(ii) What happens if the angle of incidence i = 0^0 ? (2)

Practically there is no single ray. There is a bunch. Hence a part of the bunch shall be reflected and part transmitted through the opposite face.

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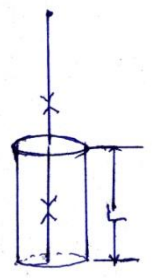
3 c) Whenever an object is heated the dimensions as well as the refractive index changes. Within a range of temperatures the changes are linear in the temperature differences. If the length and refractive index of a cylindrical object at room temperature (24^0C) is L and n respectively, then the changes ΔL *and* Δn are characterized by two properties of the body $\beta = \frac{1}{L}\frac{\Delta L}{\Delta T}$ *and* $\gamma = \frac{\Delta n}{\Delta T}$ where ΔT is the change in temperature. If β is known γ can be determined from observing the interference pattern due to reflection from the top surface and the bottom surface of a normally incident Laser beam on a cylindrical sample of length L. As the temperature changes the fringe patterns shift.



(i) Obtain a relation between γ and the fringe shift m. (3)

$\gamma = \frac{m\lambda}{2L\Delta T} - n\beta$

(ii) In a real experiment with L=1.0cm., n= 1.515 and β = 7.19 X 10^{-16} ^{0}C $^{-1}$ a Laser beam of λ = 632nm.was used. The data of the fringe shift with the temperature is given below:

m	1	2	3	4	5	6	7	8	9	10
T^0C	25.4	27.0	28.9	31.0	33.4	35.3	37.6	40.0	42.2	44.4

m	11	12	13	14	15	16	17	18	19	20
T^0C	46.6	48.9	51.6	54.0	56.2	58.6	61.2	64.0	66.4	69

Plot a graph between m and T^0C. (4)

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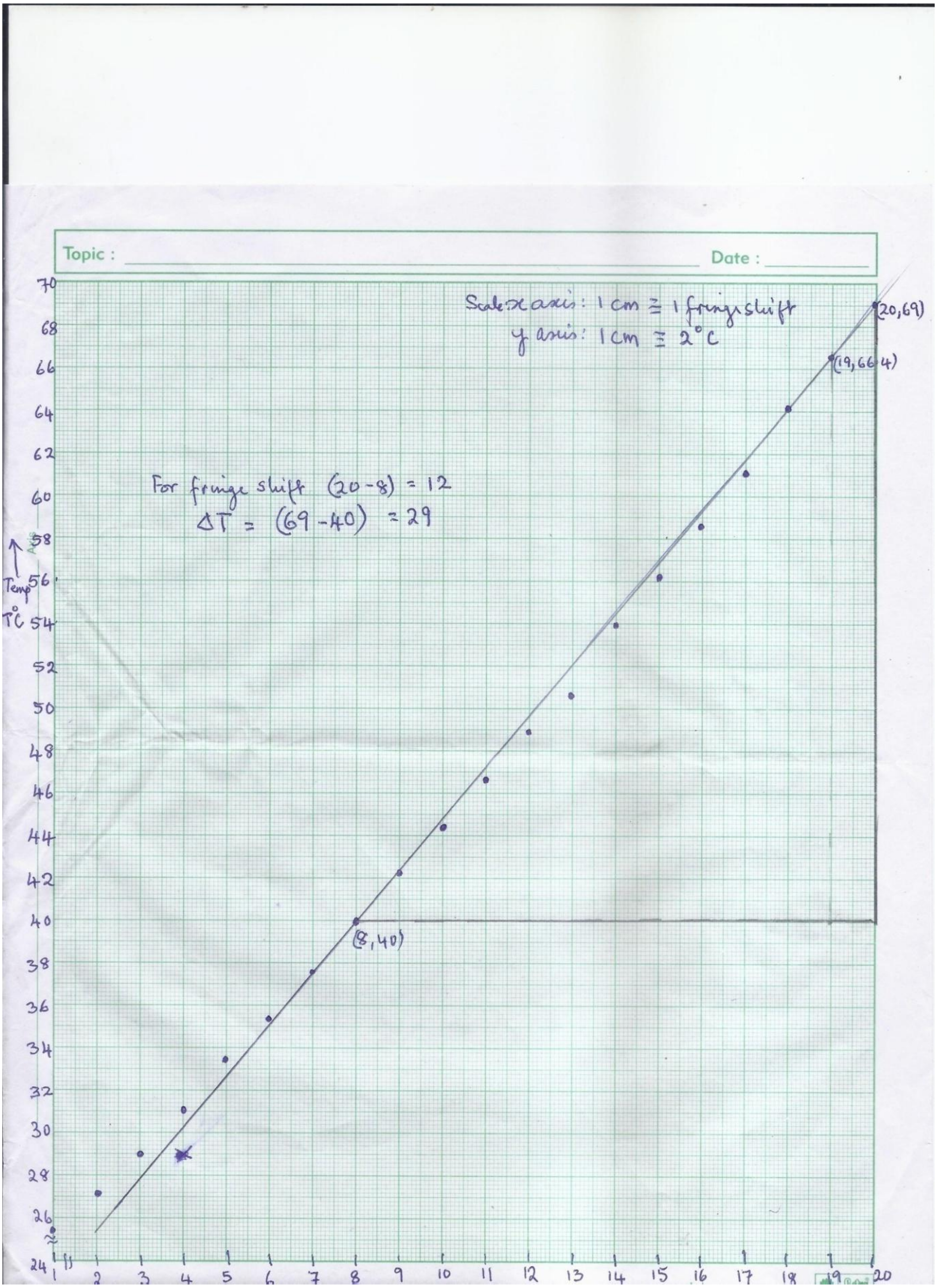

Topic :
Date :
Scale: x axis: 1 cm ≅ 1 fringe shift
y axis: 1 cm ≡ 2°C
For fringe shift (20-8) = 12
ΔT = (69-40) = 29
(20,69)
(19,66.4)
(8,40)
Temp T°C

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(iii) From the plot find γ. (3)

γ = 2.1×10^{-6} $^{0}C^{-1}$

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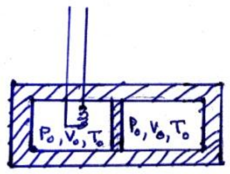
Q4) The figure shows a double chambered vessel with thermally insulated walls and partition. On each side there are n moles of an ideal monoatomic gas. Initially the pressure, volume and temperature on each side is P_0, V_0 and T_0 respectively. The heater in the first chamber supplies heat very slowly till the gas in the first chamber expands such that the pressure, volume and temperature of the gas on the left side is P_1, V_1, T_1 respectively. The pressure, volume and temperature of the gas in the right chamber is $P_2 = 27P_0/8$, V_2 and T_2 respectively



a) Complete the Table below: (5)

$P_1 = 27P_0/8$	$V_1 = 14V_0/9$	$T_1 = 21T_0/4$
$P_2 = 27P_0/8$	$V_2 = 4V_0/9$	$T_2 = 3T_0/2$

b) Find the work ΔW done on the gas in the second chamber in terms of the molar specific heat and T_0. (2)

$\Delta W = nC_vT_0/2$

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c) Find the amount of heat ΔQ that flows into the first chamber. (3)

$$\Delta Q = \frac{19}{4} n C_v T_0$$

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Q5 While performing an experiment with a liquid z (specific heat capacity 2.19kJ Kg K^{-1}), an experimenter starts with 200 ml of the liquid z in a beaker on a hot plate which is attached with scales to measure mass on it along with time. There is no lid over the beaker and the liquid is kept exposed to the surrounding. The experimenter inserts a thermometer such that it is always in contact with the liquid near the bottom of the beaker. The experimenter turns on the hot plate at t=0 min. and records the liquid temperature as well as the combined mass of the liquid, beaker and thermometer every minute. After 24 mins. he observes that there is no more liquid in the beaker. The observations are tabulated below:

Time (min)	0	1	2	3	4	5	6	7	8	9	10	11	12
Temp. (^{0}C)	24	25	28	37	50	64	77	90	102	113	124	134	143
Mass (gm)	310	310	310	310	310	310	310	310	310	310	310	307	307

Time (min)	13	14	15	16	17	18	19	20	21	22	23	24
Temp. (^{0}C)	152	158	160	160	160	159	160	160	161	160	161	-
Mass (gm)	305	302	288	264	241	214	190	165	138	110	79	69

Based on the above data

a) Plot a qualitative graph between the rate of change of temperature vs. temperature i.e. ΔT/Δt vs T. (5)

(Make an appropriate Table for the plot)

t(min)	T= (T1+T2)/2^{0}C	ΔT =T2-T1^{0}C	Δt = t2-t1(min)	ΔT/Δt ^{0}Cmin^{-1}
0				
1	24.5	1	1	1
2	26.5	3	1	3
3	32.5	9	1	9
4	43.5	13	1	13
5	57	14	1	14
6	70.5	13	1	13
7	83.5	13	1	13
8	96	12	1	12
9	107.5	11	1	11
10	118.5	11	1	11
11	138.5	10	1	10
12	147.5	9	1	9
13	155	9	1	9
14	159	6	1	6
15	160	2	1	2
16	160	0	1	0
17	160	0	1	0

18	159.5	-1	1	-1
19	159.5	1	1	1
20	160	0	1	0
21	160.5	1	1	1
22	160.5	-1	1	-1
23	160.5	1	1	1

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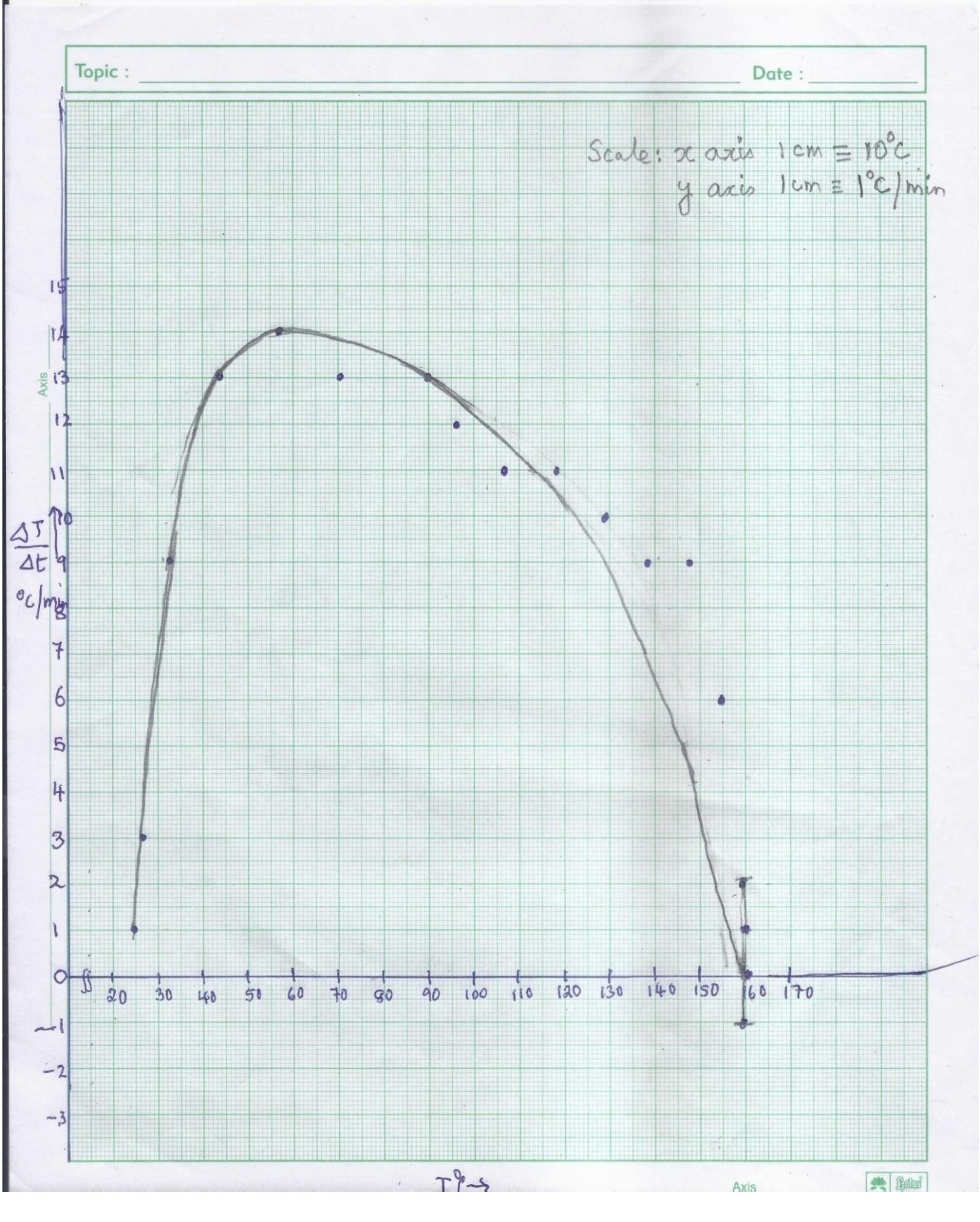

Topic :
Date :
Scale: x axis 1cm ≡ 10°C
y axis 1cm ≡ 1°C/min
ΔT/Δt °C/min
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
-1
-2
-3
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
T°→

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b)(i) From the graph identify the temperature T at which dT/dt becomes zero. (1)

T =160^0C

(ii) From(i) which intrinsic property of the liquid can be inferred ? What is the value? (2 + 1)

Intrinsic property: Boiling point

Value:160^0C

(iii) Give two possible reasons as to why the there is a 1^0 fluctuation in temperature after 15 mins.(2)

Least count of instrument is 1^0C; Statistical fluctuations such as non-uniform heating by the hot plate, convective corrections, radiation etc.

c) Plot m vs. t after t= 15 min. (4)

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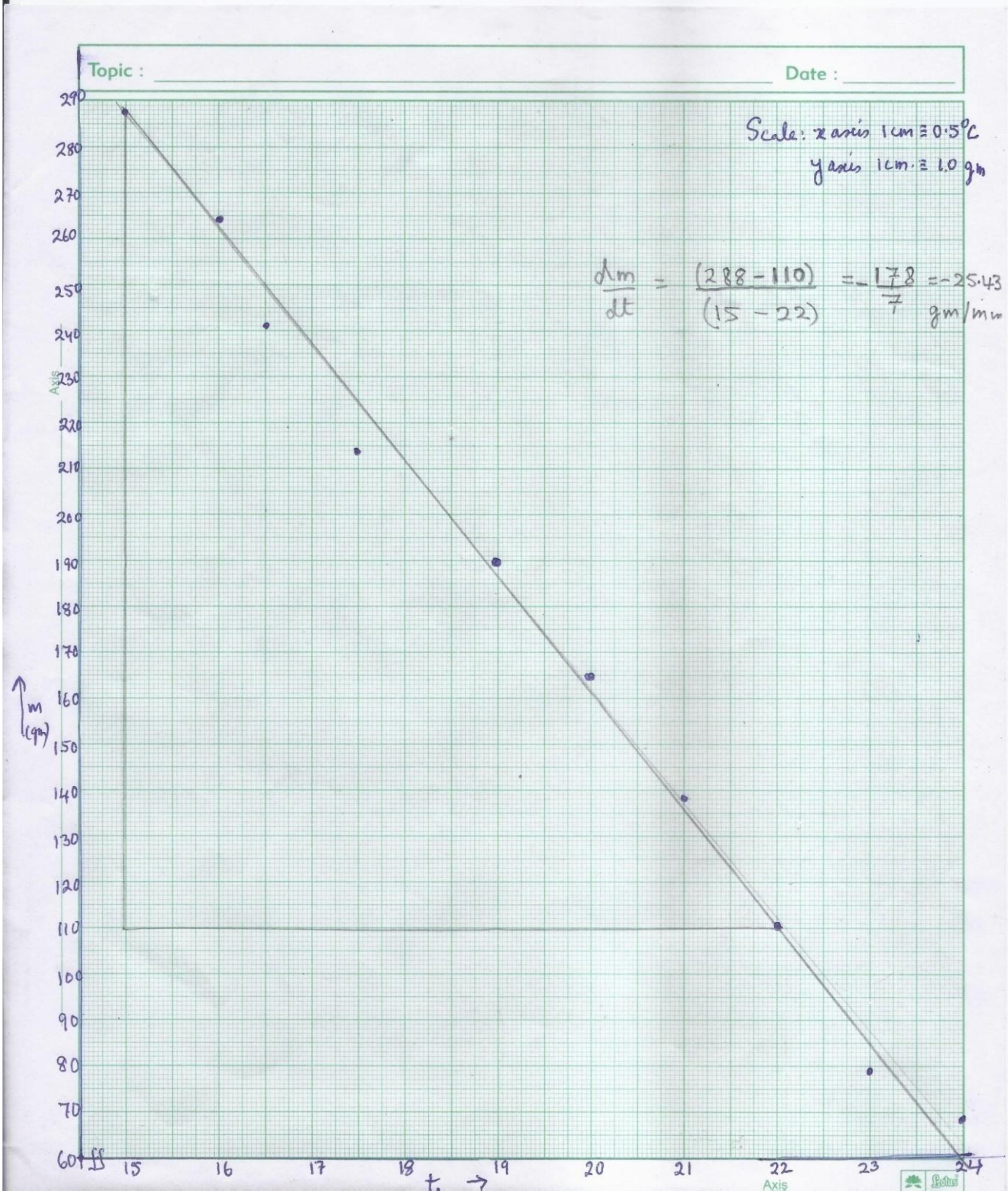

Topic :
Date :
Scale: x axis 1cm ≡ 0.5°C
y axis 1cm. ≡ 1.0 gm
dm/dt = (288 − 110)/(15 − 22) = −178/7 = −25.43 gm/min
m (gm)
t →
290
280
270
260
250
240
230
220
210
200
190
180
170
160
150
140
130
120
110
100
90
80
70
60
15
16
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d) It is established from other experiments that at t=19min, the liquid z absorbs heat from the hot plate at the rate 84.0W. Using this information what intrinsic property of the liquid, other than its boiling point, may be inferred? Find its value. (2 +3)

Intrinsic property: Latent heat Value: 198kJ Kg^{-1}

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Q6) A Schwarzschild black hole is characterized by its mass M and a mathematical spherical surface of radius $R_S = \frac{2GM}{c^2}$ called the event horizon. If the radial distance of an object r from the black hole is such that $r < R_s$, then the object is "swallowed" by the black hole and r rapidly decreases to the singular point $r = 0$.

a) Suppose a black hole of mass M "captures" a proton to form a "black hole proton atom (BHP)" in circular orbit. Find the smallest radius r_B of this atom. (3)

$$r_B = \frac{\hbar^2}{GM\, m^2}$$

b) Obtain a numerical upper bound on M such that a stable BHP may exist. (2)

$M < 2.X\ 10^{11}$Kg.

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c) Find the minimum energy E_{min} , in Mev ,required to dissociate this BHP atom from the ground state. (2)

$E_{min} = 55$ MeV

d) In 1974, Stephen Hawking showed that quantum effects cause black holes to radiate like a black body with temperature $T_{BH} = \frac{10^{23}K}{M}$. Discuss then the possibility of the existence of a stable BHP atom. (3)

For $M = 10^{11}$ Kg. $T_{BH} = \frac{10^{23}K}{10^{11}} = 10^{12}K$ At this temperature thermal energies $\simeq kT_{BH} = 82\ MeV$ The dissociation energy required is 55 MeV. Thus the BHP is thermally unstable.