UNIT 8 VECTOR ALGEBRA

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8.1 INTRODUCTION

Vectors are used extensively in almost all branches of physics, mathematics and engineering. The usefulness of vectors in engineering mathematics results from the fact that many physical quantities for example velocity of a body, forces acting on a body may be represented by vectors.

In several respects, the rules of vector calculations are as simple as rules governing the systems of real numbers. However vector analysis is a shorthand which simplifies many calculations considerably.

We have listed important results of vector algebra. We have begun this unit by giving basic definitions and then the operations on vectors such as addition, subtraction, multiplication of a vector by scalar, etc. have been taken up. We have also introduced the representation of the vectors in component forms which has been used to discuss vector products.

Objectives

After studying this unit, you should be able to

- distinguish between scalars and vectors,
- define a null vector, a unit vector, negative of a vector and equality of vectors,
- identify coinitial vectors, like and unlike vectors, free vectors,
- add and subtract vectors,
- multiply a vector by a scalar, and
- compute scalar and vector product of two vectors.

8.2 BASIC CONCEPTS

We know that many a times a single number is not sufficient to characterize a physical quantity. For example, a moving object may have a known speed but we cannot describe its motion completely without knowing the direction in which it is moving. So we need two things (i) magnitude, and (ii) direction to represent certain physical quantities such as velocity, force, acceleration etc. In this unit, we shall introduce vectors as quantities, which have both magnitude and direction.

Definition 1

A physical quantity is called a scalar if it can be completely specified by a single number (with a suitable choice of units of measure). For example mass, time, work, volume, energy etc. are scalar quantities.

Definition 2

Quantities which are specified by a magnitude and a direction are called vector quantities. For example velocity, force, acceleration, momentum etc. are vector quantities.

We know that a directed line segment is a line segment with an arrow-head showing direction (Figure 8.1).

Figure 8.1 : Directed Line Segment *AB*

A directed line segment is characterized by

(i) **Length**

Length of directed line segment *AB* is the length of line segment *AB*.

(ii) **Support**

The support of a directed line segment *AB* is the line *P* of infinite length of which *AB* is a portion. It is also called *line of action* of that directed line segment.

(iii) **Sense**

The sense of a directed line segment *AB* is from its *tail* or *the initial point A* to its *head* or *the terminal point B*.

Thus we may define a vector as follows.

A directed line segment is called a vector. Its length is called the length or magnitude of the vector and its direction is called the direction of the vector.

In Figure 8.1, the direction of directed line *AB* is from *A* to *B*. *The two end points of a directed line segments are not interchangeable* and directed line segments *AB* and *BA* are different as the segments *AB* and *BA* have equal length, same support but opposite sense.

The vector which is represented by the directed line segment *AB* is denoted by ***AB***. We also denote a vector as ***a***. If *A* is the initial point and *B* is the terminal point of a vector $\boldsymbol{a}$, then we write $\boldsymbol{a} = \boldsymbol{AB}$ and the length of the vector $\boldsymbol{a}$ is written as $|\, \boldsymbol{a} \,|$.

Some class of vectors.

(i) **Zero (null) Vector**

A null vector is a vector whose magnitude is zero. A null vector has no specific direction; its direction is arbitrary and can be chosen at will. It is usually written as $\boldsymbol{O}$ or $\boldsymbol{AA}$ or $\boldsymbol{BB}$.

(ii) **Proper Vector**

A vector whose magnitude is not zero is called a proper vector.

This means that $\boldsymbol{a}$ is a proper vector iff $|\, \boldsymbol{a} \,| \neq 0$.

(iii) **Unit Vector**

A vector of magnitude unity is called a unit vector. Generally a unit vector is denoted by a single letter with a cap ($\wedge$) over. Thus a unit vector in the direction of $\boldsymbol{a}$ is written as $\hat{\boldsymbol{a}}$. Again $\boldsymbol{a} = |\boldsymbol{a}|\, \hat{\boldsymbol{a}}$ or $\hat{\boldsymbol{a}} = \dfrac{\boldsymbol{a}}{|\boldsymbol{a}|}$.

(iv) **Equal Vectors**

Two vectors $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$ are said to be equal written as $\boldsymbol{a}_1 = \boldsymbol{a}_2$ iff they can have equal lengths, same or parallel supports and the same sense.

(v) **Collinear Vectors**

Vectors with the same support or parallel supports are called collinear vectors. The vectors which are not collinear are called non-collinear vectors.

(vi) **Like and Unlike Vectors**

Two collinear vectors which have the same sense are called like vectors and two collinear vectors which are in the opposite sense are called unlike vectors.

(a) Like Vectors **(b) Unlike Vectors**

Figure 8.2

Note that two vectors cannot be equal if they have

(i) different magnitudes, or

(ii) inclined supports, or

(iii) different senses.

Example 8.1

ABCDEF is a regular hexagon. If $\boldsymbol{P} = \boldsymbol{AB}$, $\boldsymbol{Q} = \boldsymbol{BC}$, and $\boldsymbol{R} = \boldsymbol{CD}$, name the vectors represented by $\boldsymbol{AF}$, $\boldsymbol{ED}$ and $\boldsymbol{FE}$.

Solution

Since ABCDEF is a regular hexagon,

$\therefore \quad AB = BC = CD = AF = ED = FE.$

Figure 8.3

Also $AB \parallel ED$, $BC \parallel FE$ and $CD \parallel AF$.

Further sense of AB is the same as that of ED, sense of BC is the same as that of FE and the sense of CD is the same as that of AF.

$\therefore \quad \boldsymbol{ED} = \boldsymbol{P}$, $\boldsymbol{FE} = \boldsymbol{Q}$, and $\boldsymbol{AF} = \boldsymbol{R}$.

SAQ 1

Tick (✓) the correct answer.

(i) Which of the following is a scalar quantity

(a) Displacement

(b) Kinetic energy

(c) Velocity

(d) Momentum

(ii) The unit vector along $\hat{\boldsymbol{i}} + \hat{\boldsymbol{j}}$ is

(a) $\hat{\boldsymbol{k}}$ (b) $\hat{i} + \hat{j}$

(c) $\dfrac{\hat{i} + \hat{j}}{\sqrt{2}}$ (d) $\dfrac{\hat{i} + \hat{j}}{2}$

8.3 COMPONENTS OF A VECTOR

In Unit 6, we have studied the cartesian coordinate system in a plane. Two mutually perpendicular straight lines were taken as the coordinate axis, their point of intersection was called the origin and any point in the plane was represented by an ordered pair (x, y) of real numbers.

Let us introduce a coordinate system in space.

Let O be a fixed point in space. We consider three planes (Figure 8.4) passing through O and mutually perpendicular to one another. The lines of intersection of these planes, i.e. the lines $X'OX$, $Y'OY$ and $Z'OZ$ are called the X-axis, the Y-axis and the Z-axis. The point is called the **origin**. $X'OX$, $Y'OY$ and $Z'OZ$ taken together and in this particular order so as to form a right hand triad are called coordinate axes. Right hand triad means that if the right hand is placed along positive direction of X-axis with palm towards positive direction of Y-axis, then the thumb must point in the positive direction of the Z-axis. Since the three coordinate axes are mutually perpendicular to one another, therefore, these are called **rectangular axes**.

The plane containing $X'OX$, and $Y'OY$ is called the XOY plane (or XY-plane); the plane containing $Y'OY$ and $Z'OZ$ is called the YOZ plane (or YZ-plane) and the plane containing $Z'OZ$ and $X'OX$ is called the ZOX plane (or ZX-plane). The three planes taken together in this very order are called coordinate planes.

Let P be any point in space. Through P draw planes parallel to three coordinate planes meeting X-axis in the point A, Y-axis in the point B and Z-axis in the point C.

The directed lengths OA, OB and OC are respectively known as the x-coordinate, the Y-coordinate and Z-coordinate of the point P. If $OA = x$, $OB = y$ and $OC = z$, then the ordered 3-triple (x, y, z) is called coordinates of the point P. It is conventional to write $P \equiv (x, y, z)$.

Figure 8.4

Thus to every point P in space, we have associated an ordered 3-tuple of real numbers. Conversely, if we are given an ordered 3-tuple (x, y, z) of real numbers, then we locate the points A, B and C on the coordinate axes in such a way that $OA = x$, $OB = y$ and $OC = z$. Then we complete the parallelopiped as shown in Figure 8.5. Then the uniquely determined point P corresponds to the 3-tuple (x, y, z). *So, there exists a one-one correspondence between the set of points in space and the set of ordered 3-tuple of real numbers.*

Figure 8.5

Some Facts about the Rectangular Axes

(i) The coordinates of the origin O are (0, 0, 0).

(ii) If a point P (x, y, z) lies in the YZ-plane then $x = 0$ and conversely, if $x = 0$ then P must lie in the YZ-plane. Thus the YZ-plane has the property that x-coordinate of any point in it is zero and conversely every point whose x-coordinate is zero lies in the plane. Hence **the equation of *YZ*-plane is *x* = 0**.

Similarly, **the equation of ZX-plane is y = 0** and **the equation of XY-plane is z = 0**.

(iii) If P (x, y, z) is any point in space and L, M, N are feet of perpendiculars from it on YZ, ZX and XY-planes respectively then the coordinates of L, M, N as points of space are $(0, y, z)$, $(x, 0, z)$, $(x, y, 0)$ respectively and the coordinates of these points as points of the planes, YZ, ZX and XY are (y, z) (x, z) and (x, y) respectively.

(iv) If a point P (x, y, z) lies on the X-axis then $y = 0$ and $z = 0$ and conversely, if $y = 0$ and $z = 0$ then P must lie on X-axis. Hence **the equations of *X*-axis are *y* = 0 and *z* = 0**.

Similarly, **the equations of *Y*-axis are *z* = 0, *x* = 0 and the equations of *Z*-axis are *x* = 0, *y* = 0**.

(v) Since the plane ANPM is perpendicular to X-axis (Figure 8.5), therefore, the line PA is perpendicular to X-axis and hence A is the foot of perpendicular from P (x, y, z) to X-axis. But the coordinate of A as the point of X-axis is x. Thus if P (x, y, z) is any point in space and if A, B, C are feet of perpendiculars from P to coordinate axes respectively then the coordinates of these points as points of coordinate axes are x, y and z respectively. However, the coordinates of these points as points of space are $(x, 0, 0)$, $(0, y, 0)$ and $(0, 0, z)$ respectively.

(vi) *Alternatively*, coordinates of a point P can be fixed as follows :

Figure 8.6

Let N be the foot of perpendicular from P on the XY-plane and let A be the foot of perpendicular (in XY-plane) drawn from N on $X'OX$, then P has the coordinates (x, y, z) where $x = OA$, $y = AN$ and $z = NP$; all the three lengths OA, AN and NP being directed lengths.

Definition 3

Let P, Q be any arbitrary points in space and l be any directed line. If M, N are feet of perpendicular from P and Q respectively on l then the points M and N are called ***projections of points****, P and Q on l and directed length MN is called the* ***projection of [PQ]*** *on l.*

Note : (i) The projection of a segment $[PQ]$ on a directed line l is a real number which may be positive, zero or negative.

This fact is clear from Figure 8.7 because the directed length MN is + ve, zero and – ve in Figure 8.7 (i), (ii) and (iii) respectively.

(i) **(ii)**

(iii)

Figure 8.7

(ii) The projection of a segment $[PQ]$ on l is zero iff line PQ is perpendicular to l.

(iii) If line PQ is parallel to the line l then $PMNQ$ becomes a rectangle and hence $MN = PQ$, i.e. projection of $[PQ]$ on l is equal to the directed length PQ.

Theorem 1

If $P = (x_1, y_1, z_1)$ and $Q = (x_2, y_2, z_2)$ then projections of $[PQ]$ on coordinate axes are $x_2 - x_1, y_2 - y_1, z_2 - z_1$ respectively.

Proof

Let M, N be the feet of perpendiculars from points P and Q respectively on X-axis, so that the coordinates of M, N as points of X-axis are x_1, x_2 respectively.

Figure 8.8

$\therefore$ Projection of $[PQ]$ on X-axis $= MN = ON - OM = x_2 - x_1$.

Similarly, projection of $[PQ]$ on Y-axis $= y_2 - y_1$ and projection of $[PQ]$ on Z-axis $= z_2 - z_1$.

We can prove that the distance between two points $P = (x_1, y_1, z_1)$ and $Q = (x_2, y_2, z_2)$ in space is given by $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$.

Definition 4

Let $P = (x_1, y_1, z_1)$ be the initial point and $Q = (x_2, y_2, z_2)$ be the terminal point of a vector $\boldsymbol{a}$.

Then the numbers

$$a_1 = x_2 - x_1,\ a_2 = y_2 - y_1,\ a_3 = z_2 - z_1 \qquad \ldots (8.1)$$

are called the components of the vector $\boldsymbol{a}$ with respect to that coordinate system, i.e. the projection of $[PQ]$ on the three coordinate axis are the components of the vector $\boldsymbol{a} = \boldsymbol{PQ}$.

Figure 8.9

By definition $|\,\boldsymbol{a}\,|$ is the distance of PQ and

$$\therefore \qquad |\boldsymbol{a}| = PQ = \sqrt{a_1^2 + a_2^2 + a_3^2} \qquad \ldots (8.2)$$

For example a vector $\boldsymbol{a}$ with initial point as $P : (3, 1, 4)$ and terminal point $Q : (1, -2, 4)$ has the components $a_1 = 1 - 3 = -2$, $a_2 = -2 - 1 = -3$, $a_3 = 4 - 4 = 0$ and the magnitude is

$$|\boldsymbol{a}| = \sqrt{(-2)^2 + (-3)^2 + (0)^2} = \sqrt{4 + 9} = \sqrt{13}$$

Conversely if $\boldsymbol{a}$ has the components $-2, -3, 0$ and if we choose the initial point of $\boldsymbol{a}$ as $(-1, 5, 8)$ then the corresponding terminal point is

$(-1 - 2, 5 - 3, 8 - 0)$, i.e. $(-3, 2, 8)$.

You may observe from Eq. (8.1) that if we choose the initial point of a vector to be origin, then its components are equal to the coordinates of the terminal point and the vector is then called the position vector of the terminal point w. r. to our coordinate system and is usually denoted by $\boldsymbol{r}$ (Figure 8.10).

We see that

$$\boldsymbol{r} = \boldsymbol{OA} = \boldsymbol{OM} + \boldsymbol{MA_o} + \boldsymbol{A_oA}$$

$$= x\,\hat{\boldsymbol{i}} + y\,\hat{\boldsymbol{j}} + z\,\hat{\boldsymbol{k}}$$

where $\hat{\boldsymbol{i}}, \hat{\boldsymbol{j}}, \hat{\boldsymbol{k}}$ are unit vectors parallel to the axes of x, y, z respectively.

Figure 8.10

The vectors $\hat{\boldsymbol{i}}, \hat{\boldsymbol{j}}, \hat{\boldsymbol{k}}$ are mutually perpendicular. From Eq. (8.1), we can also see that the components a_1, a_2, a_3 of a vector $\boldsymbol{a}$ are independent of the choice of initial point of $\boldsymbol{a}$. This is because if we translate (displace without rotation) $\boldsymbol{a}$, then corresponding coordinates of P and Q are altered by the same amount. Hence given a fixed cartesian coordinate system, each vector is uniquely determined by the ordered triple of its components w. r. t. that coordinate system.

We may introduce the null vector as the vector with components 0, 0, 0.

Figure 8.11

Further, two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are equal iff components of these vectors are equal, i.e. $a_1 = b_1, a_2 = b_2, a_3 = b_3$ where a_1, a_2, a_3 are the components of $\boldsymbol{a}$ and b_1, b_2, b_3 are the components of $\boldsymbol{b}$ w. r. t. the same cartesian coordinate system.

From Eq. (8.1) we note that a_1, a_2, a_3 are the projection of $\boldsymbol{a}$ as

$$\boldsymbol{a} = a_1 \hat{\boldsymbol{i}} + a_2 \hat{\boldsymbol{j}} + a_3 \hat{\boldsymbol{k}}\,.$$

In Figure 8.11 if α, β, γ are the angles which PQ makes with the three axes, then

$$a_1 = |\boldsymbol{a}| \cos \alpha,\ a_2 = |\boldsymbol{a}| \cos \beta,\ a_3 = |\boldsymbol{a}| \cos \gamma\,.$$

The three angles α, β, γ are called the direction angles and cosines of these angles are called direction cosines.

Theorem 2

If $\boldsymbol{a}$ and $\boldsymbol{b}$ are the position vectors of the points A and B then the position vector of AB is $\boldsymbol{b} - \boldsymbol{a}$.

Proof

Let O be the origin.

Figure 8.12

Then $\qquad \boldsymbol{OA} = \boldsymbol{a}; \ \boldsymbol{OB} = \boldsymbol{b}$

but $\qquad \boldsymbol{OA} + \boldsymbol{AB} = \boldsymbol{OB}$

i.e. $\qquad \boldsymbol{AB} = \boldsymbol{OB} - \boldsymbol{OA} = \boldsymbol{b} - \boldsymbol{A}$

$\qquad\qquad$ = Position vector of B – Position vector of A

Theorem 3

Find the position vector of the point which divides the line joining two given points in the ratio of *m* : *n*.

Proof

Let A and B be the two points and C divides AB in the ratio of $m : n$ then $\boldsymbol{OA} = \boldsymbol{a}$, $\boldsymbol{OB} = \boldsymbol{b}$.

Figure 8.13

$$\boldsymbol{OC} = \boldsymbol{OA} + \boldsymbol{AC} = \boldsymbol{OA} + \left(\frac{m}{m+n}\right)\boldsymbol{AB} \ \left(\because \ AC = \frac{m}{m+n} AB\right)$$

$$= \boldsymbol{a} + \frac{m}{m+n}(\boldsymbol{b} - \boldsymbol{a})$$

$$= \frac{m\boldsymbol{b} + n\boldsymbol{a}}{m+n}$$

Corollary

If $m = n = 1$, then $\boldsymbol{OC} = \dfrac{\boldsymbol{a} + \boldsymbol{b}}{2}$ and C is the mid point of A and B.

Example 8.2

Prove that the diagonals of a parallelogram bisect each other.

Solution

Let $ABCD$ be the parallalogram and $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$, $\boldsymbol{d}$ be the position vectors of A, B, C, D respectively.

Then $\boldsymbol{AB} = \boldsymbol{b} - \boldsymbol{a}$

$\boldsymbol{DC} = \boldsymbol{c} - \boldsymbol{d}$

Figure

As $\boldsymbol{AB} = \boldsymbol{DC}$ $\quad\therefore\quad$ $\boldsymbol{b} - \boldsymbol{a} = \boldsymbol{c} - \boldsymbol{d}$

($AB = DC$ and $AB \parallel DC$ as $ABCD$ is a parallelogram.)

$\therefore$ $\boldsymbol{a} + \boldsymbol{c} = \boldsymbol{b} + \boldsymbol{d}$

i.e. $$\frac{\boldsymbol{a} + \boldsymbol{c}}{2} = \frac{\boldsymbol{b} + \boldsymbol{d}}{2} \qquad \ldots$$

(i)

Position vector of the mid point of the diagonal $\boldsymbol{AC} = \dfrac{\boldsymbol{a} + \boldsymbol{c}}{2} = \dfrac{\boldsymbol{b} + \boldsymbol{d}}{2} =$ position vector of the mid point of BD.

SAQ 2

Tick (✔) the correct answer.

(i) If $|A + B| = |A - B|$ and A and B are finite, then

(a) A is parallel to B

(b) $A = B$

(c) $|A| = B$

(d) A and B are mutually perpendicular.

(ii) A vector $\boldsymbol{X}$ when added to two vectors $A = 3\hat{\boldsymbol{i}} - 5\hat{\boldsymbol{j}} + 7\hat{\boldsymbol{k}}$ and $B = 2\hat{\boldsymbol{i}} + 4\hat{\boldsymbol{j}} - 3\hat{\boldsymbol{k}}$ gives a unit vector along Y-axis as their resultant. Find the vector X.

8.4 OPERATIONS ON VECTORS

By 'algebraic operations on vectors', we mean various ways of combining vectors and scalars, satisfying different laws, called **laws of calculations**. Let us take this one by one.

8.4.1 Addition of Vectors

Let ***a*** and ***b*** be two vectors. Let the vector ***a*** be the directed line segment ***AB*** and the vector ***b*** be the directed segment ***BC*** (so that the terminal point ***B*** of ***a*** is the initial point of ***b***) (see Figure 8.14). Then the directed line segment ***AC*** (i.e. ***AC***) represents the sum (or resultant) of ***a*** and ***b*** and is written as ***a*** + ***b***.

Thus $\quad \boldsymbol{AC} = \boldsymbol{AB} + \boldsymbol{BC} = \boldsymbol{a} + \boldsymbol{b}$.

Figure 8.14 : Addition of Two Vectors

The method of drawing a triangle in order to define the vector sum (***a*** + ***b***) is called *triangle law of addition of two vectors*, which states as follows.

If two vectors are represented by two sides of a triangle, taken in order, then their sum (or resultant) is represented by the third side of the triangle taken in the reverse order.

Since any side of a triangle is less than the sum of the other two sides of the triangle; hence modulus of ***AC*** is less than the sum of moduli of ***AB*** and ***BC***.

It may be noted that the vector sum does not depend upon the choice of initial point of the vector as can be seen from Figure 8.15.

Figure 8.15 : Vector Sum is Independent of Initial Point

If in some fixed coordinate system, ***a*** has the components a_1, a_2, a_3 and ***b*** has the components b_1, b_2, b_3, then the components c_1, c_2, c_3 of the sum vector ***c*** = ***a*** + ***b*** are obtained by the addition of corresponding components of ***a*** and ***b***; thus

$$c_1 = a_1 + b_1, c_2 = a_2 + b_2, c_3 = a_3 + b_3 \qquad \ldots (8.3)$$

This fact is represented in Figure 8.16 in the case of plane. In space the situation is similar.

Figure 8.16 : Vector Addition in Terms of Components in a Plane

8.4.2 Properties of Vector Addition

From the definition of vector addition and using Eq. (8.3), it can be shown that vector addition has the following properties.

(i) **Vector Addition is Commutative**

If ***a*** and ***b*** are any two vectors, then

$$\boldsymbol{a} + \boldsymbol{b} = \boldsymbol{b} + \boldsymbol{a}$$

Let ***OA*** = ***a*** and ***AB*** = ***b*** (Figure 8.17)

$\because$ ***OB*** = ***OA*** + ***AB***

$$= \boldsymbol{a} + \boldsymbol{b} \qquad \ldots (8.4)$$

Let us complete the parallelogram *OABC*. Then ***OC*** = ***AB*** = ***b*** and ***CB*** = ***OA*** = ***a***

$\therefore$ ***OB*** = ***OC*** + ***CB***

$$= \boldsymbol{b} + \boldsymbol{a} \qquad \ldots (8.5)$$

From Eqs. (8.4) and (8.5), we have

$$\boldsymbol{a} + \boldsymbol{b} = \boldsymbol{b} + \boldsymbol{a}$$

Figure 8.17 : Commutative Vector Addition

(ii) **Vector Addition is Associative**

If $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$ are any three vectors, then

$$\boldsymbol{a} + (\boldsymbol{b} + \boldsymbol{c}) = (\boldsymbol{a} + \boldsymbol{b}) + \boldsymbol{c}$$

The above equation can be easily verified from Figure 8.18.

Figure 8.18 : Associative Law of Vector Addition

Note that the sum of three vectors $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$ is independent of the order in which they are added and is written as

$$\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c}.$$

(iii) **Existence of Additive Identity**

For any vector $\boldsymbol{a}$,

$$\boldsymbol{a} + \mathbf{0} = \boldsymbol{a} = \mathbf{0} + \boldsymbol{a},$$

where $\mathbf{0}$ is a null (or zero) vector.

Thus $\mathbf{0}$ is called additive identity of vector addition.

(iv) **Existence of Additive Inverse**

For any vector $\boldsymbol{a}$, there exists another vector $-\boldsymbol{a}$ such that

$$\boldsymbol{a} + (-\boldsymbol{a}) = \mathbf{0},$$

where $-\boldsymbol{a}$ denotes the vector having the length $|\boldsymbol{a}|$ and the direction opposite to that of $\boldsymbol{a}$.

In view of the above property, the vector $(-\boldsymbol{a})$ is called the additive inverse of vector $\boldsymbol{a}$.

Let us take an example from geometry to illustrate the use of vector addition.

Example 8.3

Show that the sum of three vectors determined by the medians of a triangle directed from the vertices is zero.

Solution

In Δ *ABC*, *AD*, *BE* and *CF* are the median.

Figure 8.19 : Medians of a Triangle

Now $\quad \boldsymbol{AD} + \boldsymbol{BE} + \boldsymbol{CF}$

$$= (\boldsymbol{AB} + \boldsymbol{BD}) + (\boldsymbol{BC} + \boldsymbol{CE}) + (\boldsymbol{CA} + \boldsymbol{AF})$$

$$= \boldsymbol{AB} + \boldsymbol{BC} + \boldsymbol{CA} + \frac{1}{2}\boldsymbol{BC} + \frac{1}{2}\boldsymbol{CA} + \frac{1}{2}\boldsymbol{AB}$$

$$= \frac{3}{2}(\boldsymbol{AB} + \boldsymbol{BC} + \boldsymbol{CA}) \qquad \text{. . . (8.6)}$$

By triangle law of addition,

$$\boldsymbol{AB} + \boldsymbol{BC} = \boldsymbol{AC} \qquad \text{. . . (8.7)}$$

From Eqs. (8.6) and (8.7), we get

$$\boldsymbol{AD} + \boldsymbol{BE} + \boldsymbol{CF}$$

$$= \frac{3}{2}(\boldsymbol{AC} + \boldsymbol{CA})$$

$$= \frac{3}{2}[\boldsymbol{AC} + (-\boldsymbol{AC})]$$

$$= \frac{3}{2}(0) = \mathbf{0}$$

8.4.3 Multiplication of Vectors by Scalars

Definition 5

Let $\boldsymbol{a}$ be any vector and m be any given scalar. Then the vector $m\boldsymbol{a}$ (product of vector $\boldsymbol{a}$ and scalar m) is a vector whose

(i) magnitude $|m\boldsymbol{a}| = |m| . |\boldsymbol{a}|$

$$= m\,|\boldsymbol{a}| \text{ if } m \geq 0$$

$$= -m\,|\boldsymbol{a}| \text{ if } m < 0$$

(ii) support is the same or parallel to that of support of $\boldsymbol{a}$ if $\boldsymbol{a} \neq 0$ and $m > 0$ and

(iii) $m\boldsymbol{a}$ has the direction of vector $\boldsymbol{a}$ if $\boldsymbol{a} \neq 0$ and $m > 0$ and $m\boldsymbol{a}$ has the direction opposite to vector $\boldsymbol{a}$ if $\boldsymbol{a} \neq 0$ and $m < 0$.

Further, if $\boldsymbol{a} = 0$ or $m = 0$ (or both), then $m\boldsymbol{a} = \mathbf{0}$. Geometrically, we can represent $m\boldsymbol{a}$ as follows :

Let $\boldsymbol{AB} = \boldsymbol{a}$, then $\boldsymbol{AC} = m\boldsymbol{a}$ if $m > 0$. Here we choose the point $\boldsymbol{C}$ on $\boldsymbol{AB}$ on the same side of $\boldsymbol{A}$ as $\boldsymbol{B}$ (see Figure 8.20(a)).

Figure 8.20(a)

Now if $m < 0$ and $\boldsymbol{AB} = \boldsymbol{a}$, then $\boldsymbol{AC} = m\boldsymbol{a}$ where we have choosen the point $\boldsymbol{C}$ on $\boldsymbol{AB}$ on the side of $\boldsymbol{A}$ opposite to that of $\boldsymbol{B}$ (see Figure 8.20(b)).

Figure 8.20(b)

Further, if $\boldsymbol{a}$ has the component a_1, a_2, a_3 then $m\boldsymbol{a}$ has the components $m\boldsymbol{a}_1, m\boldsymbol{a}_2, m\boldsymbol{a}_3$ (w. r. t. the same coordinate system).

From the definition, we have the following *properties of* ***multiplication of a vector*** *by a scalar*.

(I) $m(\boldsymbol{a} + \boldsymbol{b}) = m\boldsymbol{a} + m\boldsymbol{b}$

Figure 8.21

Here $\boldsymbol{OA'} = m\boldsymbol{a}$ and $\boldsymbol{A'B'} = m\boldsymbol{b}$

$\because$ $\boldsymbol{OB'} = m\boldsymbol{a} + m\boldsymbol{b}$

Also, $\boldsymbol{OB'} = m(\boldsymbol{a} + \boldsymbol{b})$

(II) $(m + n)\,\boldsymbol{a} = m\boldsymbol{a} + n\boldsymbol{a}$ (Distributive Law)

(III) $m(n\boldsymbol{a}) = (mn)\,\boldsymbol{a} = mn\boldsymbol{a}$ (Associative Law)

(IV) $1\boldsymbol{a} = \boldsymbol{a}$ (Existence of multiplicative identity)

(V) $0\boldsymbol{a} = \boldsymbol{0}$

(VI) $(-1)\,\boldsymbol{a} = -\boldsymbol{a}$

Also if $\hat{\boldsymbol{a}}$ is the unit vector having the same direction as $\boldsymbol{a}$, then

$$\boldsymbol{a} = |\boldsymbol{a}|\,\hat{\boldsymbol{a}} \text{ and } \hat{\boldsymbol{a}} = \frac{\boldsymbol{a}}{|\boldsymbol{a}|}.$$

Let us take up some examples to illustrate the above properties.

Example 8.4

If $\boldsymbol{a}$ is a non-zero vector, find a scalar λ such that $|\lambda\,\boldsymbol{a}| = 1$.

Solution

We have to determine λ such that

$$|\lambda \boldsymbol{a}| = 1$$

$$\Rightarrow \quad |\lambda|\,|\boldsymbol{a}| = 1$$

$$\Rightarrow \quad |\lambda| = \frac{1}{|\boldsymbol{a}|} \quad (\because\ \boldsymbol{a} \text{ is non-zero}, \therefore |\boldsymbol{a}| \neq 0)$$

$$\Rightarrow \quad |\lambda| = \pm\frac{1}{|\boldsymbol{a}|} \quad \begin{array}{l}\text{the + sign is to be taken when } \lambda > 0 \text{ and} \\ \text{the } - \text{ sign is to be taken when } \lambda < 0\end{array}$$

We can now define the difference of two vectors.

Difference of Two Vectors

The difference $\boldsymbol{a} - \boldsymbol{b}$ of two vectors is defined as the sum $\boldsymbol{a} + (-\boldsymbol{b})$, where $(-\boldsymbol{b})$ is the negative of $\boldsymbol{b}$. We can geometrically represent the difference $\boldsymbol{a} - \boldsymbol{b}$ as in Figure 8.22.

Figure 8.22 : Difference of Two Vectors

It is evident that

$$\boldsymbol{a} - \boldsymbol{a} = \boldsymbol{0}$$

and $$\boldsymbol{a} + \boldsymbol{0} = \boldsymbol{a}.$$

If two vectors are given in their component forms then to obtain their difference, subtract the vectors component wise.

For example, if $\boldsymbol{a} = a_1\hat{\boldsymbol{i}} + a_2\hat{\boldsymbol{j}} + a_3\hat{\boldsymbol{k}}$, and $\boldsymbol{b} = b_1\hat{\boldsymbol{i}} + b_2\hat{\boldsymbol{j}} + b_3\hat{\boldsymbol{k}}$, then $\boldsymbol{a} - \boldsymbol{b} = (a_1 - b_1)\hat{\boldsymbol{i}} + (a_2 - b_2)\hat{\boldsymbol{j}} + (a_3 - b_3)\hat{\boldsymbol{k}}$.

Let us take a few examples.

Example 8.5

If the sum of two unit vectors is a unit vector, prove that the magnitude of their difference is $\sqrt{3}$.

Solution

Let $\boldsymbol{OA}$ and $\boldsymbol{AB}$ be two unit vectors $\hat{\boldsymbol{a}}$ and $\hat{\boldsymbol{b}}$.

Then by triangle law of addition,

$$\hat{\boldsymbol{a}} + \hat{\boldsymbol{b}} = \boldsymbol{OB}$$

Figure 8.23

We are given $|\hat{\boldsymbol{a}}| = 1, |\hat{\boldsymbol{b}}| = 1, |\hat{\boldsymbol{a}} + \hat{\boldsymbol{b}}| = 1$

$\therefore$ $OA = AB = OB = 1$

Let $\boldsymbol{AC} = -\hat{\boldsymbol{b}}.$

Then $AC = |\boldsymbol{AC}| = |-\hat{\boldsymbol{b}}| = |\hat{\boldsymbol{b}}| = 1$

Since $OA = AB = AC$, then by geometry $\Delta\, BOC$ is a right-angled triangle, with $\angle\, BOC = \pi / 2$.

Now $\hat{\boldsymbol{a}} - \hat{\boldsymbol{b}} = \hat{\boldsymbol{a}} + (-\hat{\boldsymbol{b}}) = \boldsymbol{OA} + \boldsymbol{AC} = \boldsymbol{OC}$

$\therefore$ $|\hat{\boldsymbol{a}} - \hat{\boldsymbol{b}}| = |\boldsymbol{OC}| = OC$

Now

$$\boldsymbol{BC}^2 = \boldsymbol{OB}^2 + \boldsymbol{OC}^2 \Rightarrow OC = \sqrt{BC^2 - OB^2} = \sqrt{2^2 - 1^2} = \sqrt{4-1} = \sqrt{3}$$

So far we have defined addition and subtraction of vectors as well as multiplication of vectors by scalars. We shall now introduce multiplication of vectors by vectors.

8.5 PRODUCT OF TWO VECTORS

When one vector is multiplied with another vector, result can be a scalar or a vector. There are in general two different ways in which vectors can be multiplied. These are the *scalar* or *dot* or *inner* product which is a mere number (or scalar) having magnitude alone and the other is called *vector* or *cross* product, which is a vector having a definite direction. We shall now take up these two products one by one.

8.5.1 Scalar or Dot Product

The scalar or dot or inner product of two vectors ***a*** and ***b*** in the three-dimensional space is written as ***a . b*** (read as '***a*** dot ***b***') and is defined as

$$\boldsymbol{a} \,.\, \boldsymbol{b} = \begin{bmatrix} |\boldsymbol{a}|\,|\boldsymbol{b}| \cos\gamma, & \text{when } \boldsymbol{a} \neq \boldsymbol{0}, \boldsymbol{b} \neq \boldsymbol{0} \\ 0 & \text{when } \boldsymbol{a} = \boldsymbol{0} \text{ or } \boldsymbol{b} = \boldsymbol{0} \end{bmatrix}, \qquad \ldots (8.8)$$

where $\gamma\ (0 \le \gamma \le \pi)$ is the angle between ***a*** and ***b*** (computed when the vectors have their initial points coinciding) (Refer Figure 8.24).

Figure 8.24 : Angle between Vectors

The value of the dot product is a scalar (a real number), and this motivates the term "scalar product". Since the cosine in Eq. (8.8) may be positive, zero or negative, the same is true for the dot product.

Angle γ in Eq. (8.8) lies between 0 and π and we know that $\cos \gamma = 0$, if and only if $\gamma = \pi / 2$, we thus have the following important result :

Two non-zero vectors are orthogonal (perpendicular) if and only if their dot product is zero.

If we put $\boldsymbol{b} = \boldsymbol{a}$ in Eq. (8.8), we have $\boldsymbol{a} \,.\, \boldsymbol{a} = |\,\boldsymbol{a}^2\,|$ and this shows that the length (or modulus or magnitude) of a vector can be written in terms of scalar product as

$$|\boldsymbol{a}| = \sqrt{\boldsymbol{a} \,.\, \boldsymbol{a}} \ (\geq 0) \qquad \qquad \ldots (8.9)$$

From Eqs. (8.8) and (8.9), we obtain the angle γ between two non-zero vectors as

$$\cos \gamma = \frac{\boldsymbol{a} \,.\, \boldsymbol{b}}{|\boldsymbol{a}|\,|\boldsymbol{b}|} = \frac{\boldsymbol{a} \,.\, \boldsymbol{b}}{\sqrt{\boldsymbol{a} \,.\, \boldsymbol{a}}\,\sqrt{\boldsymbol{b} \,.\, \boldsymbol{b}}}$$

The scalar product has the following properties :

(I) $(q_1\boldsymbol{a} + q_2\boldsymbol{b}) \,.\, \boldsymbol{c} = q_1\boldsymbol{a} \,.\, \boldsymbol{c} + q_2\boldsymbol{b} \,.\, \boldsymbol{c}$ (Linearity)

(II) $\boldsymbol{a} \,.\, \boldsymbol{b} = \boldsymbol{b} \,.\, \boldsymbol{a}$ (Symmetry or Commutative Law)

(III) $\left.\begin{array}{l} \boldsymbol{a} \,.\, \boldsymbol{a} \geq 0 \\ \text{Also } \boldsymbol{a} \,.\, \boldsymbol{a} = 0 \text{ if and only if } \boldsymbol{a} = 0 \end{array}\right\}$ (Positive Definiteness)

In property (I), with $q_1 = 1$ and $q_2 = 1$, we have

(IV) $(\boldsymbol{a} + \boldsymbol{b}) \,.\, \boldsymbol{c} = \boldsymbol{a} \,.\, \boldsymbol{c} + \boldsymbol{b} \,.\, \boldsymbol{c}$ (Distributivity)

Hence scalar product is commutative and distributive with respect to vector addition. From the definition of scalar product, we get

(V) $|\boldsymbol{a} \,.\, \boldsymbol{b}| \leq |\boldsymbol{a}|\,|\boldsymbol{b}|$ $(\because \ |\cos \gamma| \leq 1)$ (Schwarz Inequality)

Also using the definition and simplifying, we get

(VI) $|\boldsymbol{a} + \boldsymbol{b}|^2 + |\boldsymbol{a} - \boldsymbol{b}|^2 = 2\,(|\boldsymbol{a}|^2 + |\boldsymbol{b}|^2)$ (Parallelogram Equality)

Further, if $\hat{\boldsymbol{i}}, \hat{\boldsymbol{j}}, \hat{\boldsymbol{k}}$ are unit vectors forming an orthogonal triad, then, from definition of scalar product, we have

(VII) $\left\{\begin{array}{lll} \hat{\boldsymbol{i}} \,.\, \hat{\boldsymbol{i}} = 1 \,, & \hat{\boldsymbol{j}} \,.\, \hat{\boldsymbol{j}} = 1 \,, & \hat{\boldsymbol{k}} \,.\, \hat{\boldsymbol{k}} = 1 \\ \hat{\boldsymbol{i}} \,.\, \hat{\boldsymbol{j}} = 0 \,, & \hat{\boldsymbol{j}} \,.\, \hat{\boldsymbol{k}} = 0 \,, & \hat{\boldsymbol{k}} \,.\, \hat{\boldsymbol{i}} = 0 \end{array}\right\}$

If vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are represented in terms of components, say,

$$\boldsymbol{a} = a_1\hat{\boldsymbol{i}} + a_2\hat{\boldsymbol{j}} + a_3\hat{\boldsymbol{k}} \text{ and } \boldsymbol{b} = b_1\hat{\boldsymbol{i}} + b_2\hat{\boldsymbol{j}} + b_3\hat{\boldsymbol{k}} \,,$$

then their scalar product is given by the formula

$$\boldsymbol{a} \,.\, \boldsymbol{b} = \boldsymbol{a}_1\boldsymbol{b}_1 + \boldsymbol{a}_2\boldsymbol{b}_2 + \boldsymbol{a}_3\boldsymbol{b}_3 \qquad \text{(using (VII))}$$

and $\cos \gamma = \left(\dfrac{\boldsymbol{a} \,.\, \boldsymbol{b}}{|a||b|} \right) = \dfrac{(\boldsymbol{a}_1\boldsymbol{b}_1 + \boldsymbol{a}_2\boldsymbol{b}_2 + \boldsymbol{a}_3\boldsymbol{b}_3)}{(\sqrt{\boldsymbol{a}_1^2 + \boldsymbol{a}_2^2 + \boldsymbol{a}_3^2}\ \sqrt{\boldsymbol{b}_1^2 + \boldsymbol{b}_2^2 + \boldsymbol{b}_3^2}\,)}$, where γ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$. Before we take up some applications of scalar products, we give below the geometrical interpretation of scalar product of two vectors.

Geometrical Interpretation of Dot Product

The scalar product of two vectors is the product of the modulus of either vector and the resolution (projection) of the other in its direction.

Let

$$\boldsymbol{OA} = \boldsymbol{a},\ \boldsymbol{OB} = \boldsymbol{b} \text{ and } \angle\, BOA = \gamma$$

By definition, $\boldsymbol{a} \,.\, \boldsymbol{b} = |\,\boldsymbol{a}\,|\,|\,\boldsymbol{b}\,| \cos \gamma$, where $|\,\boldsymbol{a}\,| = OA$ and $|\,\boldsymbol{b}\,| = OB$.

From the point B draw BC perpendicular on OA (Figure 8.25(b)) and from the point A, draw AD perpendicular on OB (Figure 8.25(a)).

(a) Projection of *a* in the Direction of *b* **(b) Projection of *b* in the Direction of *a***

Figure 8.25

$\therefore \qquad OC$ = Projection of OB on $OA = OB \cos \gamma = |\,\boldsymbol{b}\,| \cos \gamma$

and

$\qquad OD$ = Projection of OA on $OB = OA \cos \gamma = |\,\boldsymbol{a}\,| \cos \gamma$

Thus $\boldsymbol{a} \,.\, \boldsymbol{b} = |\,\boldsymbol{a}\,|\ |\,\boldsymbol{b}\,| \cos \gamma = |\,\boldsymbol{a}\,| (|\,\boldsymbol{b}\,| \cos \gamma) = |\,\boldsymbol{a}\,|$ (Projection of $\boldsymbol{b}$ in the direction of $\boldsymbol{a}$).

Also, $\boldsymbol{a} \,.\, \boldsymbol{b} = |\,\boldsymbol{a}\,|\ |\,\boldsymbol{b}\,| \cos \gamma = |\,\boldsymbol{b}\,| (|\,\boldsymbol{a}\,| \cos \gamma) = |\,\boldsymbol{b}\,|$ (Projection of $\boldsymbol{a}$ in the direction of $\boldsymbol{b}$).

Hence the result.

Remember that if $\mathbf{a}$ *and* $\mathbf{b}$ *are two vectors and* $\mathbf{a} \neq 0$*, then*

$p = |\boldsymbol{b}| \cos \gamma = \dfrac{\boldsymbol{a} \,.\, \boldsymbol{b}}{|\boldsymbol{a}|}$ *is called the* ***component*** *of* $\mathbf{b}$ *in the direction of* $\mathbf{a}$*, or the* ***projection*** *of* $\mathbf{b}$ *in the direction of* $\mathbf{a}$*, where* γ *is the angle between* $\mathbf{a}$ *and* $\mathbf{b}$*. If* $\boldsymbol{a} = \mathbf{0}$*, then* γ *is undefined and we set* $p = 0$*.*

It follows that $|\,P\,|$ is the length of the orthogonal projection of $\boldsymbol{b}$ on a straight line l in the direction of $\boldsymbol{a}$. Here p may be positive, zero or negative (Figure 8.26).

Figure 8.26 : Components of *b* in the Direction of *a*

In particular, if $\boldsymbol{a}$ is a unit vector, then we simply have

$$p = \boldsymbol{a} \cdot \boldsymbol{b}$$

The following examples illustrate the applications of a dot product.

Example 8.6

Give a representation of *work done by a force* in terms of scalar product.

Solution

Consider a particle $\boldsymbol{P}$ on which a constant force $\boldsymbol{F}$ acts. Let the particle be given a displacement $\boldsymbol{d}$ by the application of this force. Then the work done $\boldsymbol{W}$ by $\boldsymbol{F}$ in this displacement is defined as the product of $|\,\boldsymbol{d}\,|$ and the component of $\boldsymbol{F}$ in the direction of $\boldsymbol{d}$, i.e.,

$$\boldsymbol{W} = |\,\boldsymbol{F}\,|\,(|\,\boldsymbol{d}\,| \cos \alpha) = \boldsymbol{F} \cdot \boldsymbol{d},$$

where α is the angle between $\boldsymbol{F}$ and $\boldsymbol{d}$.

Figure 8.27 : Work Done by a Force

Example 8.7

Find the projection of $\boldsymbol{b}$ on the line of $\boldsymbol{a}$ if $\boldsymbol{a} = \hat{\boldsymbol{i}} + \hat{\boldsymbol{j}} + \hat{\boldsymbol{k}}$ and $\boldsymbol{b} = 2\hat{\boldsymbol{i}} + 4\hat{\boldsymbol{j}} + 5\hat{\boldsymbol{k}}$.

Solution

Here, $\boldsymbol{a} \cdot \boldsymbol{b} = 1.2 + 1.4 + 1.5 = 2 + 4 + 5 = 11$

Also $\quad |\boldsymbol{a}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

$\therefore$ Projection of $\boldsymbol{b}$ on the line of $\boldsymbol{a}$

$$= \frac{\boldsymbol{a} \cdot \boldsymbol{b}}{|\boldsymbol{a}|} = \frac{11}{\sqrt{3}} = \frac{11\sqrt{3}}{3}$$

Example 8.8

Show by vector method that the diagonals of a rhombus are at right angles.

Solution

Let *ABCD* be a rhombus.

Let *A* be taken on the origin.

Let $\boldsymbol{b}$ and $\boldsymbol{d}$ be the position vectors of vertices *B* and *D* respectively referred to the origin *A* (Figure 8.28).

Figure 8.28

Then the position vector of *C* (using triangle or parallelogram law of addition) is $\boldsymbol{b} + \boldsymbol{d}$.

Also $\quad \boldsymbol{BD} = \boldsymbol{d} - \boldsymbol{b}$

Now *ABCD* is a rhombus.

$$\therefore \quad \boldsymbol{AB} = \boldsymbol{AD} \Rightarrow \boldsymbol{AB}^2 = \boldsymbol{AD}^2$$

$$\Rightarrow \boldsymbol{b}^2 = \boldsymbol{d}^2$$

Now $\quad \boldsymbol{AC} = \boldsymbol{b} + \boldsymbol{d}$.

$$\therefore \quad \boldsymbol{AC} \cdot \boldsymbol{BD} = (\boldsymbol{b} + \boldsymbol{d}) \cdot (\boldsymbol{d} - \boldsymbol{b})$$

$$= (\boldsymbol{d} + \boldsymbol{b}) \cdot (\boldsymbol{d} - \boldsymbol{b})$$

$$= \boldsymbol{d}^2 - \boldsymbol{b}^2 = 0$$

Hence $\boldsymbol{AC}$ is perpendicular to $\boldsymbol{BD}$.

Thus diagonals *AC* and *BD* of the rhombus *ABCD* are at right angles.

Dot multiplication of two vectors gives the product as a scalar. Various applications suggest another kind of multiplication of vectors such that the product is again a vector. We next take up such **vector product** or **cross product** between two vectors. Cross products play an important role in the study of electricity and magnetism.

8.5.2 Vector Product of Two Vectors

The ***vector product*** *or the* ***cross product*** *of two vectors* $\boldsymbol{a}$ *and* $\boldsymbol{b}$*, denoted by* $\boldsymbol{a} \times \boldsymbol{b}$*, is defined as*

$$\boldsymbol{a} \times \boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \sin \theta\ \hat{n}, \qquad \ldots (8.9)$$

where $|\boldsymbol{a}|$ *and* $|\boldsymbol{b}|$ *are the magnitudes of the vectors* $\boldsymbol{a}$ *and* $\boldsymbol{b}$ *respectively,* θ *is the angle between vectors* $\boldsymbol{a}$ *and* $\boldsymbol{b}$ *and* $\hat{\boldsymbol{n}}$ *is a unit vector perpendicular to both* $\boldsymbol{a}$ *and* $\boldsymbol{b}$ *and is such that* $\boldsymbol{a}, \boldsymbol{b}, \hat{\boldsymbol{n}}$*, in this order, form a right-handed triple or right-hand triod* (Figure 8.29).

Figure 8.29 : Vector Product

The term *right-handed* comes from the fact that the vectors $\boldsymbol{a}$, $\boldsymbol{b}$, $\hat{\boldsymbol{n}}$, in this order, assume the same sort of a orientation as the thumb, index finger, and middle finger of right hand when these are held as shown in Figure 8.30(a). Let us now look at the screw shown in Figure 8.30(b). You may observe that if $\boldsymbol{a}$ is rotated in the direction of $\boldsymbol{b}$ through an angle θ ($< \pi$), then $\hat{\boldsymbol{n}}$ advances in a direction pointing towards the reader or away from the reader accordingly as the screw is right-handed or left-handed. On the same account an ordered vector triad, $\boldsymbol{a}$, $\boldsymbol{b}$, $\hat{\boldsymbol{n}}$ is **right-handed** or **left-handed**.

(a) Right-handed Triple of Vector $\boldsymbol{a}, \boldsymbol{b}, \hat{\boldsymbol{n}}$ **(b) Right-handed Screw**

Figure 8.30

You may note here that

(i) $0 \leq \theta \leq \pi$.

(ii) $\hat{\boldsymbol{n}}$ is perpendicular to the plane which contains $\boldsymbol{a}$ and $\boldsymbol{b}$ both.

(iii) Vector product of two vectors is always a vector quantity.

(iv) The sine of the angle between two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ is given by $\sin \theta = |\boldsymbol{a} \times \boldsymbol{b}| / (|\boldsymbol{a}|\,.\,|\boldsymbol{b}|)$.

Vector product $\boldsymbol{a} \times \boldsymbol{b}$ is also called the **cross product** because of the notation used and is read as $\boldsymbol{a}$ cross $\boldsymbol{b}$.

From the definition of vector product, you know that $\boldsymbol{b} \times \boldsymbol{a}$ is a vector of magnitude $|\boldsymbol{a}|\,|\boldsymbol{b}| \sin\theta$ and is normal to $\boldsymbol{a}$ and $\boldsymbol{b}$ and in a direction such that $\boldsymbol{b}$, $\boldsymbol{a}$ and $\boldsymbol{b} \times \boldsymbol{a}$ form a right-handed system (Figure 8.31). This is possible only if $\boldsymbol{b} \times \boldsymbol{a}$ is opposite to the direction of $\boldsymbol{a} \times \boldsymbol{b}$. Since $\boldsymbol{b} \times \boldsymbol{a}$ and $\boldsymbol{a} \times \boldsymbol{b}$ have equal magnitudes, thus

$$\boldsymbol{b} \times \boldsymbol{a} = -\,\boldsymbol{a} \times \boldsymbol{b},$$

which shows that the cross-product of vectors is not commutative.

Figure 8.31

Hence the order of the vectors in a vector product is of great importance and must be carefully observed.

It may be observed that the unit vector normal to both $\boldsymbol{a}$ and $\boldsymbol{b}$, namely, $\hat{\boldsymbol{n}}$ is given by

$$\hat{\boldsymbol{n}} = \frac{\boldsymbol{a} \times \boldsymbol{b}}{|\boldsymbol{a} \times \boldsymbol{b}|}$$

If the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are parallel (or collinear) then $\theta = 0$ or $180^\circ \Rightarrow \sin\theta = 0$.

Hence $\boldsymbol{a} \times \boldsymbol{b} = \boldsymbol{0}$ is the condition for the two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ to be parallel.

In particular $\boldsymbol{a} \times \boldsymbol{a} = \boldsymbol{0}$.

You know that $\hat{\boldsymbol{i}}, \hat{\boldsymbol{j}}, \hat{\boldsymbol{k}}$, represent unit vectors along the axes of a cartesian coordinate system. Also since $\hat{\boldsymbol{i}}, \hat{\boldsymbol{j}}, \hat{\boldsymbol{k}}$ in this order, form a right-handed system of mutually perpendicular vectors, therefore, $\hat{\boldsymbol{i}} \times \hat{\boldsymbol{j}}$ is a vector modulus as unity and direction parallel to $\hat{\boldsymbol{k}}$.

Thus $\qquad \hat{\boldsymbol{i}} \times \hat{\boldsymbol{j}} = \hat{\boldsymbol{k}} = -\,\hat{\boldsymbol{j}} \times \hat{\boldsymbol{i}}$

Similarly $\qquad \hat{\boldsymbol{j}} \times \hat{\boldsymbol{k}} = \hat{\boldsymbol{i}} = -\,\hat{\boldsymbol{k}} \times \hat{\boldsymbol{j}}$

and $\qquad \hat{\boldsymbol{k}} \times \hat{\boldsymbol{i}} = \hat{\boldsymbol{j}} = -\,\hat{\boldsymbol{i}} \times \hat{\boldsymbol{k}}$

Also $\qquad \hat{\boldsymbol{i}} \times \hat{\boldsymbol{i}} = \hat{\boldsymbol{j}} \times \hat{\boldsymbol{j}} = \hat{\boldsymbol{k}} \times \hat{\boldsymbol{k}} = \boldsymbol{0}$.

From the definition of cross product, it follows that for any constant λ

$$(\lambda \boldsymbol{a}) \times \boldsymbol{b} = \lambda\,(\boldsymbol{a} \times \boldsymbol{b}) = \boldsymbol{a} \times (\lambda \boldsymbol{b})$$

Further, cross-multiplication is distributive w. r. t. vector addition, i.e.

$$\boldsymbol{a} \times (\boldsymbol{b} + \boldsymbol{c}) = \boldsymbol{a} \times \boldsymbol{b} + \boldsymbol{a} \times \boldsymbol{c}$$

$$(\boldsymbol{a} + \boldsymbol{b}) \times \boldsymbol{c} = \boldsymbol{a} \times \boldsymbol{c} + \boldsymbol{b} \times \boldsymbol{c}$$

Cross multiplication has a very unusual and important property, namely, *cross multiplication is not associative*, i.e. in general

$$\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) \neq (\boldsymbol{a} \times \boldsymbol{b}) \times \boldsymbol{c}$$

For instance, $\hat{\boldsymbol{i}} \times (\hat{\boldsymbol{i}} \times \hat{\boldsymbol{j}}) = \hat{\boldsymbol{i}} \times \hat{\boldsymbol{k}} = -\hat{\boldsymbol{j}}$

whereas $(\hat{\boldsymbol{i}} \times \hat{\boldsymbol{i}}) \times \hat{\boldsymbol{j}} = \mathbf{0} \times \hat{\boldsymbol{j}} = \mathbf{0}$

From the definition of cross-product and dot product of two vectors, we have

$$\begin{aligned} |\boldsymbol{a} \times \boldsymbol{b}|^2 &= |\boldsymbol{a}|^2\, |\boldsymbol{b}|^2 \sin^2\theta \\ &= |\boldsymbol{a}|^2\, |\boldsymbol{b}|^2 (1 - \cos^2\theta) \\ &= |\boldsymbol{a}|^2\, |\boldsymbol{b}|^2 - [|\boldsymbol{a}|\, |\boldsymbol{b}| \cos\theta]^2 \\ &= (\boldsymbol{a} \,.\, \boldsymbol{a})\, (\boldsymbol{b} \,.\, \boldsymbol{b}) - (\boldsymbol{a} \,.\, \boldsymbol{b})^2 \end{aligned}$$

From the above identity, we obtain a useful formula for the modulus of a vector product as

$$|\boldsymbol{a} \times \boldsymbol{b}| = \sqrt{(\boldsymbol{a} \,.\, \boldsymbol{a})\, (\boldsymbol{b} \,.\, \boldsymbol{b}) - (\boldsymbol{a} \,.\, \boldsymbol{b})^2}$$

Before taking up geometrical interpretation of vector product we list all the properties of vector product discussed above for the ready reference.

(I) $\boldsymbol{a} \times \boldsymbol{b} = -\, \boldsymbol{b} \times \boldsymbol{a}$,

i.e. vector product is not commutative.

(II) $\boldsymbol{a} \times \boldsymbol{b} = (|\boldsymbol{a}|\, |\boldsymbol{b}| \sin 0)\, \hat{n} = 0,$ if $\boldsymbol{a}$ is parallel to $\boldsymbol{b}$.

Vector product of parallel vectors is zero. In particular, $\boldsymbol{a} \times \boldsymbol{a} = 0$ and $\boldsymbol{a} \times 0 = 0$.

(III) If m and n are scalars then $(m\boldsymbol{a} \times n\boldsymbol{b}) = mn\, (\boldsymbol{a} \times \boldsymbol{b})$

(IV) For a right handed triad of unit vectors $\hat{\boldsymbol{i}}, \hat{\boldsymbol{j}}, \hat{\boldsymbol{k}}$

$$\hat{\boldsymbol{i}} \times \hat{\boldsymbol{i}} = \hat{\boldsymbol{j}} \times \hat{\boldsymbol{j}} = \hat{\boldsymbol{k}} \times \hat{\boldsymbol{k}} = \mathbf{0}$$

$$\hat{\boldsymbol{i}} \times \hat{\boldsymbol{j}} = \hat{\boldsymbol{k}},\ \hat{\boldsymbol{j}} \times \hat{\boldsymbol{k}} = \hat{\boldsymbol{i}},\ \hat{\boldsymbol{k}} \times \hat{\boldsymbol{i}} = \hat{\boldsymbol{j}}$$

$$\hat{\boldsymbol{j}} \times \hat{\boldsymbol{i}} = -\hat{\boldsymbol{k}},\ \hat{\boldsymbol{k}} \times \hat{\boldsymbol{j}} = -\hat{\boldsymbol{i}},\ \hat{\boldsymbol{i}} \times \hat{\boldsymbol{k}} = -\hat{\boldsymbol{j}}$$

(V) Vector product is distributive w. r. t. vector addition, i.e., $\boldsymbol{a} \times (\boldsymbol{b} + \boldsymbol{c}) = \boldsymbol{a} \times \boldsymbol{b} + \boldsymbol{a} \times \boldsymbol{c}$.

(VI) $\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) \neq (\boldsymbol{a} \times \boldsymbol{b}) \times \boldsymbol{c}$ i.e., vector product is not associative.

We next take up geometrical interpretation of vector product.

Geometrical Interpretation of Vector Product

Consider a parallelogram *OACB* with $\boldsymbol{a}$ and $\boldsymbol{b}$ as adjacent sides.

Let $\boldsymbol{a} = \boldsymbol{OA},\ \boldsymbol{b} = \boldsymbol{OB}$

Figure 8.32

and let BN be perpendicular to OA from B

Let $\quad \angle BON = \theta$

$\therefore \quad BN = OB \sin\theta$

$= |\, b \,| \sin\theta$

Now $\quad |\boldsymbol{a} \times \boldsymbol{b}| = |\boldsymbol{a}|\,|\boldsymbol{b}| \sin\theta$

$= OA \,.\, BN =$ Area of the parallelogram $OACB$.

Thus, the magnitude of $\boldsymbol{a} \times \boldsymbol{b}$ is equal to the area of the parallelogram whose adjacent sides are the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$.

The sign may also be assigned to the area. When a person travels along the boundary and the area lies to his left side, the area is positive and if the area lies to his right side, then the area is negative.

Thus $\quad \boldsymbol{a} \times \boldsymbol{b} =$ vector area of the parallelogram $OACB$

and $\quad \boldsymbol{b} \times \boldsymbol{a} =$ vector area of the parallelogram $OBCA$.

Example 8.9

Show that the vectors $\boldsymbol{A} = \hat{i} - 5\hat{j}$ and $\boldsymbol{B} = 2\hat{i} - 10\hat{j}$ are parallel to each other.

Solution

Here $\boldsymbol{A} = \hat{i} - 5\hat{j}$; $\boldsymbol{B} = 2\hat{i} - 10\hat{j}$. If vectors $\boldsymbol{A}$ and $\boldsymbol{B}$ are parallel, then $\boldsymbol{A} \times \boldsymbol{B} = \boldsymbol{0}$.

Now, $$\boldsymbol{A} \times \boldsymbol{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -5 & 0 \\ 2 & -10 & 0 \end{vmatrix}$$

$$= \hat{i}\,[-5 \times 0 - 0 \times (-10)] + \hat{j}\,[0 \times 2 - 1 \times 0] + \hat{k}\,[1 \times (-10) - (-5) \times 2]$$

$$= \hat{i}\,(0) + \hat{j}\,(0) + \hat{k}\,(0)$$

$$= \boldsymbol{0}$$

Hence, vectors $\boldsymbol{A}$ and $\boldsymbol{B}$ are parallel to each other.

Example 8.10

Find a unit vector perpendicular to both of the vectors $\boldsymbol{a} = 2\hat{\boldsymbol{i}} + \hat{\boldsymbol{j}} - \hat{\boldsymbol{k}}$ and $\boldsymbol{b} = \hat{\boldsymbol{i}} - \hat{\boldsymbol{j}} + 2\hat{\boldsymbol{k}}$.

Solution

$\boldsymbol{a} \times \boldsymbol{b} = (2i + j - k) \times (i - j + 2k)$

$= 2\,(i \times i) - 2\,(i \times j) + 4\,(i \times k) + (j \times i) - (j \times j) + 2\,(j \times k)$

$- (k \times i) + (k \times j) - 2\,(k \times k)$

$= 2 \times 0 - 2k - 4j - k - 0 + 2i - j - i - 2\,.\,0$

$= (2 - 1)i - (4 + 1)j + (-2 - 1)k$

$= \hat{\boldsymbol{i}} - 5\,\hat{\boldsymbol{j}} - 3\,\hat{\boldsymbol{k}}$

Unit vectors of $\boldsymbol{a} \times \boldsymbol{b} = \boldsymbol{c} = \dfrac{\hat{\boldsymbol{i}} - 5\,\hat{\boldsymbol{j}} - 3\,\hat{\boldsymbol{k}}}{|\boldsymbol{a} \times \boldsymbol{b}|}$ (say)

$$= \frac{\hat{\boldsymbol{i}} - 5\,\hat{\boldsymbol{j}} - 3\,\hat{\boldsymbol{k}}}{\sqrt{1 + 25 + 9}} = \frac{\hat{\boldsymbol{i}} - 5\,\hat{\boldsymbol{j}} - 3\,\hat{\boldsymbol{k}}}{\sqrt{35}}$$

$\boldsymbol{a} \times \boldsymbol{b}$ is a vector perpendicular to both the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ $\therefore$ $\boldsymbol{c}$ is the unit vector perpendicular to both $\boldsymbol{a}$ and $\boldsymbol{b}$.

SAQ 3

(i) If $\boldsymbol{c} = 3\boldsymbol{a} + 4\boldsymbol{b}$ and $2\boldsymbol{c} = \boldsymbol{a} - 3\boldsymbol{b}$, then show that $\boldsymbol{c}$ and $\boldsymbol{a}$ are like vectors and $|\,\boldsymbol{c}\,| > |\,\boldsymbol{a}\,|$.

(ii) If *ABCDEF* is a regular hexagon, prove that $\boldsymbol{AB} + \boldsymbol{AC} + \boldsymbol{AD} + \boldsymbol{AE} + \boldsymbol{AF} = 3\,\boldsymbol{AD} = 6\,\boldsymbol{AO}$ where O is the center of the hexagon.

(iii) If $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$, $\boldsymbol{d}$ are the vectors forming the consecutive sides of a quadrilateral, prove that $\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c} + \boldsymbol{d} = 0$.

(iv) Show that the vectors $\boldsymbol{A} = 2\hat{i} - 3\hat{j} - \hat{k}$ and $\boldsymbol{B} = -6\hat{i} + 9\hat{j} + 3\hat{k}$ are parallel.

(v) If $\boldsymbol{A} = 4\hat{i} + 6\hat{j} - 3\hat{k}$ and $\boldsymbol{B} = -2\hat{i} - 5\hat{j} + 7\hat{k}$, find the angle between the vectors $\boldsymbol{A}$ and $\boldsymbol{B}$.

SAQ 4

(i) Find the components and magnitude of the vector $\boldsymbol{PQ}$ where P and Q are the points (– 1, – 2, 4) and (2, 0, – 2) respectively.

(ii) Show that the vectors $\boldsymbol{a} = 3i - 2j + k$, $\boldsymbol{b} = i - 3j + 5k$, $\boldsymbol{c} = 2i + j - 4k$ form a right angled triangle.

(iii) Prove that the line joining the mid-points of two sides of a triangles is parallel and half to the third.

(iv) If the mid-points of consecutive sides of any quadrilateral are connected by straight lines, prove that the resulting quadrilateral is a parallelogram.

SAQ 5

(i) Find the projection of the vector

$$i + j + k \quad \text{on} \quad 4i + 4j + 5k$$

(ii) Find the scalar *m* so that the vectors $2i + j - mk$ is perpendicular to the sum of the vectors $i + j + 2k$ and $3i + 2j + k$.

(iii) Find a vector of magnitude 9 which is perpendicular to both the vectors $4i - j + 3k$ and $-2i + j - 2k$.

(iv) Show that $(\boldsymbol{a} - \boldsymbol{b}) \times (\boldsymbol{a} + \boldsymbol{b}) = 2\boldsymbol{a} \times \boldsymbol{b}$.

8.6 SUMMARY

In this unit, you have learnt

- A physical quantity completely specified by a single number (with a suitable choice of units of measure) is called a *scalar*.
- Quantities specified by a magnitude and a direction are called *vector quantities*.
- Length, support and sense characterize a *directed line segment*. Length of a directed line segment is called *magnitude or modulus* or norm of the vector it represents. *Direction* of a vector is from its initial to terminal point.
- A vector whose length is zero is called a *null vector*.
- A vector whose length is unity is called *unit vector*.
- All vectors having the same initial point are called *coinitial vectors*.
- Vectors having the same or parallel line of action are called *like* or *parallel* or *collinear vectors* and vectors are called *unlike* if they have opposite directions.
- Two vectors are said to be equal if they have the same length, same or parallel supports and the same sense.
- Projections of a vector on the axes of an orthogonal cartesian coordinate system are called its components. If $\boldsymbol{a} = a_1 \hat{\boldsymbol{i}} + a_2 \hat{\boldsymbol{j}} + a_3 \hat{\boldsymbol{k}}$ in components form, then

$$|\boldsymbol{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

and direction cosines of the vector are $\dfrac{a_1}{|\boldsymbol{a}|}, \dfrac{a_2}{|\boldsymbol{a}|}, \dfrac{a_3}{|\boldsymbol{a}|}$.

- If initial point of a vector is chosen to be the origin of a cartesian coordinate system, then components of the vector are the coordinates of its terminal point and the vector is called *position vector* of its terminal point.
- Two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ can be added graphically using *triangles law* or *parallelogram law of vector addition*.
- *Vector addition is commutative and associative*. There exist *additive identity* (0) *and additive increase* (negative of the vector).
- The difference $\boldsymbol{a} - \boldsymbol{b}$ of two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ is obtained by adding (–$\boldsymbol{b}$) to $\boldsymbol{a}$.
- Given $\boldsymbol{a} = a_1\,\hat{\boldsymbol{i}} + a_2\,\hat{\boldsymbol{j}} + a_3\,\hat{\boldsymbol{k}}, \boldsymbol{b} = b_1\,\hat{\boldsymbol{i}} + b_2\,\hat{\boldsymbol{j}} + b_3\,\hat{\boldsymbol{k}}$,
  $\boldsymbol{a} \pm \boldsymbol{b} = (a_1 \pm b_1)\,\hat{\boldsymbol{i}} + (a_2 \pm b_2)\,\hat{\boldsymbol{j}} + (a_3 \pm b_3)\,\hat{\boldsymbol{k}}$.
- If m is a scalar, and $\boldsymbol{a}$ is a vector, then $m\boldsymbol{a}$ is a vector whose magnitude
  $= |m|\,|\boldsymbol{a}|$, support is same or parallel to $\boldsymbol{a}$ and direction of vector $m\,\boldsymbol{a}$ is same as $\boldsymbol{a}$ if $m > 0$ and opposite to $\boldsymbol{a}$ if $m < 0$.
- The scalar product of two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ is defined as
  $\boldsymbol{a} \,.\, \boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos\gamma$, where γ $(0 \le \gamma \le \pi)$ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.
  In component form, $\boldsymbol{a} \,.\, \boldsymbol{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$.
- Two non-zero vectors are perpendicular to each other if and only if their dot product is zero.
- $|\boldsymbol{a}| = \sqrt{\boldsymbol{a} \,.\, \boldsymbol{a}}$ and $\cos\gamma = \dfrac{\boldsymbol{a} \,.\, \boldsymbol{b}}{|\boldsymbol{a}|\,|\boldsymbol{b}|}$ for angle γ between $\boldsymbol{a}$ and $\boldsymbol{b}$.
- The vector product of two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ is defined as

  $$\boldsymbol{a} \times \boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \sin\gamma\ \hat{\boldsymbol{n}}, \qquad (0 \le \gamma \le \pi),$$

  where γ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$ and $\hat{\boldsymbol{n}}$ is a unit vector perpendicular to both $\boldsymbol{a}$ and $\boldsymbol{b}$ in the direction such that $\boldsymbol{a}, \boldsymbol{b}, \hat{\boldsymbol{n}}$ form a right-handed triad.
- $\boldsymbol{a} \times \boldsymbol{b} = \boldsymbol{0}$ is the condition for vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ to be parallel.
- Geometrically, $\boldsymbol{a} \times \boldsymbol{b}$ represents the vector area of the parallelogram having adjacent sides represented by $\boldsymbol{a}$ and $\boldsymbol{b}$.

8.7 ANSWERS TO SAQs

SAQ 1

(i) (b)

(ii) (c)

SAQ 2

(i) (d)

(ii) $-5\hat{i} + 2\hat{j} - 4\hat{k}$

SAQ 3

(v) 148.8^{o}

SAQ 4

(i) $\boldsymbol{PQ} = 3\hat{\boldsymbol{i}} + 2\hat{\boldsymbol{j}} - 6\hat{\boldsymbol{k}}$ and $|\boldsymbol{PQ}| = 7$.

SAQ 5

(i) $\frac{13}{57}(4i + 4j + 5k)$

(ii) $m = 3$

(iii) $3(-i + 2j + 2k)$

MATHEMATICS-I

The concepts of mathematics are the backbone of any engineering problem solving. This makes the study of mathematical concepts a must for you. Accordingly, Mathematics-I is the first basic mathematics course for you which has in all 8 units covering topics on Algebra (Units 1, 2 and 3), Trigonometry (Units 4 and 5), Coordinate Geometry (Units 6 and 7) and Vector Algebra (Unit 8).

In Unit 1, the notion of surds, logarithms and quadratic equations have been introduced and the basic laws of surds and logarithms and the various techniques of solving quadratic equations have been discussed. In Unit 2, the notion of sequences and series have been defined and particularly this unit is devoted to the study of arithmetic series and geometric series. Binomial Theorem and Computer Mathematics have been introduced in Unit 3. Binomial theorem for Positive Integral Index is also discussed in detail in this unit.

Unit 4 deals with various systems of measuring angles and Trigonometric (circular) functions. In this unit, various properties of trigonometric functions have been developed. In Unit 5, relations between sides and angles of a triangle are discussed and application of trigonometry for solving some problems of height and distances are included.

In Unit 6, the Cartesian Coordinate System has been developed and Unit 7 has been devoted to representing Lines, Circle and Conic Sections by algebraic equations.

In Unit 8, we have presented the basic concepts of vectors, the different operations on vectors and the vector products of two vectors have been discussed.

For the want of clarity in concepts, number of solved examples have been included in each unit. To help you to check your understanding and to assess yourself, each unit contains SAQs. The answers to these SAQs are given at the end of each unit. We suggest that you look at them only after attempting the exercises.

At the end, we wish you all the best for your all educational endeavours.