UNIT 2 SEQUENCES AND SERIES

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2.1 INTRODUCTION

In this unit, we shall be introducing the notion of sequences and series of various types. In particular we shall study arithmetic sequences (series) and geometric sequences (series).

Objectives

After studying this unit, you should be able to

- introduce the concept of sequences, and series,
- define Arithmetic Progression and Geometric Progression,
- compute the sum of a finite (or infinite) AP or GP,
- define Arithmetic Mean (or Means) and Geometric Mean (or Means) between two numbers, and
- insert and compute the AMs and GMs between two numbers.

2.2 SEQUENCES

Definition 1

Let N be the set of natural numbers and N_n be the set of first numbers, i.e. $N_n = \{1, 2, 3, \ldots, n\}$ and X be a non-empty set.

Then a map $f : N \to X$ is called a sequence. If $X = R$ or a subset of R, R the set of real numbers then f is called a **real sequence** and if $X = C$ or a subset of C, C the set of complex numbers then f is called a **complex sequence**.

If domain $f = N$, we call it an **infinite sequence**. And if domain $f = N_n$ a subset of N, then it is called a **finite sequence**.

Thus, if f is a sequence, then for any $k \in N_n$ or N, we have $f(k) = a_k \in X$.

Hence $a_1, a_2, a_3, \ldots, a_n$ or $a_1, a_2, a_3, \ldots, a_n, \ldots$ determine the sequence f.

These sequences are usually denoted by $\{a_k\}_{k=1}^{n}$ and $\{a_k\}_{k=1}^{8}$.

The different elements in a sequence are called its terms which are usually denoted by $T_1, T_2, T_3, \ldots, T_n$ or $T_1, T_2, \ldots, T_n, \ldots$. accordingly as the sequence is finite or infinite. T_n, the n^{th} term is called the **general term**.

Definition 2

A sequence following some definite rule (or rules) is called a **progression**.

For example, consider the following sequences,

(i) 3, 5, 7, 9, . . . , 21

(ii) 8, 5, 2, – 1, – 4, ...

(iii) 2, 6, 18, 54, ...

(iv) 1, 4, 9, 16, ...

We observe that each term (except first) in (i) is formed by adding 2 to the preceding term; each term in (ii) is formed by subtracting 3 from the preceding term; each term in (iii) is formed by multiplying the preceding term by 3, each term in (iv) is formed by squaring the next natural number. Thus all are sequences, (i) is a finite sequence whereas others are infinite sequences.

If the terms of a sequence are connected by plus (or minus) signs, a series is formed for example

$$3 + 5 + 7 + . . . + 21$$

$$8 + 5 + 2 + (-1) + . . .$$

2.3 ARITHMETIC PROGRESSION

A sequence is called an **Arithmetic Progression** (abbreviated AP) if any only if the difference of any term from its preceding term is constant.

This constant is denoted by d and is called the **common difference** (CD).

Thus $a_1, a_2, \ldots, a_n$ or $a_1, a_2, \ldots, a_n, \ldots$ is an AP iff $a_{k+1} - a_k = d$, a constant independent of k, $k = 1, 2, . . . , n$ or $k = 1, 2, . . , n, . . .$ as the case may be.

Theorem 1

Find the general term of an A. P.

Proof

Let a be the first term and d be the CD of an AP. Let $T_1, T_2, \ldots, T_n$ denote $1^{st}, 2^{nd}, \ldots n^{th}$ terms respectively, then we have

$$T_2 - T_1 = d$$
$$T_3 - T_2 = d$$
$$T_4 - T_3 = d$$
$$\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots$$
$$T_n - T_{n-1} = d$$

By adding we have $\quad T_n - T_1 = (n-1)\, d$

i.e. $\quad T_n = T_1 + (n-1)\, d$

$$= a + (n-1)\, d$$

If the last terms of an AP consisting of n terms is l, then $l = a + (n-1)\, d$

i.e. $$d = \frac{l - a}{n - 1}$$

Example 2.1

A sequence $\{a_n\}$ is given by the formula $a_n = 10 - 3n$. Prove that it is an AP.

Solution

$$a_n = 10 - 3n$$

$$a_{n+1} = 10 - 3\,(n+1) = 10 - 3n - 3 = 7 - 3n$$

$$\therefore\ a_{n+1} - a_n = (7 - 3n) - (10 - 3n)$$

$$= 7 - 3n - 10 + 3n = -3$$

which is independent of *n*. Hence the sequence is an A. P.

Theorem 2

If the terms of an arithmetic sequence be increased, decreased, multiplied or divided by the same non-zero constant, they remain in arithmetic sequence.

Proof

Consider an AP

$$a,\ a + d,\ a + 2d,\ a + 3d, \ldots \qquad \ldots (2.1)$$

(i) When each term of (2.1) is increased by a non-zero constant k, we obtain the sequence $a + k$, $a + d + k$, $a + 2d + k$, $a + 3d + k$, ...

i.e. $(a + k)$, $(a + k) + d$, $(a + k) + 2d$, $(a + k) + 3d$, . . . which is clearly an AP whose first term is $a + k$, and CD is d.

(ii) Can be similarly proved.

(iii) When each term is multiplied by k, we obtain the sequence ak, $ak + kd$, $ak + 2kd$, $ak + 3kd$, . . . which is an AP.

(iv) Can be similarly proved.

Theorem 3

Find the sum of n terms of an A. P.

Proof

Let a be the first term, d the common difference and l the last term i.e. the n^{th} term. If S_n denotes the sum of n terms then

$$S_n = a + (a + d) + (a + 2d) + \ldots + (l - 2d) + (l - d) + l \qquad \ldots (2.2)$$

Rewriting S_n in the reverse order, we have

$$S_n = l + (l - d) + (l - 2d) + \ldots + (a + 2d) + (a + d) + a \qquad \ldots (2.3)$$

Adding Eqs. (2.2) and (2.3), we have

$$2S_n = (a + l) + (a + l) + \ldots + (a + l) \qquad \{n \text{ times}\}$$

$$= n\,(a + l)$$

$$\therefore\ S_n = \frac{n(a + l)}{2} = \frac{n}{2}\,[a + a + (n - 1)\,d]\,;\ \{\because\ l = a + (n - 1)\,d\}$$

$$= \frac{n}{2}\,[2a + (n-1)\,d]$$

2.3.1 Arithmetic Mean

When three numbers are in AP, the middle one is said to be the **Arithmetic Mean** (written as AM) between the other two.

Similarly if $a, A_1, A_2, \ldots, A_n, b$ are in AP then $A_1, A_2, \ldots, A_n$ are called the **n-Arithmetic Means** between a and b.

Theorem 4

Find an Arithmetic Mean between two numbers a and b.

Let A be the AM between a and b.

Then a, A, b are in AP

$$\Rightarrow \quad A - a = b - A \quad \Rightarrow \quad 2\,A = a + b \quad \Rightarrow \quad A = \frac{a+b}{2}$$

Thus the AM between two numbers is half their sum.

Theorem 5

Find n arithmetic means between two numbers.

Let $A_1, A_2, \ldots, A_n$ be the n-arithmetic means between a and b.

Then $a, A_1, A_2, \ldots, A_n, b$ are in AP.

Let d be the CD. Then b is the $(n + 2)^{\text{th}}$ term.

$$\therefore \quad b = a + (n + 2 - 1)\,d = a + (n+1)\,d$$

i.e. $\quad b - a = (n+1)\,d$

i.e. $\quad d = \dfrac{b-a}{n+1}$

$$\therefore \quad A_1 = a + d = a + \frac{b-a}{n+1} = \frac{na+b}{n+1}$$

$$A_2 = a + 2d = a + 2\left(\frac{b-a}{n+1}\right)$$

$$A_n = a + nd = a + n\left(\frac{b-a}{n+1}\right) = \frac{a+nb}{n+1}$$

The sum of these n-arithmetic means is $A_1 + A_2 + \ldots + A_n$

$$= a + A_1 + A_2 + \ldots + A_n + b - (a+b)$$

$$= \text{Sum of } (n+2) \text{ term of an AP} - (a+b)$$

$$= \frac{(n+2)}{2}(a+b) - (a+b)$$

$$= (a+b)\left[\frac{n+2}{2} - 1\right] = n\left(\frac{a+b}{2}\right)$$

Thus we find that the sum of n AM′s between two numbers is n times the single AM between the two numbers.

Example 2.2

If the third term of an AP is 18 and the seventh term is 30, find the series.

Solution

Let a be the first term and d be the common difference of the given AP.

$$\left.\begin{aligned} T_3 = 18 &\Rightarrow a + 2d = 18 \\ T_7 = 30 &\Rightarrow a + 6d = 30 \end{aligned}\right\} \Rightarrow 4d = 12 \Rightarrow d = 3$$

$$a + 2d = 18,\ d = 3 \Rightarrow a = 12$$

$$\therefore\ T_n = a + (n-1)\,d = 12 + (n-1)\,3 = 12 + 3n - 3 = 9 + 3n$$

$\therefore$ The series is

$$12 + 15 + 18 + \ldots + (3n + 9) + \ldots$$

Example 2.3

If a^2, b^2, c^2 are in AP, prove that $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in AP.

Solution

$$\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b} \text{ are in AP.}$$

iff $\frac{a}{b+c} + 1, \frac{b}{c+a} + 1, \frac{c}{a+b} + 1$ are in AP (adding 1 to each term)

iff $\frac{a+b+c}{b+c}, \frac{a+b+c}{c+a}, \frac{a+b+c}{a+b}$ are in AP.

iff $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in AP (dividing each term by $(a+b+c)$)

iff $(c+a)(a+b), (b+c)(a+b), (b+c)(c+a)$ are in AP (multiplying each term by $(c+a)(a+b)(b+c)$)

iff $ac + a^2 + bc + ab, ab + ac + b^2 + bc, bc + c^2 + ab + ac$ are in AP

iff a^2, b^2, c^2 are in AP (subtracting $ab + bc + ca$ from each term)

$\therefore$ if a^2, b^2, c^2 are in AP

then $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in AP.

Example 2.4

Find the sum of 19 terms of an AP whose n^{th} term is $2n + 1$.

Solution

Let a be the first term and d be the common difference.

Then $T_1 = a = 2.1 + 1 = 3$ (as $n = 1$)

$T_2 = 2.2 + 1 = 5$ (as $n = 2$)

$\therefore\ d = T_2 - T_1 = 5 - 3 = 2$

$$S_n = \frac{n}{2}[2a + (n-1)\,d]$$

$$= \frac{n}{2}[6 + 2\,(n-1)] = \frac{n}{2}[6 + 2n - 2]$$

$$= \frac{n}{2}(4 + 2n) = n\,(2 + n)$$

$$\therefore \quad S_{19} = 19\,(2 + 19) = 19 \times 21 = 399$$

Example 2.5

The sum of three consecutive numbers in an AP is 18 and their product is 192. Find the numbers.

Solution

Let the three numbers be $a - d,\, a,\, a + d$

Then $\quad a - d + a + a + d = 18 \;\Rightarrow\; 3\,a = 18 \;\Rightarrow\; a = 6$

and $\quad (a - d)\,a\,(a + d) = 192$

$$a\,(a^2 - d^2) = 192$$

$$6\,(36 - d^2) = 192 \;\Rightarrow\; 36 - d^2 = 32$$

$$\Rightarrow\; d^2 = 36 - 32 = 4$$

$$\Rightarrow\; d = \pm\,2$$

Taking $d = 2$, we have the numbers 4, 6, 8.

Taking $d = -\,2$, we have the numbers 8, 6, 4.

Hence the required numbers are 4, 6, 8 or 8, 6, 4.

Example 2.6

Insert six arithmetic means between 2 and 16. Also prove that their sum is 6 times the arithmetic mean between 2 and 16.

Solution

Let A be the arithmetic mean between 2 and 16 then

$$A = \frac{2 + 16}{2} = 9 \qquad \text{. . . (2.4)}$$

Let $A_1, A_2, \ldots, A_6$ be the six arithmetic means between 2 and 16, then $2, A_1, A_2, \ldots, A_6, 16$ are in AP. Let d be the common difference. Then 16 is the 8th term

$$\therefore\; 16 = a + 7d \;\Rightarrow\; 16 = 2 + 7d \;\Rightarrow\; d = 2$$

$$\therefore\; A_1 = 2 + 2 = 4,\ (A_1 = a + d)\ ,\ A_6 = a + 6d = 2 + 12 = 14$$

Now,

$$A_1 + A_2 + \ldots + A_6 = \frac{6}{2}(4 + 14) \qquad \left\{\because\; S_n = \frac{n}{2}(a + b)\right\}$$

$$= 3 \times 18 = 54 \qquad \text{. . . (2.5)}$$

From Eq. (2.4)

$$= 6A$$

$$= 6 \times 9$$

$$= 54 \qquad \text{(Proved)}$$

SAQ 1

(a) Which term of the sequence – 3, – 7, – 11, – 15, . . . is 403? Also find which term if any of the given sequence is – 500?

(b) The fourth term of an AP is equal to 3 times the first term and the seventh term exceeds twice the third term by 1. Find the first term and the common difference.

(c) Prove that a, b, c are in AP iff $\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are in AP.

(d) If p times the p^{th} term of an AP is equal to q times the q^{th} term, show that the $(p + q)^{th}$ term is zero.

SAQ 2

(a) The sum of n terms of two arithmetic series are in the ratio of $(7n + 1) : (4n + 27)$. Find the ratio of their 11^{th} terms.

(b) If a, b, c, x are real numbers and $(x - a + b)^2 + (x - b + c)^2 = 0$, prove that a, b, c are in AP.

(c) The ratio of the 2^{nd} to 7^{th} of n AM between – 7 and 65 is 1 : 7. Find n.

(d) The sum of two numbers is $\frac{13}{6}$. An even number of arithmetic means are inserted between them and their sum exceeds their number by 1. Find the number of means inserted.

[**Hint :** Take $2n$ to be the number of means inserted.]

2.4 GEOMETRIC PROGRESSION

A sequence (finite or infinite) of non-zero terms is called a geometric progression (GP) iff the ratio of any term to its preceding term is constant.

We assume that none of the terms of the sequence is zero. This constant ratio (non-zero) is denoted by r and is called common ratio (CR).

Thus $a_1, a_2, \ldots, a_n$ or $a_1, a_2, a_3, \ldots, a_n, \ldots$ is a GP iff $\frac{a_{k+1}}{a_k} = r$ (constant independent of k) for $k = 1, 2, 3, \ldots, n-1$ or $k = 1, 2, 3, \ldots$

$\therefore$ In a GP $T_n = T_{n-1} \cdot r$

For example, the sequence $\frac{1}{2}, 1, 2, 4, 8, \ldots, 512$ is a finite GP with $r = 2$ and the sequence $1, -\frac{1}{3}, \frac{1}{9}, -\frac{1}{27}, \ldots$ is an infinite GP with $r = -\frac{1}{3}$.

Theorem 6

General term of a GP

Let a be the first term and $r \neq 0$ be the CR.

$$T_2 = T_1 \cdot r$$
$$T_3 = T_2 \cdot r$$
$$T_4 = T_3 \cdot r$$
$$\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots$$
$$T_n = T_{n-1} \cdot r$$

Multiplying all these we have

$$T_2\ T_3\ T_4 \ldots T_n = T_1\ T_2\ T_3 \ldots T_{n-1} \cdot r^{n-1}$$

i.e. $$T_n = T_1\ r^{n-1} = ar^{n-1}$$

If last term of a GP with n terms is denoted by l, then $l = ar^{n-1}$.

Theorem 7

Sum of *n* Terms of a GP

Let a be the first term and r be the common ratio of a GP and S_n the sum of n terms.

Then $$S_n = a + ar + ar^2 + \ldots + ar^{n-1} \qquad \ldots (2.6)$$

$$rS_n = ar + ar^2 + \ldots + ar^{n-1} + ar^n \qquad \ldots (2.7)$$

Subtracting Eq. (2.7) from Eq. (2.6), we have

$$(r-1)\ S_n = ar^n - a = a\ (r^n - 1)$$

i.e. $$S_n = a\left(\frac{r^n - 1}{r - 1}\right) \text{ or } a\left(\frac{1 - r^n}{1 - r}\right)$$

This formula fails when $r - 1 = 0$, i.e. $r = 1$. When $r = 1$, then

$$S_n = a + a + a + \ldots n \text{ times} = na$$

Sum of an Infinite GP

Consider a GP $a, ar, ar^2, \ldots$ with $|r| < 1$. We know that

$$S_n = a\left(\frac{1 - r^n}{1 - r}\right) = \frac{a}{1 - r} - \frac{ar^n}{1 - r}$$

Since $|r| < 1 \Rightarrow r^n$ goes on decreasing as n goes on increasing. Ultimately when $n \to \infty, r^n \to 0$.

$$\therefore \qquad S_n \to \frac{a}{1-r}$$

In other words if S denotes the sum of infinite terms then $S = \dfrac{a}{1-r}$.

2.4.1 Geometric Mean

When three numbers are in GP, the middle one is said to be the Geometric Mean (GM) between the other two.

Similarly if $a, a_1, a_2, \ldots, a_n, b$ are in GP then $a_1, a_2, \ldots, a_n$ are called the n geometric means between a and b.

Theorem 8

To find the GM between two members *a* and *b*

Let G be the GM. Then a, G, b are in GP.

i.e. $\dfrac{G}{a} = \dfrac{b}{G}$, i.e. $G^2 = ab$, i.e. $G = \sqrt{ab}$

(Conventionally G is taken to be the +ve square root of ab).

To find *n* GMs between the numbers *a* and *b*

Let $a_1, a_2, \ldots, a_n$ be the GMs between a and b.

Then $a, a_1, a_2, \ldots, a_n, b$ are in GP.

Let r be the common ratio.

Then $$b = ar^{n+2-1} = ar^{n+1} \Rightarrow r^{n+1} = \frac{b}{a} \Rightarrow r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

$$\therefore \; G_1 = ar = a\left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

$$G_2 = ar^2 = a\left(\frac{b}{a}\right)^{\frac{2}{n+1}}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$G_n = ar^n = a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$$

Cor.

$$a_1 \times a_2 \times \ldots \times a_n = ar \,.\, ar^2 \,.\, ar^3 \ldots ar^n$$

$$= a^n \; r^{1+2+3+\ldots+n}$$

$$= a^n \; r^{\frac{n(n+1)}{2}} \quad (\because 1 + 2 + 3 + \ldots + n \text{ is AP of } n \text{ terms})$$

$$= a^n \left[\left(\frac{b}{a} \right)^{\frac{1}{n+1}} \right]^{\frac{n(n+1)}{2}}$$

$$= a^n \left(\frac{b}{a} \right)^{\frac{n}{2}} = a^{\frac{n}{2}}\ b^{\frac{n}{2}} = (\sqrt{ab})^n$$

= n^{th} power of the single GM between *a* and *b*.

Example 2.7

Find the n[th] term and the 12[th] term of the sequence 2, – 6, 18, – 54, ...

Solution

The given sequence is a G. P with $a = 2$ and $r = -3$.

$$\therefore\ \ T_n = a\ r^{n-1} = 2\ (-3)^{n-1} = (-1)^{n-1}\ .\ 2\ .\ 3^{n-1}$$

$$T_{12} = (-1)^{11}\ .\ 2\ .\ 3^{11} = -\ 2\ .\ 3^{11}$$

Example 2.8

Find the sum of the series $\frac{2}{9} + \frac{1}{3} + \frac{1}{2} + \ldots + \frac{81}{32}$.

Solution

The given series is a GP where *a*, the first term is $\frac{2}{9}$ and *r* the common ratio is $\frac{3}{2}$.

Last terms is $\frac{81}{32} = T_n$ say.

Then $\frac{81}{32} = ar^{n-1} = \frac{2}{9}\left(\frac{3}{2}\right)^{n-1}$

i.e. $\left(\frac{3}{2}\right)^{n-1} = \frac{81}{32} \times \frac{9}{2} = \left(\frac{3}{2}\right)^6 \quad \therefore\ \ n - 1 = 6 \Rightarrow n = 7$

Hence the sum of the 7 term of the series

$$= \frac{\frac{2}{9}\left[\left(\frac{3}{2}\right)^7 - 1\right]}{\frac{3}{2} - 1} = \frac{2}{9} \times \frac{2}{1}\left(\frac{2187}{128} - 1\right)$$

$$= \frac{2059}{288}$$

Example 2.9

Find the sum of 50 terms of the sequence 7, 7.7, 7.77, 7.777, . . .

It is not a GP but we can relate it to a GP. Let S_{50} denote the sum of 50 term of this sequence.

$$S_{50} = 7 + 7.7 + 7.77 + 7.777 + \ldots \text{ to 50 terms}$$

$$= \frac{7}{9}\,(9 + 9.9 + 9.99 + 9.999 + \ldots) \text{ to 50 terms}$$

$$= \frac{7}{9}\,[(10-1) + (10-.1) + (10-.01) + \ldots] \text{ to 50 terms}$$

$$= \frac{7}{9}\,(10 + 10 + \ldots \text{ to 50 terms}) - \underbrace{\frac{7}{9}\,(1 + .1 + .01 + \ldots \text{ to 50 terms})}_{\text{This is a GP}}$$

$$= \frac{7}{9}\,(10 \times 50) - \frac{7}{9}\,1.\left[\frac{1-(.1)^{50}}{1-.1}\right]$$

$$= \frac{7}{9} \times 500 - \frac{7}{9}\;\frac{1-\dfrac{1}{10^{50}}}{\dfrac{9}{10}} = \frac{7}{9} \times 500 - \frac{7}{9} \times \frac{10}{9}\left(1 - \frac{1}{10^{50}}\right)$$

$$= \frac{7}{81}\left[4500 - 10 + \frac{10}{10^{50}}\right]$$

$$= \frac{7}{81}\left[4490 + \frac{1}{10^{49}}\right]$$

Example 2.10

If the first term of a GP. exceeds the second term by 2 and the sum of infinite terms is 50, find the GP.

Solution

Let a be the first term and r with $|\, r \,| < 1$ be the common ratio.

Then $\quad T_1 = T_2 + 2 \;\Rightarrow\; a = a\,r + 2 \;\Rightarrow\; a = \dfrac{2}{1-r} \quad \ldots (2.8)$

Also $\quad S_\infty = 50 = \dfrac{a}{1-r} \quad \ldots (2.9)$

From Eqs. (2.8) and (2.9), we have

$$50\,(1-r) = a = \frac{2}{1-r}$$

$$\Rightarrow\; (1-r)^2 = \frac{2}{50} = \frac{1}{25} \Rightarrow 1 - r = \pm\frac{1}{5} \Rightarrow r = 1 - \frac{1}{5} = \frac{4}{5} \text{ or } \frac{6}{5}$$

$$r \neq \frac{6}{5} \text{ as } |\, r \,| < 1;\; r = \frac{4}{5}$$

$$\therefore\; a = \frac{2}{1-\dfrac{4}{5}} = \frac{2}{\dfrac{1}{5}} = 10$$

The GP is $10, 8, \dfrac{32}{5}, \dfrac{128}{25}, \ldots$

Example 2.11

Insert 4 GMs between 3 and 96. Show that their product is the 4^{th} power of the GM between them.

Solution

Let a be the single GM and a_1, a_2, a_3, a_4 be the four geometric means inserted between the two numbers. Let r be the common ratio, then 96 is the 6th term

$$\therefore \ 96 = ar^5 \ \Rightarrow \ r^5 = \frac{96}{a} = \frac{96}{3} = 32 \ \Rightarrow \ r = 2$$

$$\therefore \ G_1 = ar = 6$$

$$G_2 = ar^2 = 12 \qquad (G = \sqrt{3 \times 96} = \sqrt{288} = 12\sqrt{2})$$

$$G_3 = ar^3 = 24$$

$$G_4 = ar^4 = 48$$

$$G_1 \ G_2 \ G_3 \ G_4 = 6 \times 12 \times 24 \times 48 = 12^4 \times 2^2 = (12\sqrt{2})^4 = G^4$$

SAQ 3

(a) The second term of a GP is $\frac{25}{4}$ and the 8th term is $\frac{16}{625}$, find the GP.

(b) If the pth, qth and rth terms of a GP are a, b, c respectively, prove that $a^{q-r} \,.\, b^{r-p} \,.\, c^{p-q} = 1$.

(c) If a, b, c are in GP, show that $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are also in GP.

(d) Determine the number of terms of a GP $\{a_n\}$ if $a_1 = 3,\ a_n = 96$ and $S_n = 189$.

SAQ 4

(a) Sum to n terms the series 5 + 55 + 555 +. . .

(b) Sum the series $(x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + \ldots$ upto n terms.

SAQ 5

(a) The sum of four numbers in GP is 60 and the AM of the first and the last is 18. Find the numbers.

(b) Find the sum of an infinite sequence $7, -1, \frac{1}{7}, -\frac{1}{49}, \ldots$

(c) Show that AM between two distinct numbers is always greater than their GM.

(d) If p, q, r are in AP, a is GM between p and q and b is GM between q and r', prove that a^2, q^2, b^2 are in AP.

2.5 SUMMARY

AP and GP

If a is the first term and d the common difference of an AP then

(i) n^{th} term of an AP is $T_n = a + (n-1)d$,

(ii) $S_n =$ Sum of the n terms $= \frac{n}{2}[2a + (n-1)d]$

(iii) AMs between a and b is $\frac{a+b}{2}$.

(iv) $A_1, A_2, \ldots, A_n$ are n AMs iff $a, A_1, \ldots, A_n, b$ are in AP.

(v) If 1st term is a and r is the common ratio of a GP, then n^{th} term $T_n = ar^{n-1}$

Sum of first n terms $S_n = \frac{a(1-r^n)}{1-r}$ or $\frac{a(r^n-1)}{(r-1)}, r \neq 1$

Sum to infinity of the G. P. $= \frac{a}{1-r}$

(vi) G is the GM between a and b then $G = \sqrt{a\,b}$.

2.6 ANSWERS TO SAQs

SAQ 1

(a) – 403 is the 101st term, and there is no term which is – 500.

(b) 1st term = 3 and common difference = 2.

SAQ 2

(a) Required ratio is $\frac{4}{3}$

(c) $n = 11$

(d) 12

SAQ 3

(a) $\frac{125}{8}, \frac{25}{4}, \frac{5}{2}, 1, \ldots$

(d) Number of terms = 6

SAQ 4

(a) $\frac{5}{81}(10^{n+1} - 9n - 10)$

(b) $\frac{1}{x-y}\left\{\frac{x^2(1-x^n)}{1-x} - \frac{y^2(1-y^n)}{1-y}\right\}$

SAQ 5

(b) $\frac{49}{8}$