UNIT 1 PROPERTIES OF MATTER

Structure

1.1 INTRODUCTION

You may recall from your school physics course that matter exists in any one of the three states namely solid, liquid and gas. At the atomic level, these states of matter are distinguished from each other in terms of the nature of bonding among their atoms or molecules. In solids, atoms or molecules are strongly bound to each other and are almost fixed at their positions. In liquids, atoms or molecules are not bound to each other as strongly as in the case of solids and they are somewhat free to move inside the whole mass of the liquid. In gases, atoms or molecules are not bound to each other and are completely free to move around in the entire volume of the container. Due to unique nature of their atomic or molecular bondings, solids, liquids and gases possess characteristics macroscopic properties. In this unit, you will study some of these properties of matter such as surface tension, viscosity, hydrostatic pressure and elasticity. These properties have wide ranging applications in engineering, industry and even in our day-to-day lives.

Due to weaker intermolecular forces, the molecules of a liquid can move about freely. This gives rise to surface tension – a characteristic property of stationary liquids which helps us understand phenomenon like the formation of water droplets, how the mosquitoes stand and walk on still water, and why mercury spreads in the form of spherical globules on the flat ground. You will learn surface tension in Section 1.2.

One of the important discoveries of science is that fluids – liquids and gases – exert pressure. We talk of hydrostatic pressure in case of liquid at rest and atmospheric pressure in case of air in our atmosphere. This property of fluids has contributed significantly in the industrial progress and sophistication in machines. You will learn the characteristics of fluids at rest in Section 1.3. In Section 1.4, you will study viscosity – a property of fluids in motion – which is a measure of fluids' resistance to flow. Understanding of viscosity enables us to explain lubrication, fluid flow in pipes or blood vessels and sedimentation rates.

In Section 1.5, you will study an important property of solids called elasticity. Elasticity of solids and related parameters are of vital importance for the construction of buildings, machines, bridges etc. For example, elastic properties of steel help us determine the size and shape of a steel beam needed to safely support a given load.

Objectives

After studying this unit, you should be able to

- use the concept of surface tension to explain various day-to-day observations,
- determine the hydrostatic pressure at a point inside a liquid,
- state and explain the Pascal's law and the Archimedes Principle,
- derive the equation of continuity and Bernoulli's equation for fluids and discuss their applications,
- understand the concept of viscosity and explain the associated terms like critical velocity and Reynolds's number,
- define elasticity and other parameters related to it,
- state the Hooke's law, and
- define the Young's modulus, bulk modulus, modulus of rigidity and Poisson's ratio.

1.2 SURFACE TENSION

You might have observed some of the interesting characteristics of liquids. For example, drops of mercury on a plane surface always assume spherical shape. Similarly, raindrops and dewdrops on the leaves of plants appear spherical. Did you ever ask yourself : **Why a small amount of liquid gather together into a spherical drop**? Further, you also might have observed that small insects could move on the surface of water without sinking! Similarly, sewing needle floats on the surface of water. On the other hand, heavier objects can neither walk nor float on the water surface; they simply sink. On the basis of these observations, you may conclude that :

- there exists a force along the surface of the liquid which tends to shrink the liquid surface so that it has minimum area, and
- the surface of the liquid behaves like a stretched membrane.

A logical question you may ask now is : **Why does a liquid surface behave in this manner**? What causes such behaviour of liquids? The phenomenon responsible for above characteristics of liquids is called **surface tension.** Due to this phenomenon, the free surface of a liquid behaves like a stretched elastic membrane tending to contract so as to have minimum surface area.

Now, let us first understand this phenomenon qualitatively. You know that all substances are made of molecules and the molecules interact with each other. The intermolecular force which enables same types of molecules of a given substance to attract each other is called **cohesive force** and this process is called **cohesion**. In solids, the cohesive forces are very strong. On the other hand, cohesive forces are weak for liquids and weakest for gases. That is why solids have definite shape, liquids have definite free surface and gases have neither. The force of attraction or repulsion between unlike molecules is called **adhesive force** and the process is called **adhesion**. Adhesive forces come into play at the common surface of two different substances. For example, glue adheres to wood, solder adheres to brass, water adheres to glass etc. On the basis of these concepts, you can understand the surface tension related characteristics of liquids mentioned above. Let us discuss some of them now.

Due to the cohesive force, molecules at the surface of a liquid attract each other more strongly than the molecules in the interior of the liquid (Figure 1.1). It is so because the surface molecules do not have neighbouring molecules above the surface and, therefore, there are lesser number of molecules to share the cohesive force. This enhancement of cohesive forces among the molecules at the surface gives rise to a well-defined surface to liquids and the liquid surface behaves like a stretched membrane (something similar to a rubber sheet) tending to have minimum surface area. You may ask : **How does this explain spherical shape of a liquid drop**? Well, there is a little geometry involved here. You may recall that for a given volume, the surface area of a sphere is minimum. Thus, a drop of liquid must attain a spherical shape to have minimum surface area.

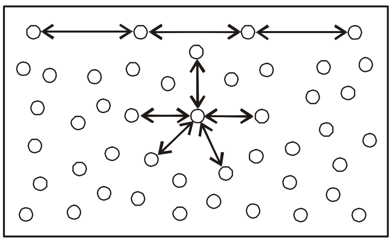


Figure 1.1 : Two-dimensional View of the Molecules on the Surface and Interior of a Liquid

When small insects move on a liquid surface or when we place a sewing needle on a liquid surface, the surface of water is slightly depressed under the feet of the insect or under the needle. At these points, the cohesive force between the surface molecules gives rise to a restoring force which is equal in magnitude and opposite in direction of the weight of the insect or the needle. As a result, cohesive force tends to restore the horizontal surface of the liquid. It is, however, important to mention here that when the weight of the insect or the needle become very large, the restoring force due to cohesion can no longer support them and they sink in the liquid.

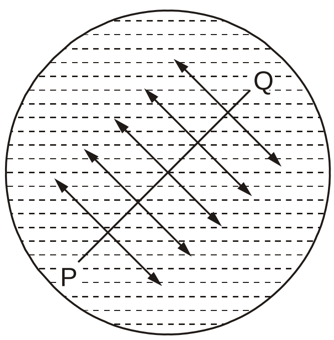


Figure 1.2 : Cohesive Force between Molecules on the Two Sides of a Line, *PQ*, on a Liquid Surface

To obtain an analytical expression for the surface tension, let us consider an imaginary line *PQ* drawn on the free surface of a liquid (Figure 1.2). Due to cohesive force, the molecules just lying on one side of the line *PQ* try to pull away from the molecules lying just on the other side so as to decrease the surface area. Surface tension is essentially a manifestation of the cohesive force among the surface molecules of the liquid. Hence, it is measured as the force per unit length acting perpendicularly on either side of any imaginary line in the liquid surface in equilibrium; the direction of the force being tangential to the surface. Mathematically, we write

$$\text{Surface Tension } (\sigma) = \frac{\text{Force}}{\text{Length}} = \frac{F}{l} \qquad \text{. . . (1.1)}$$

where F is the force acting on either side of the line *PQ* which is of length l. The unit of surface tension in SI system is Newton per metre (Nm^{-1}) and its dimensions are MT^{-2}.

It is important to mention here that the value of surface tension depends on the temperature of the liquid : the value diminishes with the rise in temperature. At 20°C, the values of surface tensions of water and mercury are given below :

$$(\sigma)_{\text{water}} = 72.6 \times 10^{-3}\ \text{N m}^{-1}$$

$$(\sigma)_{\text{mercury}} = 465 \times 10^{-3}\ \text{N m}^{-1}$$

Let us now discuss some more day-to-day observations which can be explained on the basis of surface tension.

Examples of Surface Tension

(a) You might have noticed that umbrellas, raincoats, tents and canvas have tiny holes in them because they are made of fabrics. Despite this, during rain, the water does not pass through these pores. Did you ever ask yourself why it is so? It is because the surface tension of water prevents it from passing through the fabric.

(b) One of the important consequences of surface tension is that the free surface of a liquid tries to have the minimum possible area. For a given volume, a sphere has the least surface area. Hence the liquid assumes a spherical shape. That is why the raindrops and the mercury globules are spherical in nature. **However, the spherical shape of liquid gets distorted if the mass is somewhat larger**. This is caused by the force of gravity. If the force of gravity is counter balanced, even a large mass of liquid will assume a perfectly spherical shape. An experiment to verify this was done by Platau. In Platau's experiment, a large drop of olive oil is introduced in a mixture of alcohol and water. The mixture has the same density as olive-oil. It is observed that the drop assumes a perfectly spherical shape (Figure 1.3). In this experiment, the effect of gravitational force on the drop of olive-oil is balanced by the upward thrust of the water-alcohol mixture on it.

(c) If you blow a soap bubble at the end of a thin glass tube and allow it to stay in this state for some time, you will observe that it gradually shrinks in volume. This happens because the surface tension of the surface of the bubble tends to reduce the surface area to a minimum.

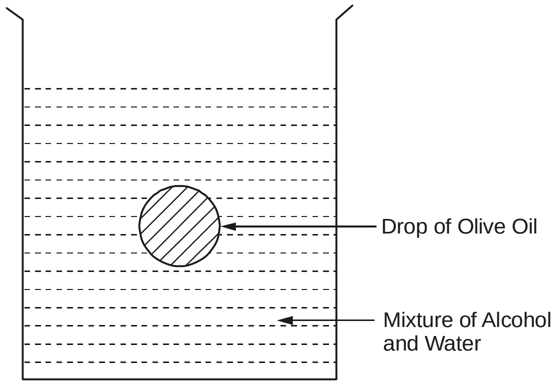


Figure 1.3 : Platau's Experiment; the Shape of Even a Large Drop of Liquidremains Spherical if the Effect of Gravity is Neutralised

(d) When you dip a camel-hair brush in water, you may observe that the individual hair gets separated and projects in different directions. However, as soon as the brush is removed from the water, the hairs are drawn together as if they are now connected by a stretched membrane.

(e) If you place a drop of oil on a water surface, you will observe that the oil drop is unable to maintain its spherical shape and it spreads over the surface of water. **Can you guess why does it happen**? If you are thinking that it is caused due to difference in the values of the surface tensions of the two (oil and water) liquids, you are correct. The surface tension of oil is much less than that of water. *The greater tension of water surface stretches the oil surface in all directions and oil drop spreads on the water surface.*

You know from the above discussion that, due to surface tension, a liquid tends to have minimum surface area. At this stage, an obvious question is : **What happens when the surface area of a liquid is increased**? This question can be answered in terms of the work done or the surface energy of the liquid surface. You will learn it now.

1.2.1 Surface Energy

If we wish to increase the area of the liquid surface, we will have to do work (apply force) to stretch it and this work will have to be done against the surface tension. To find an expression for the work to be done, consider a rectangular wire frame *MNPO* (Figure 1.4) in which the wire *OP* is movable. Let *l* is the

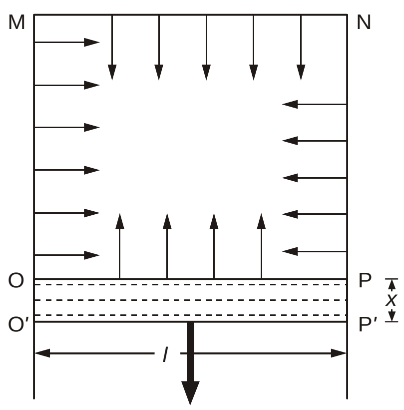


Figure 1.4 : Soap Film on a Rectangular Frame

length of the wire *OP*. If we dip this frame in soap solution, a soap film is formed. Due to surface tension, the soap film pulls the wire OP inward. From Eq. (1.1), this force can be written as :

$$\text{Force} = \sigma \times 2\,l \qquad \text{. . . (1.2)}$$

where σ denotes the surface tension of the film. Note that we have multiplied the length of the wire by a factor of 2 because the soap film touches the wire *OP* above as well as below. Suppose that a force is applied on the wire *OP* such that, the film stretches from position *OP* to *O′ P′* by a small distance *x*. Thus, the work done on the wire *OP* can be written as :

$$\begin{aligned}\text{Work Done} &= \text{Force} \times \text{Displacement}\\ &= \sigma \times 2l \times x\\ &= \sigma \times \Delta s \qquad \text{. . . (1.3)}\end{aligned}$$

where $\Delta s\ (= 2l \times x)$ is the total increase in the surface area of the soap film. **This work is stored as the surface energy in the soap film**. Further, Eq. (1.3) can be written as :

$$\begin{aligned}\text{Surface Tension } (\sigma) &= \frac{\text{Work Done}}{\text{Area}}\\ &= \frac{\text{Surface Energy}}{\text{Area}}\end{aligned}$$

Therefore, we can also define surface tension as surface energy per unit area of the surface.

You must have noticed that, in discussing surface tension, we used the concepts of mechanics such as force, work and energy. Similarly, other static and dynamic properties of liquids can be explained using the concepts of mechanics such as force, pressure, density and velocity. In the next section, you will study one of the most important properties namely hydrostatic pressure. You will also learn the Pascal’s law and Archimedes principle which govern the behaviour of fluids at rest.

1.3 FLUID STATICS

If one of your friends claims that she/he can lift an elephant, you will think she/he is joking. Actually, your friend is not joking : a person can indeed lift an elephant by his/her own body weight by standing on the platform of a hydraulic lift. A hydraulic lift is a machine which is based on the mechanical properties of a fluid in hydrostatic equilibrium. The study of the properties of a fluid in hydrostatic equilibrium is called **fluid statics**.

You may ask : **What is hydrostatic equilibrium**? This means that in a given volume of a fluid at rest, the individual fluid molecules may move continuously but the mass of the fluid as a whole has zero velocity and zero acceleration. In this section, we wish to study the behaviour of such fluids by asking ourselves : **What happens when force/pressure is applied on a fluid at rest**? Or, does fluids at rest exert pressure on a body immersed in it? Answers to these questions were discovered by Pascal and Archimedes. Before we discuss the Pascal’s law and the Archimedes principle, it is necessary to recapitulate the concept of pressure.

Pressure

You are familiar with the concepts of force and pressure from your school physics. To appreciate the difference between these two quantities, think about the following situations.

Why is it that camels walk easily in deserts on sand but for us it is quite difficult? Similarly, when we press a balloon filled with air, it does not burst easily. However, if we prick the balloon with a pin, the balloon bursts easily. *The clue to explain these observations is that besides force, we must also consider the area over which the force acts*. And, therein lies the concept of pressure which is defined as the ratio of the force to the area on which it acts; that is,

$$\text{Pressure } (P) = \frac{\text{Force } (F)}{\text{Area } (A)} \qquad \text{. . . (1.4)}$$

The unit of pressure is Nm^{-2} or $\text{kg m}^{-1}\text{s}^{-2}$. This unit is also known as **Pascal** denoted by Pa.

1.3.1 Pressure-Depth Relation

When an object is immersed in a fluid at rest, the fluid exerts pressure on it. The pressure on the object at any point inside the liquid depends on its depth from the free surface of the liquid. *It is so because the fluid pressure refers to the weight of the fluid above each square meter at that level*.

To find a relation between pressure and depth in a fluid, consider point A which is at the depth h below the free surface of water in a container (Figure 1.5). We consider a column of fluid over unit area at this point. Therefore, the height of this column is equal to the magnitude of its volume. (This is because volume is equal to the area of cross-section times height, that is, $V = a \times h$. Since we are considering a unit cross-sectional area, $a = 1$. Thus, $V = h$.) Now, mass of liquid in this column of unit cross-sectional area can be written as :

$$\text{Mass} = \text{Density} \times \text{Volume}$$

$$m = \rho \times V$$

$$= \rho \times h$$

Therefore, weight of the liquid in this column is :

$$\text{Weight} = m \times g$$

$$= \rho \times h \times g$$

where g is the acceleration due to gravity. The value of g is 9.8 m s^{-2}. Now, the pressure exerted by this column of fluid at point A is the force exerted by the liquid column at point A per unit area. Equivalently, we can say that the pressure at point A is equal to the weight of the liquid in a column of unit cross-sectional area and height h (vertical distance of the point A from the free surface of liquid). Thus, we can write :

$$P = h \rho g \qquad \text{. . . (1.5)}$$

Eq. (1.5) implies that the pressure of fluid is proportional to the height of the column : as the height of the liquid column (or the depth of the point under consideration) increases, the pressure increases. *Also note that the fluid pressure does not depend upon the total mass or total volume of the liquid*.

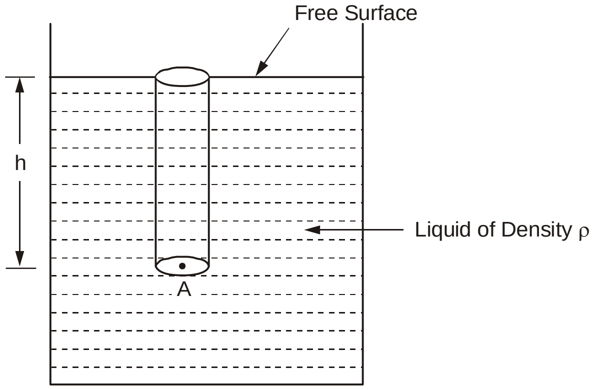


Figure 1.5 : Liquid Pressure at a Point *A* inside the Liquid

The pressure-depth relation of a fluid can be demonstrated using an apparatus shown in Figure 1.6. It consists of a cylindrical container in which there are small holes on the surface along a line parallel to its axis. Keeping all the holes closed, the container is filled with water and put on a flat surface. When all the holes are opened simultaneously, it is observed that the water jets coming out from different holes touches the surface at different distances from the axis of the container : the jet from the lowest hole falls farthest and that from the top most one falls closest to the base. This clearly shows that the pressure exerted by the liquid near the lower hole is highest and *vice-versa*.

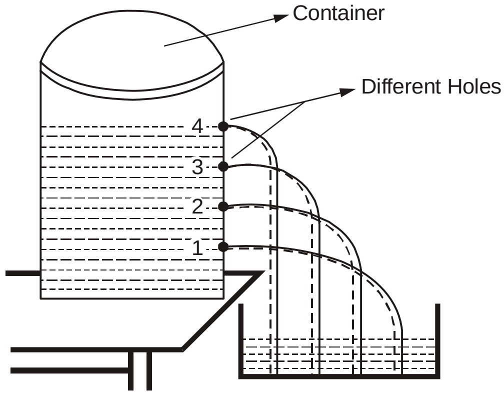


Figure 1.6 : Pressure of Liquid Coming out from Different Holes Depends on their Depth from the Free Surface of the Liquid

Atmospheric Pressure

As liquids exert pressure on object immersed in it, the air in the earth's atmosphere also exerts pressure on all of us at all times. *The pressure exerted by the atmospheric air at any point is equal to the weight of air contained in a column of unit cross-sectional area and extending up to the top of the atmosphere*.

The atmospheric pressure at the surface of the earth is approximately 10^5 Pa. At an altitude of 5 km, the pressure is approximately one half of the pressure at the surface. And, at an altitude of 31 km, pressure is only one percent of the pressure at the surface. The question is : **Why does atmospheric pressure decreases with altitude**? This is due to the fact that the height of the air column decreases with altitude which results in the decrease of weight of the air in a column of unit cross-sectional area. An instrument called **barometer** is used to measure the atmospheric pressure. Barometer measures the atmospheric pressure at a point in terms of the height of a mercury column. The standard atmospheric pressure at the sea level is equivalent to the pressure due to a mercury column of height 76 cm or 760 mm. The numerical value of the atmospheric pressure at the sea level is $1.013 \times 10^5 \ \text{N m}^{-2}$ or 10^5 Pa.

The hydrostatic pressure has some other characteristics. These were discovered on the basis of experimentations and a few laws were formulated. You will now learn some of them such as the Pascal's law and the Archimedes principle.

SAQ 1

Calculate value of the atmospheric pressure in Nm^{-2} at a point where the height of mercury column (in a barometer) is 76 cm. Take the density of mercury equal to 13,600 kgm^{-3} and the value of *g* equal to 9.8 ms^{-2}.

1.3.2 Pascal's Law

According to this law, "*when pressure is applied anywhere on the surface of an enclosed liquid at rest, an equal and uniform pressure is transmitted over the whole liquid; the pressure is transmitted throughout the liquid and acts in a direction at right angles to the surface of the liquid everywhere*".

Pascal's law can be demonstrated by taking a spherical flask filled with water and fitted with a piston (Figure 1.7). As shown in the figure, the flask has a number of small holes all around its circular surface. When the piston is gradually pushed-in, the water spreads out through different holes at almost the same speed.

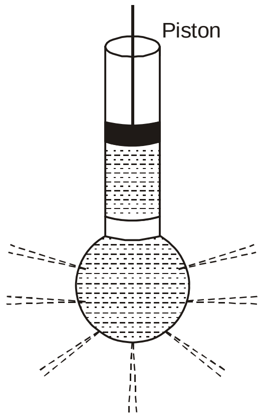


Figure 1.7 : Demonstration of Pascal's Law

Pascal's law has been put to a variety of uses in our everyday life. Some of the machines which operates on Pascal's law are the **hydraulic press** used to compress metal steels, wool etc., the **hydraulic brakes** used in cars, aircrafts etc., and the **hydraulic jack** used to lift vehicles in workshops. These machines are collectively referred to as **hydraulic machines**. Let us discuss the working of one of these hydraulic machines.

A **hydraulic jack** is used to lift heavy vehicles (like cars, trucks, buses etc.) to required heights in automobile workshops so that mechanic can do work conveniently under the vehicles. A force F_1 is applied on the piston of smaller area of cross-section A_1 (Figure 1.8). This pressure is *transmitted* to a piston of larger cross-sectional area (that is, the platform which lifts the vehicle) A_2. Now,

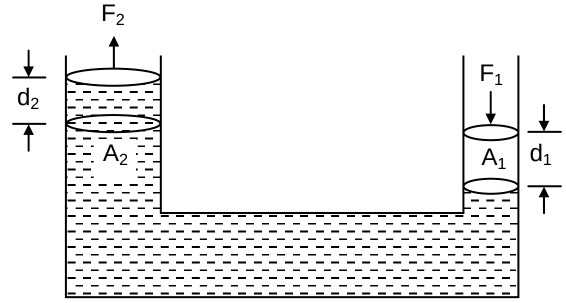


Figure 1.8 : A Schematic Diagram Showing the Principle of Operation of a Hydraulic Jack

according to Pascal's law, the pressure would remain the same in the larger piston. But, due to larger area of cross-section of this piston, the force F_2 exerted by it will be larger. *That is, if pressure remains the same, force will be proportional to the area of cross-section of the pistons as per the relation*, $F = PA$. However, you must note that the work done on the smaller piston will be the same as the work done by the larger piston; $F_1 d_1 = F_2 d_2$ where d_1 and d_2 are the displacements of the smaller and larger pistons respectively such that $d_1 > d_2$.

1.3.3 Archimedes' Principle

While you take bath, you might have noticed that a mug filled with water and fully immersed in bucket of water does not feel as heavy as when it is taken out completely from the bucket. You can feel the increase in weight as the object comes out of the water. **Do you know : why does this happen**? This happens because of **buoyancy**. Buoyancy (an upward thrust) arises because the fluid pressure increases with depth. Due to this, there is an unbalanced upward force called the **buoyant force** on the bottom of the object (like mug full of water) immersed in the liquid. The effect of buoyant force on objects immersed partially or completely in fluids is summarised in **Archimedes' principle**.

According to the Archimedes' principle, a body immersed in a fluid appears to loose weight and the apparent loss of weight is equal to the weight of the fluid displaced by it (*the body*). The apparent loss in weight of the body is caused by the upward thrust of the liquid on it, that is, the buoyancy.

Archimedes' principle enables us to determine the density of a material and it also helps us know whether or not a given object will float in a liquid. In order to prove that the floatation depends on the relative densities of the object and the liquid, let us consider a (solid) body of volume V and density ρ_S which is immersed in a liquid having density ρ_L. Let W_a is the weight of the body in air and W_L is the weight of the body when it is fully immersed in the liquid. Hence, the apparent loss of weight of the body $= W_a - W_L$. Now, according to the Archimedes' Principle, the weight $(= V \rho\ g)$ of the liquid displaced by the immersed body is equal to the apparent loss in the weight of the body; that is,

$$W_a - W_L = \rho_L\, g V$$

or,
$$\rho_s\, g V - W_L = \rho_L\, g V$$

or,
$$W_L = (\rho_s - \rho_L)\, V g \qquad \text{. . . (1.6)}$$

From Eq. (1.6), you may note that if $\rho_S < \rho_L$, W_L will be negative. *Since effective weight of the body cannot be negative,* ***the body*** *will rise to the surface instead of* *going to the bottom and a part of it will come out of the liquid. The body will stop rising when its effective weight becomes zero.* ***This is the condition of floatation***.

Further, the Archimedes' principle can also be used for the determination of density of a solid. You must have noticed that when a piece of wood or plastic or cork is put on the surface of water, it does not immerse fully; a part of the body remains outside above the free surface of water. Let V be the volume of the body placed on the surface of water so that its volume outside the water is $V - V_{sub}$, where V_{sub} is the volume of the submerged portion of the body. The loss in weight of the body will be equal to the weight of the water displaced, that is, $V_{sub}\, \rho_L\, g$. Therefore, we can write :

$$W_a - W_L = V_{sub}\, \rho_L\, g$$

or,

$$W_L = (\rho_S V - \rho_L V_{sub})\, g \qquad \text{. . . (1.7)}$$

Since the body is floating, we have $\rho_S < \rho_L$ and $V > V_{sub}$, we can find a value of V_{sub} (which is not fixed) such that $W_L = 0$ i.e., the apparent weight of the body within the liquid (or gas) is zero. In such as condition; we have from Eq. (1.7) :

$$\frac{V_{sub}}{V} = \frac{\rho_S}{\rho_L} \qquad \text{. . . (1.8)}$$

Eq. (1.8) can be used to determine the density of a liquid. To do so, we can take a block of wood which has a graduated scale on one side and let it float in the liquid whose density is to be determined. We note down the mark up to which the block dips in the liquid and determine V_{sub}. Thus, knowing V, V_{sub} and ρ_s, we can determine ρ_L using Eq. (1.8).

Till now, you studied the properties of fluids at rest and some of their applications. The next logical step is to understand the behaviour of fluids in motion. This is the subject matter of the next section. You should solve an SAQ before proceeding further.

SAQ 2

(a) A solid floats with one fourth of its volume above the surface of water. Calculate the density of the solid.

(b) A copper cube of mass 0.50 kg is weighed in water. The mass of the cube is found to be 0.40 kg. Is the cube hollow or solid? Take the densities of water and copper as 10^3 kgm^{-3} and 8.96×10^3 kgm^{-3} respectively.

(c) A piece of ice floats on water. What fraction of its volume will be above the surface of water?

Take the density of ice to be 0.92×10^3 kgm^{-3}.

1.4 FLUIDS IN MOTION

Whether it is the flow of water in rivers or in pipes, or the flow of blood in the blood vessels, all these motions of fluids are governed by the principles of fluid dynamics. In reality, the motion of fluids is a rather complex phenomenon. To keep our discussion of fluid motion simpler, we make certain simplifying assumptions as given below :

(a) **The fluid is incompressible**. This condition is ordinarily satisfied by liquids but not by gases. However, if a gas is not subjected to large change in pressure, it can also be considered incompressible.

(b) **The fluid is non-viscous**. That is, fluid motion does not suffer any friction. This condition is similar to ignoring rolling or sliding friction in mechanics. (The motion of viscous fluid has been discussed in the next section on viscosity.)

(c) **The flow of fluid is steady**. This means that the fluid velocity does not change with time at a given position. Note that this does not mean that the velocity of fluid is same at all positions in the body of the fluid.

(d) **The fluid motion is irrotational**. This implies that if a paddle wheel is placed in the flowing fluid, it will not rotate.

Above simplifying assumptions enable us to analyse the behaviour of a fluid in motion without using complex mathematical techniques. Further, before you study the dynamics of a incompressible, non-viscous, steady and irrotational fluid, it is advisable to understand the concepts of streamline motion and turbulent motion.

Streamline Motion

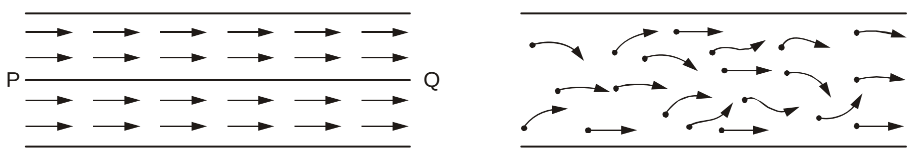
The flow of liquid is said to be streamline (motion) if

(a) the liquid particles move along fixed paths known as streamlines, and

(b) the velocity of the particles passing through a given point, one after the other, on a streamline remains unchanged in magnitude and direction at that point.

You may ask : What is a streamline? **A streamline is defined as the curve whose tangent at any point gives the direction of the liquid velocity at that point**. In other words, streamlines are the curves parallel to the direction of the fluid velocity at all points. Refer to Figure 1.9(a) which shows the flow of liquid through a straight tube. In steady flow, the streamlines such as *PQ* coincides with the line of flow.

One of the most important properties of streamlines is that the two streamlines never cross each other. Further, in a streamline motion, it is assumed that the entire thickness of the stream of the liquid is made up of a large number of plane layers, one flowing over the other. Such a flow is, therefore, also called **laminar flow**.



(a) **(b)**

Figure 1.9 : (a) Streamline and (b) Turbulent Flow of Liquid

Turbulent Motion

In turbulent motion, the velocity of a fluid particle passing through a given point is not the same as that of the particle which preceded it. In this kind of flow, the streamlines cross each other and the flow path becomes zigzag as shown in Figure 1.9(b).

With this background knowledge, you are now in a position to learn two basic equations namely the equation of continuity and the Bernoulli's equation governing fluid dynamics.

1.4.1 Equation of Continuity

The equation of continuity for a flowing fluid is essentially a consequence of the **conservation of mass**. To appreciate what does conservation of mass mean for fluid in motion, let us consider a pipe filled completely with an incompressible fluid. If more fluid enters the pipe from one end, an equal amount must leave from the other end, that is, the mass of the fluid in the pipe is conserved.

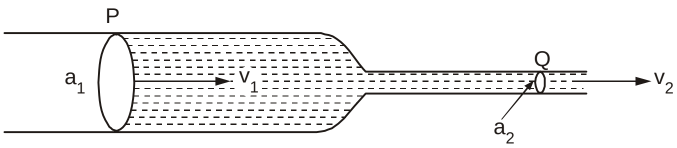


Figure 1.10 : Liquid Flowing through a Tube of Different Cross-sectional Areas

To obtain the equation of continuity, consider a tube which has different areas of cross-sections a_1 and a_2 at points P and Q respectively (Figure 1.10). Let a liquid flowing through this tube has velocities v_1 and v_2 respectively at points P and Q. The distance travelled by a particle of the liquid crossing point P in a small time interval δt is $v_1\,\delta t$. Considering all the particles crossing point P in this time interval δt, we can write the volume of liquid crossing point P as $a_1\ v_1\ \delta t$.

Further, if ρ_1 is the density of the liquid, mass of liquid crossing P in time δt is $\rho\ a_1\ v_1\ \delta t$. Similarly, in time δt, mass of liquid crossing point Q is $\rho\ a_2\ v_2\ \delta t$.

Now, according to the law of conservation of mass, the mass of liquid entering at point P must be equal to the mass leaving the point Q because there is no accumulation of mass between the points P and Q in the tube. Therefore, we can write :

$$\rho\ a_1\ v_1\ \delta t = \rho\ a_2\ v_2\ \delta t$$

or, $$a_1\ v_1 = a_2\ v_2$$

or, $$a\ v = \text{constant.} \qquad \text{. . . (1.9)}$$

Eq. (1.9) is called the **equation of continuity** for streamline flow of a liquid. One of the important consequences of the equation of continuity can be obtained if we write Eq. (1.9) as :

$$v_2 = v_1\ \frac{a_1}{a_2}$$

This implies that if $a_2 < a_1$ then $v_2 > v_1$. *That is, the velocity of liquid flow increases if the tube becomes narrower and vice-versa*. This explains why water from a tube falls at a larger distance if its outlet is made narrower.

An important advancement in the understanding of fluid dynamics was made by Bernoulli. In view of the fact that, like mechanical particles, fluid particles also obey the Newton's laws of motion, Bernoulli employed work-energy principles to investigate the behaviour of fluid in motion. This gave rise to the Bernoulli's equation which you will study now.

1.4.2 Bernoulli's Equation

To derive Bernoulli's equation, let us consider a tube *AB* in which a liquid is flowing as streamline flow (Figure 1.11). Let the density of the liquid is ρ and the heights of points *A* and *B* above ground level be h_1 and h_2 respectively. Let a_1 and a_2 be the areas of cross-sections of the tube at points *A* and *B* and p_1, p_2 be the pressures and v_1, v_2 be the velocities of flow at these points respectively.

Since the flow of liquid is streamline and in view of the equation of continuity, the net result of liquid flow in a time interval δt is transfer of mass, say *m*, from point *A* to point *B*. The question is : **How much work is done on the liquid of mass *m* and what is the change in its kinetic energy when it is transferred from point A to point B**?

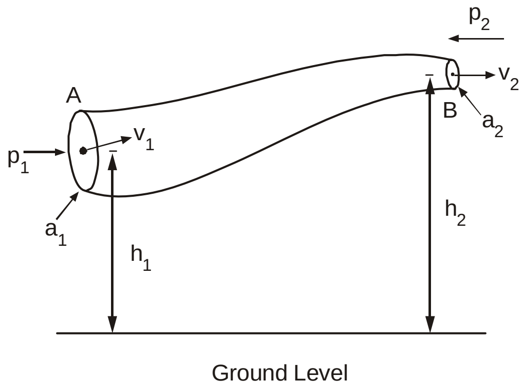


Figure 1.11 : Streamline Flow of Liquid through a Tube of Different Cross-sectional Areas at difficult Heights from the Ground Level

To find an answer to these questions, we need to

(a) calculate the change in the kinetic energy of mass *m* of the liquid in moving from point *A* to *B*, and

(b) calculate the work done on the system (mass *m* of the liquid) by (i) pressure difference between points *A* and *B*, and (ii) the gravity.

The change in kinetic energy of mass *m* is given by :

$$\Delta KE = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 \qquad \text{. . . (1.10)}$$

To obtain an expression for the work done on the liquid due to pressure at points *A* and *B*, let the pressure force at *A* displaces the fluid by a distance δx_1 and the corresponding displacement at point *B* is δx_2. So, the work done at $A = p_1 \; a_1 \; \delta x_1$ and work done at point $B = - \; p_2 \; a_2 \; \delta x_2$. The negative sign indicates that at point *B*, the pressure force $p_2 \, a_2$ is directed opposite to the displacement δx_2. So, the net work done on the liquid due to pressure forces is $(p_1 \; a_1 \; \delta x_1 - p_2 \; a_2 \; \delta x_2)$. Now, since $a_1 \; \delta x_1$ and $a_2 \; \delta x_2$ are the volumes of equal mass *m*, we may write, $a_1 \; \delta x_1 = a_2 \; \delta x_2 = \frac{m}{\rho}$. Thus, the net work done on the liquid of mass *m* by the liquid pressure can be written as :

$$(p_1 - p_2)\frac{m}{\rho} \qquad \ldots (1.11)$$

There is yet another kind of force acting on the fluid : the gravitational force, which contributes to the work done on the liquid of mass *m*. You may note from Figure 1.11 that mass *m* moves through a vertical distance $(h_2 - h_1)$ in going from the point *A* to *B*. Thus, work must be done on the liquid to move it from *A* to *B* against the gravitational force. The net work done on the system by the gravitational force can be written as :

$$-mg\,(h_2 - h_1) \qquad \ldots (1.12)$$

According to the work-energy principle, the change in kinetic energy of the system is equal to the net work done on the system by external forces. So, from Eqs. (1.10), (1.11) and (1.12), we can write :

$$\frac{1}{2}m\,(v_2^2 - v_1^2) = (p_1 - p_2)\frac{m}{\rho} + [-mg\,(h_2 - h_1)]$$

or,

$$\frac{p_1}{\rho} + gh_1 + \frac{v_1^2}{2} = \frac{p_2}{\rho} + gh_2 + \frac{v_2^2}{2} \qquad \ldots (1.13)$$

Eq. (1.13) is called **Bernoulli's equation**. It can also be written as :

$$\frac{p}{\rho} + gh + \frac{v^2}{2} = \text{constant.}$$

Multiplying both sides by ρ, we get :

$$p + \rho gh + \frac{1}{2}\rho v^2 = \text{constant.} \qquad \ldots (1.14)$$

We have written Bernoulli's equation in the form represented by Eq. (1.14) to show that it (Bernoulli's equation) is a statement of the fact that **the available energy per unit volume of a fluid remains constant along any given tube of flow**. This follows from the fact each term in Eq. (1.14) has the dimensions of energy. The term $\frac{1}{2}\rho v^2$ is the kinetic energy per unit volume; the term ρgh is the gravitational potential energy per unit volume; and *p* represents the flow energy per unit volume. *Thus, in other words, Bernoulli's equation represents the fact that the various forms of available energy* (*kinetic, potential and flow or pressure energy*) *can be transformed from one to another; the total available energy remains constant*. Now let us discuss a few illustrations of Bernoulli's equation.

Venturi Meter

It is an instrument used for measuring the rate of flow of a liquid and it works on the principles of Bernoulli's equation. As shown in Figure 1.12, it consists of two wide bore tubes *A* and *B* joined by a narrow tube *C* (known as the throat). *A* and *C* are fitted with manometer tubes. The apparatus is interposed horizontally in the pipe in which the rate of flow of a liquid is to be measured. The horizontal placement of the apparatus ensures that the potential energy of the liquid remains the same at all points along the axis.

Let a_1 and a_2 be the areas of cross-sections of the tubes *A* and *C* and let v_1 and v_2 be the velocities of flow of the liquid through these tubes

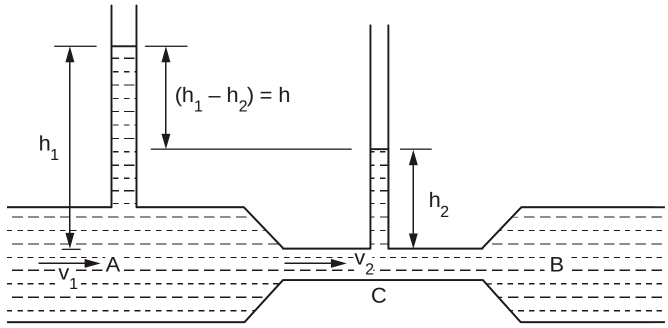


Figure 1.12 : A Venturi Meter

respectively. If V be the volume of the liquid flowing through a given section in one second, we can write :

$$V = \frac{a_1 \, a_2}{\sqrt{a_1^2 - a_2^2}} \cdot \sqrt{2\,g} \;\sqrt{h}$$

or, $$V = k\,\sqrt{h}.$$

where, $k \left(= a_1 \, a_2 \cdot \sqrt{\frac{2\,g}{a_1^2 - a_2^2}} \right)$ is a constant. Thus, we get :

$$V \propto \sqrt{h}$$

Knowing a_1, a_2 and g, the value of the constant k for a given apparatus (venturi meter) can be calculated once for all. Hence, V, the rate of flow of a liquid, can be determined by recording the value of h in the manometers.

Spinning Cricket Ball

Consider a cricket ball which has translational as well as rotational motions. Refer to Figure 1.13 which shows a cricket ball B spinning in the clockwise direction about an axis perpendicular to this paper and moving along a line from left to right. Let the air is rushing in the opposite direction (right to left). Due to the rotational motion, the air below the ball gets speeded up and that above it is slowed down. According to the Bernoulli's equation, the increase in kinetic energy of air (fluid) below the ball is compensated by decrease in the pressure energy. As a result, low pressure area is created in the region below the ball. The ball, therefore, gets deflected downwards.

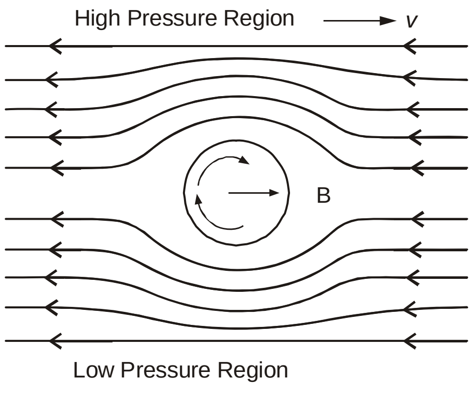


Figure 1.13 : A Cricket Ball with Rotational and Translational Motion

Flying of an Aeroplane

Do you know how an aeroplane lift itself above the ground and keep flying in the air? It is mainly due to the aerodynamic force called lift. To understand the operation of the lift force, we need to apply the Bernoulli's equation on the air (fluid) around the wings of the aeroplane. Refer to Figure 1.14 which shows the cross-section of the wing of an aeroplane. When the airplane moves, the air above and below the wings flow. Aeroplane wings are so designed (i.e. streamlined) that the total distance traveled by air flowing *over* the wing is longer than that of the distance under it. Thus, the velocity of air flow above the wing must be higher than the velocity of air flow under the wing. Now, according to Bernoulli's equation, in this situation, the pressure p_2 above the wing must be lower than the corresponding pressure p_1 under the wing. This unbalanced in pressures causes a force (lift) to act on the wings. The lift force can be resolved into two components – the vertical component, *L* enables the aeroplane to rise above the ground and the horizontal component, *D* (also called drift) enables it to keep flying in the air.

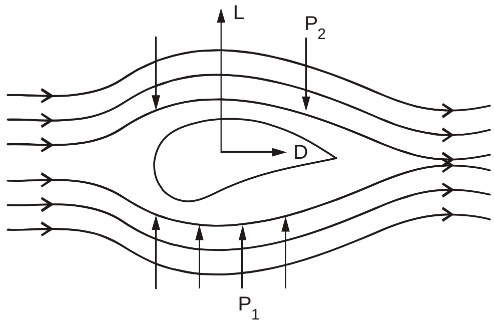
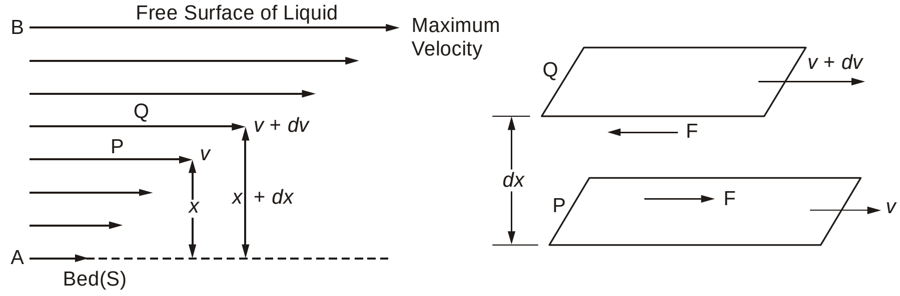


Figure 1.14 : Air Pressures in the Upper and Lower Regions of the Wing of an Aeroplane

Till now, you studied the dynamical properties of non-viscous fluids, that is, the characteristics of the frictionless fluid motion. However, the motion of real fluids is opposed by frictional or viscous forces. This characteristics of fluids to resist motion is known as viscosity about which you will learn now.

1.4.3 Viscosity

Viscosity of a fluid is defined as a measure of its resistance to flow. The greater the viscosity of the fluid, larger is the force (or pressure) required to maintain the flow. For example, when water flows in a uniform horizontal tube, there is a fall in its pressure along the tube in the direction of flow. This is because work (force × displacement) needs to be done against viscosity. *In other words, viscosity of fluids is similar to the frictional force encountered by solids in motion.*



(a) **(b)**

Figure 1.15 : (a) Laminar Flow of Liquid; and (b) Motion of Two Layers *P* and *Q* of Liquid Separated by a Distance *dx*

To understand the effect of viscosity, let us consider a liquid flowing over a horizontal surface (Figure 1.15) in such a manner that the velocity of each layer is almost the same as that of the adjacent layer. That is, the velocity changes continuously. The velocity of the layer in contact with the surface (bed) is negligibly small and can be taken to be zero. As the distance increases from the bed, the velocity goes on increasing as shown in Figure 1.15(a). The velocity of a given layer is proportional to its distance from the stationary layer. The motion of a liquid characterized by these features is called **laminar flow** and is valid for flow of viscous fluids at low velocities.

In laminar flow, a fast moving layer of liquid tends to accelerate the motion of a given layer adjacent and lower to it whereas the slow moving layer adjacent and lower to a given layer tends to retard its motion. Thus, there are two types of forces which act on a layer in opposite directions and consequently the layer of liquid is subjected to a shearing (or tangential) stress. (You will learn about shearing stress in the next section of this unit.) Forces acting on two adjacent layers tend to destroy the relative motion between them. This force is called the *dragging force* or the *viscous force* which is the characteristic of a particular liquid. *The property by virtue of which a liquid opposes relative motion between its different layers is called viscosity*.

All gases and liquids possess the property of viscosity. Gases offer a smaller resistance to flow than liquids do because the viscosity of gases is less than liquids.

Co-efficient of Viscosity

To obtain a quantitative expression for viscosity, let us consider the velocities of the layers P and Q situated at distance x and $x + dx$ respectively from the stationary horizontal surface S (Figure 1.15(b)). Let the velocities of layers P and Q be v and $v + dv$ respectively and the velocity gradient between the two layers is $\frac{dv}{dx}$. The viscous force between the two layers is proportional to :

(a) the surface area A of the layer on which it acts, and

(b) the velocity gradient $\frac{dv}{dx}$.

Therefore, we can write the viscous force F as :

$$F \propto A \frac{dv}{dx}$$

or,

$$F = -\eta A \frac{dv}{dx} \qquad \text{. . . (1.15)}$$

where η is the constant of proportionality known as the *co-efficient of viscosity*. It depends on the nature of the liquid. The negative sign in Eq. (1.15) signifies that the viscous force acts in a direction opposite to the flow of liquid. Further, if $A = 1$ and $\frac{dv}{dx} = 1$, we have $F = \eta$. That is, *the co-efficient of viscosity* (η) *of a liquid can be defined as the tangential viscous force per unit area acting between layers of a liquid in which unit velocity gradient is maintained in a direction normal to the layers*. To determine the dimensions of η, we can write from Eq. (1.15) :

$$\eta = \frac{F/A}{dv/dx}$$

$$= \frac{\text{Force / Area}}{\text{Velocity / Distance}}$$

$$= \frac{\text{MLT}^{-2}/\text{L}^2}{\text{LT}^{-1}/\text{L}}$$

$$= \text{ML}^{-1}\ \text{T}^{-1}$$

The absolute cgs unit of the co-efficient of viscosity is "Poise" and SI unit is $\text{N m}^{-2}\ \text{s}$ or Pa s or $\text{kg m}^{-1}\ \text{s}^{-1}$. Also, 1 Poise = 10^{-1} Pa s.

Above description of viscosity is valid only when the flow of fluid is laminar, that is, the velocity of liquid has a small value. The flow of fluid becomes turbulent if the value of velocity is too low or when it is too high. In other words, the flow of liquid remain laminar only for a range of velocity called critical velocities. You will learn it now.

Critical Velocity

Streamline flow occurs for small values of the velocity of the fluid. In fact, it is observed that there are actually two critical velocities, a lower one at which streamline flow is **unstable** and **turbulence is possible** and a higher one above which **turbulence is inevitable**. Here, we shall confine our discussion to the lower critical velocity, v_c.

The critical velocity v_c of the liquid flowing through a narrow tube is a function of the density ρ, the viscosity η of the liquid and the diameter d of the tube. In view of this, we can find an expression for v_c, using the method of dimensions, by writing :

$$v_c = \frac{k\eta}{\rho d}$$

where, k is a number ~ 1150 (for water). The usual way in which this expression is written is to combine v, ρ and η to give a dimensionless product :

$$k = \frac{v_c \rho d}{\eta}$$

where k is called the **Reynold's number**.

Reynold's Number

Reynold's number is a pure number which gives an idea whether the flow of fluid is laminar or turbulent. The Reynold's number is given as (see above) :

$$k = \frac{v_c \rho d}{\eta} \qquad \ldots (1.16)$$

Eq. (1.16) can also be written as :

$$k = \frac{\rho v_c^2}{\eta v_c / d} = \frac{\text{Inertial Force}}{\text{Force of Viscosity}}$$

Thus, the Reynold's number k is the ratio of the force of inertia and the force of viscosity. The flow of the fluid is said to be *laminar* when the value of k is less than 2000. For values of k above 3000, the flow is *turbulent*. For k between 2000 and 3000 the flow is unstable and may switch over from laminar to turbulent and *vice-versa*.

Applications of Viscosity

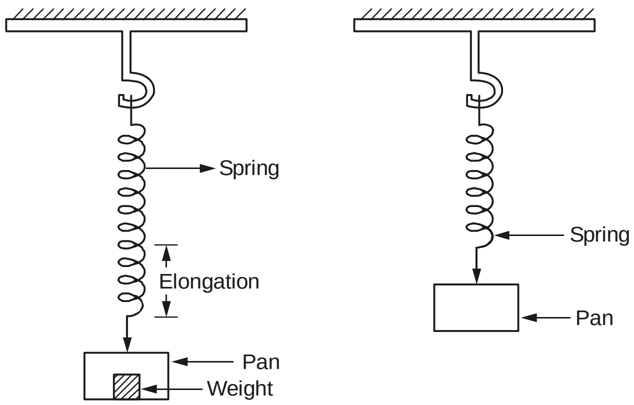
- The viscosity of liquids plays a major role in the selection of lubricating materials for various machines. Heavy machines require lubricant having high value of viscosity whereas light machines require low viscous oil. For example, a cycle requires oil of higher viscosity for lubrication as compared to a watch.
- The quality of fountain pen ink depends largely on its viscosity.
- The normal circulation of blood through arteries and veins depends on the viscosity of blood.
- The shape of the aircraft, the ship and the car is streamlined to minimise the effects of viscosity.

Till now, you learnt some characteristic properties of fluids. You must have noted that these properties of fluids have a variety of applications and they also enable us to explain some of the day-to-day observations. Further, the numerous ways in which we have been able to use solid materials is truly amazing. Be it the sewing needle or the ship or the satellite, we need solids. The question is : How do we decide which solid material is appropriate for making a particular object such as a sewing needle or a ship? Well, the choice of material is decided on the basis of some of characteristic properties of solids. One such property is known as elasticity. Elasticity plays an important role in selection of materials for construction of buildings, bridges, machines etc. You will now learn elasticity of solids.

1.5 ELASTICITY

Whenever an external force is applied on a body, it is deformed (that is, its shape or size or both changes). *The extent of deformation depends upon the nature of the material and shape of the body and the manner in which the force is applied.* As soon as the external (deforming) force is removed, the body regains its original state. This characteristic of the body to regain its original shape and size is called **elasticity**.

We come across many situations in our daily life in which the elasticity of solids is evident. For example, when a force is applied at one end of a metallic spring fixed at the other end, it elongates (Figure 1.16(a)). When the force is removed by removing weights on the pan, it comes to its original position (Figure 1.16(b)). Other examples of elasticity are : a rubber ball gets deformed when we apply a force on it; mattresses compress when we sleep; application of a force on the string of a bow produced deformation in the bow (Figure 1.17). All these objects regain their original condition (shape and size) as soon as the applied force is removed.



(a) **(b)**

Figure 1.16 : (a) Elongated Spring due to Weight on the Pan; and (b) The Spring Returns Back to its Normal Length when Weight is Removed

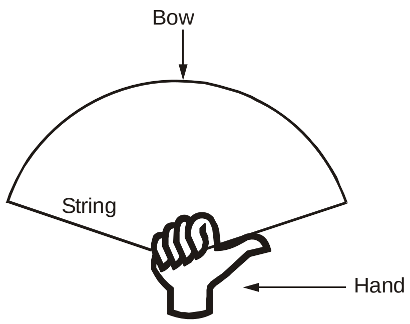


Figure 1.17 : Stretching the Spring Produces Deformation in the Bow

At this stage, you may ask : **Is it always true that a body regains its original shape and size once the deforming force is removed**? To answer this question, we need to understand the concepts of elastic limit and the difference between elastic and plastic bodies.

Elastic and Plastic Bodies

A body (such as a stretched rubber band and a loaded spring) which returns to its original (shape or size) state after the removal of the deforming force is called **perfectly elastic**. However, if we go on increasing the strength of the external (deforming) force, a situation comes when the body no longer regains its original state. This deforming force characterises the **elastic limit** of the body. On the other hand, if a body does not completely attain its original shape and size even on removing the deforming force, it is said to be **perfectly plastic**.

In physics, elasticity stands for opposition to change. Therefore, steel is more elastic than rubber because steel offers more resistance to any effort to deform it. You must note that this meaning of the term elasticity is quite opposite to its meaning in common use (wherein we say that rubber is more elastic than steel).

The elasticity of a material is described in terms of stress and strain. You will learn it now.

1.5.1 Stress and Strain

When an external force or a system of forces is applied on a body, restoring forces are developed due to displacements of molecules from their respective positions of equilibrium. The restoring force opposes the external (deforming) force. In equilibrium, the restoring force is equal in magnitude and opposite in direction to the external deforming force. Stress is defined as the **restoring force per unit area of cross-section of the body**. Mathematically, we write :

$$\text{Stress} = \frac{\text{Restoring Force } (F)}{\text{Area } (A)}$$

The unit of stress in SI system is Nm^{-2}. In the cgs system, stress is measured in dyne cm^{-2}. The dimensional formula for stress is $\text{ML}^{-1}\ \text{T}^{-2}$.

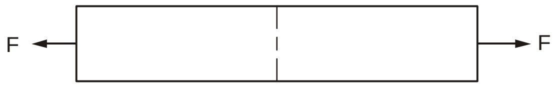


Figure 1.18 : Longitudinal Stress

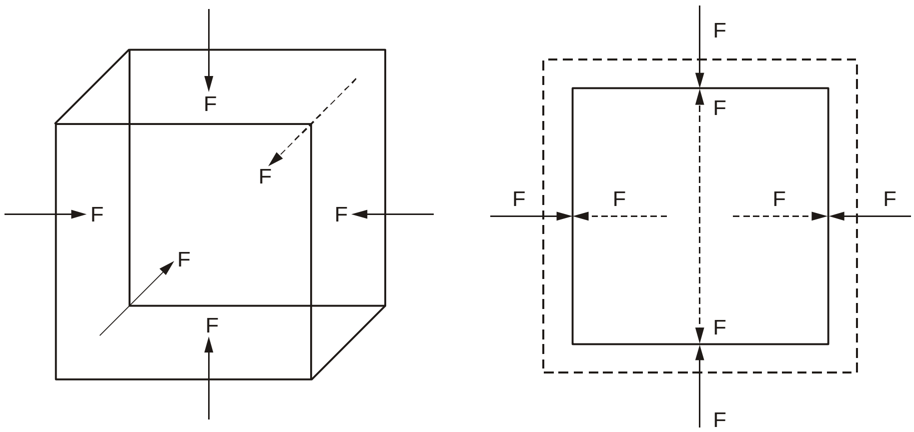
Depending upon the manner in which deforming force is applied, the following three kinds of stresses can be defined :

Longitudinal Stress

When the deforming force is applied along the length of the body (Figure 1.18), the stress produced is called longitudinal stress.

Normal (or Compression) Stress

Suppose the deforming forces are applied uniformly over the entire surface of the body normally (Figure 1.19(a)) which produce change in volume of the body without changing its shape (Figure 1.19(b)). The restoring force acting per unit area normal to the surface of the body is called normal or compression stress. For example, when we apply hydrostatic pressure uniformally over the entire surface of the body, normal stress is produced.



(a) **(b)**

Figure 1.19 : (a) Deforming Forces Acting Normally on a Body; and (b) The Deforming Force Cause Change in its Volume

You may wonder : **What is the difference between pressure and normal stress**? Pressure is defined as the external (deforming) force per unit area normal to the surface. Whereas, normal stress is defined as the internal restoring force developed per unit area normal to the surface.

Shear Stress

If the deforming force acts tangentially or parallel to the surface (Figure 1.20) so that shape of the body changes without change in its volume, the stress is called shear stress.

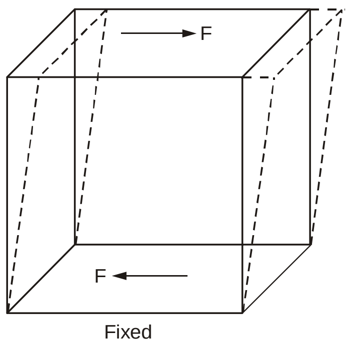


Figure 1.20 : Deforming Force Parallel to the Surface Produces Shear Stress

Strain

Strain is a measure of the deformation produced in a body as a result of an external (deforming) force applied on it. *It is defined as the fractional change in the dimensions of the body under stress*. That is, **strain is the change in dimension per unit dimension of the body**. Since strain is the ratio of two similar quantities, it is **dimensionless** quantity.

Further, corresponding to three different types of stresses, we define the following three kinds of strains :

Linear Strain

It is defined as the ratio of the increase in length (Δl) to the original length (l) of the body when a longitudinal deforming force (Figure 1.21) is applied on it. Mathematically, we write :

$$\text{Linear Strain} = \frac{\text{Change in Length } (\Delta l)}{\text{Original Length } (l)}$$

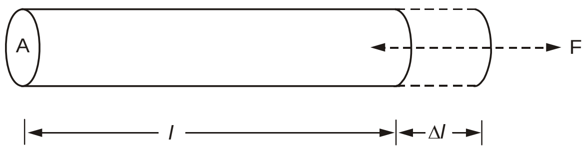


Figure 1.21 : Linear Strain

Volume Strain

It is defined as the ratio of the change in volume (ΔV) (without change of shape) to the original volume (V) of the body when a uniform pressure is applied on the body (Figure 1.22). That is :

$$\text{Volume Strain} = \frac{\text{Change in Volume } (\Delta V)}{\text{Original Volume } (V)}$$

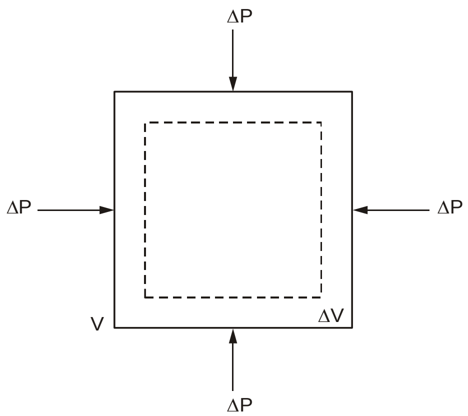


Figure 1.22 : Volume Strain

Shear Strain

When the deforming forces act tangentially (Figure 1.23) the shear strain is defined in terms of the angle θ (in radian) through which a line perpendicular to the fixed plane is turned after deformation. For small angle deformation, shear strain is given by :

$$\theta = \frac{\Delta x}{y}$$

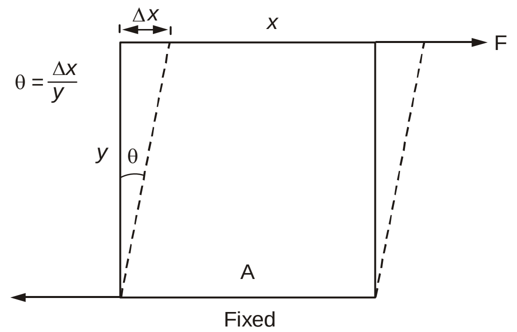


Figure 1.23 : Shear Strain

With this background knowledge about stress and strain, you are now in a position to understand elasticity of materials. Before doing that, it will be useful to study the experimentally observed variation of strain with the stress (applied deforming force) for a few solids.

1.5.2 Stress-Strain Curves for a Metal Wire and Rubber

The experimentally observed variation of strain of a material with the applied deforming force gives valuable information about its practical utility. In fact, on the basis of such observations, a few laws have also been formulated which holds true for solids in general under certain conditions. Let us discuss a few such experimental curves and, in the process, define a few important parameters of solids.

Metal Wire

Refer to Figure 1.24 which shows the variation of **stress** with the variation of **strain** in a metallic wire of uniform cross-sectional area subjected to an increasing load (deforming force).

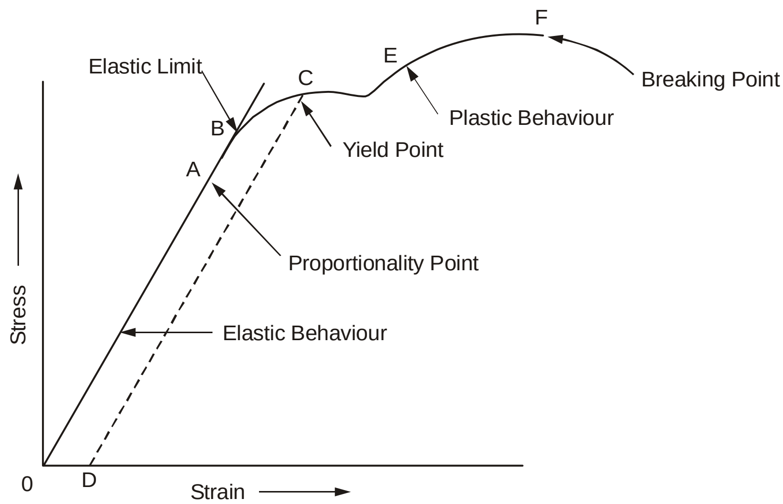


Figure 1.24 : Stress-Strain Curve for Steel Wire

Note from the figure that there are certain *distinct regions* and *special points* on this curve. Let us briefly understand their significance :

Region of Proportionality (*OA*)

Note that the region *OA* of the curve is a straight line which implies that in this region, stress is proportional to strain. This linear relation between stress and strain is a characteristic of a perfectly elastic body.

Elastic Limit (*B*)

If we increase the strain a little bit beyond the point *A*, the stress is no longer proportional to the strain. However, the wire still remains elastic. This means that, when the external force is removed, the body regains the original state. Thus, point *B* indicates the maximum value of stress for which a body (wire) shows elastic property and this point is called elastic limit.

Yield Point (*C*)

When the wire is stressed beyond the elastic limit *B*, strain increases more rapidly and the body behaves like a plastic material; that is, if the load is removed, wire will contract but it will not regain its original length. The material follows dotted line *CD* on the curve when the deforming force is removed and the residual strain *OD* is known as a **permanent set**.

When the applied stress is further increased, we reach a point *E*. Beyond the point *E*, none of the extension is recoverable and the material exhibits completely plastic behaviour.

Breaking Point (*F*)

Beyond point *E*, strain increases much more rapidly and near point *F* the length of wire increases continuously even without increase of deforming force or even by reducing the force a little. In other words the wire breaks at point *F*. This is called **breaking point**. The stress corresponding to breaking point *F* is called **breaking stress** or *tensile strength*.

If large deformation takes place between the elastic limit and the breaking point, the material is called **ductile**. If it breaks soon after the elastic limit is crossed, it is called **brittle**.

Rubber

When a rubber cord is stretched to over several times its original length, it regains its original length when the stretching force is removed. The stress-strain curve for rubber is shown in Figure 1.25. You may note that this curve is distinctly different from that for a metal wire (Figure 1.24). For example, no part of this curve is linear, that is, stress is not proportional to strain in any region of the curve. Secondly, when the deforming force is removed, the cord acquires its original length. However, the work done by the cord material in returning to its original length is less than the work done by the deforming force in deforming it. As a result, certain amount of energy is absorbed by the material in each cycle of stretching and the return back to its original length. This energy appears as heat and the phenomenon is called **elastic hysteresis**.

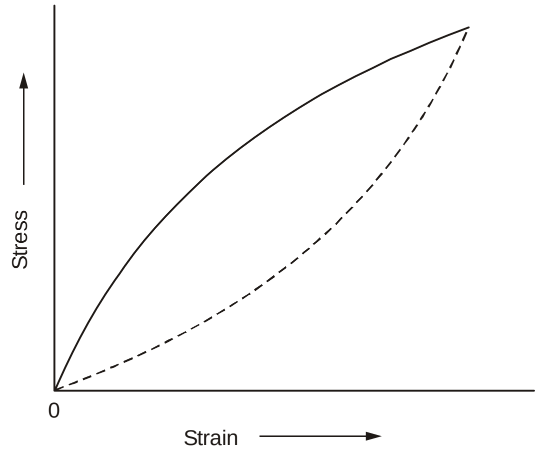


Figure 1.25 : Stress-Strain Curve for Rubber

Elastic hysteresis has an important application in shock absorbers. A part of energy transferred by deforming force is absorbed by the shock absorber and only a small part is transmitted to the body.

Experimental results similar to those discussed above clearly indicates that stress and strain of a material are related to each other. First successful attempt to establish the relation between these two parameters was made by Robert Hooke who proposed a law for this purpose. You will learn the Hooke's law now. But, before that, how about solving a few SAQs?

SAQ 3

(a) A load of 4.0 kg attached to a steel wire of radius 2.0 mm is suspended from a ceiling. Calculate the tensile stress developed in the wire. Take $g = 3.1\ \pi\ \text{ms}^{-2}$.

(b) For steel, the breaking stress is $7.9 \times 10^6\ \text{Nm}^{-2}$ and its density is $7.9 \times 10^3\ \text{kgm}^{-3}$. Determine the maximum length of a steel wire which can be suspended without breaking under its own weight. Take $g = 9.8\ \text{ms}^{-2}$.

1.5.3 Hooke's Law

According to this law, **within the elastic limit, the stress is directly proportional to the corresponding strain**. That is,

$$\text{Stress} \propto \text{Strain}$$

or,
$$\frac{\text{Stress}}{\text{Strain}} = \text{Constant } (E) \qquad \ldots (1.17)$$

The constant of proportionality, E, is a measure of elasticity of the material and is called **modulus of elasticity**. As strain is a dimensionless quantity, modulus of elasticity has the same dimensions (or units) as that of stress. *Its value is independent of stress and strain; it depends on the nature of the material*.

Modulii of Elasticity

For practical purposes, the elasticity of materials is specified in terms of a parameter called modulus of elasticity. For a given materials, we define three modulii of elasticity namely, **Young's modulus, bulk modulus** and **modulus of rigidity** corresponding to linear strain, volume strain and shearing strain respectively. Let us now discuss each of them briefly.

Young's Modulus (Y)

It is defined as the ratio of the longitudinal stress to the longitudinal strain for the material of the body. To write an expression for the Young's modulus, let a wire of length l and area of cross-section A be stretched by a longitudinal force F causing a change Δl in the length of the wire. Then we can write :

Longitudinal stress = F / A; and

Longitudinal strain = $\Delta l / l$

Therefore, the Young's modulus is written as :

$$Y = \frac{\frac{F}{A}}{\frac{\Delta l}{l}} = \frac{F\, l}{A\, \Delta l} \qquad \text{. . . (1.18)}$$

If r is the radius of the wire suspended vertically with a rigid support and M is the mass attached to its free end, we can write :

$$A = \pi\, r^2 \text{ and } F = M\, g$$

Substituting for A and F in Eq. (1.18), we get :

$$\therefore \qquad Y = \frac{M\, g\, l}{\pi\, r^2\, \Delta l} \qquad \text{. . . (1.19)}$$

The units of Young's modulus are Pascal (Pa) or Nm^{-2} in the SI system.

Bulk Modulus (B)

The ratio of normal stress to the volume strain is called the bulk modulus of the material of the body. Mathematically, we write :

$$B = \frac{\text{Normal Stress}}{\text{Volume Strain}}$$

If increase in pressure Δp causes the volume V of the body to decrease by an amount ΔV without any change in its shape, we can write :

Normal Stress = Δp

$$\text{Volume Strain} = -\frac{\Delta V}{V}$$

Negative sign in the expression for volume strain indicates that increase in pressure results in decrease in volume of the body. Thus, the bulk modulus can be written as :

$$B = -\frac{\Delta p}{\frac{\Delta V}{V}}$$

$$= -V\, \frac{\Delta p}{\Delta V} \qquad \text{. . . (1.20)}$$

The units of bulk modulus are Pa or Nm^{-2} in the SI system. The reciprocal of bulk modulus is called **compressibility**. Thus, the expression for compressibility (K) of a material can be written as (Eq. (1.20)) :

$$K = \frac{1}{B}$$

$$= -\frac{1}{V}\frac{\Delta V}{\Delta p} \qquad \text{. . . (1.21)}$$

Modulus of Rigidity

It is defined as the ratio of the shear stress to shear strain of the body. If F is the tangential force applied on a body of surface area A and θ is the resulting shear strain, we can write the modulus of rigidity as :

$$\eta = \frac{\text{Shear Stress}}{\text{Shear Strain}}$$

$$= \frac{\frac{F}{A}}{\theta}$$

$$= \frac{F}{A\,\theta} \qquad \text{. . . (1.22)}$$

where θ is in radians.

Poisson's Ratio

When a wire is stretched along its length, it is elongated and, simultaneously, there is a contraction in its diameter. The length of the wire increases in the direction of the applied force, whereas the contraction in its diameter occurs in the direction perpendicular to the direction of the applied force. This is true not only for wire but also for all other bodies under strain. The strain (change in the dimensions of the body) perpendicular to the applied force is called **lateral strain**. Poisson pointed out that within elastic limit, lateral strain is directly proportional to longitudinal strain. In other words, **the ratio of lateral strain to longitudinal strain is a constant for a material body and is known as Poisson's ratio**. It is denoted by P_R.

If α and β be the longitudinal and lateral strains respectively of a material body, its Poisson's ratio is given by :

$$P_R = \frac{\beta}{\alpha} \qquad \text{. . . (1.23)}$$

Let, due to an applied stretching force, the length l of a wire (rod or tube) increased by an amount $\Delta\, l$ and its diameter d is decreases by an amount $\Delta\, d$. Thus, longitudinal strain is $\frac{\Delta l}{l}$, and lateral strain is $\frac{\Delta d}{d}$. And, the Poisson's ratio is given as :

$$P_R = \frac{\frac{\Delta d}{d}}{\frac{\Delta l}{l}}$$

$$= \frac{l}{d} \frac{\Delta d}{\Delta l} \qquad \ldots (1.24)$$

Since Poisson's ratio is a ratio of two strains, it is a **dimensionless quantity**. The value of Poisson's ratio depends on the nature of the material and for most of the substances, it lies between 0.2 and 0.4. When a body under tension suffers no change in its volume, i.e. the body is perfectly incompressible, the value of Poisson's ratio is the highest (i.e. 0.5).

To fix your understanding of the various parameters discussed in this section, you should solve the following SAQs.

SAQ 4

(a) A load of 4.0 kg is suspended from a ceiling by a steel wire of length 2 m and radius 2.0 mm. It is observed that the length of the wire increases by 0.031 mm. Calculate the Young's modulus for steel. Take $g = 3.1\,\pi\ \text{ms}^{-2}$.

(b) A 4.0 m long copper wire of cross-sectional area 1.2 cm^2 is stretched by a force of 4.8×10^3 N. If the Young's modulus for copper is $1.2 \times 10^{11}\ \text{Nm}^{-2}$, calculate

(i) the stress,

(ii) the strain, and

(iii) increase in the length of the wire.

(c) When a solid rubber ball is taken from the surface to bottom of a lake, its volume decreases by 0.0012%. The depth of the lake is 360 m, density of the lake water is $10^3\ \text{kgm}^{-3}$. Calculate the bulk modulus of rubber. Take $g = 10\ \text{ms}^{-2}$.

Now, let us summarise what you have learnt in this unit.

1.6 SUMMARY

- Due to surface tension, the free surface of a liquid behaves like a stretched elastic membrane tending to contract so as to have minimum surface area. It is measured as the force per unit length perpendicular to an imaginary line on the liquid surface.
- Pressure is defined as the ratio of the force to the area on which it acts. That is :

$$P = \frac{F}{A}$$

The fluid pressure at a point within a fluid is given as :

$$P = h\,\rho\,g$$

- According to the Pascal's law, pressure applied at any point on the surface of a given mass of an enclosed liquid at rest is transmitted over the whole liquid.
- Archimedes' principle states that there is an apparent loss in the weight of a body immersed in a liquid (or gas) and this loss of weight is equal to the weight of the liquid (or gas) displaced by it.
- An ideal fluid is incompressible and its flow is laminar, non-viscous and irrotational. The flow of liquid is of two types. That is :

  (i) streamline motion, and

  (ii) turbulent motion.
- The equation of continuity is a consequence of the principle of conservation of mass of liquid and for streamline flow of a liquid is

$$a_1 v_1 = a_2 v_2$$

  where a_1 and a_2 are the areas of two different cross-sections of a tube of flow and v_1 and v_2 respectively are the velocities of flow through these sections.
- The Bernoulli's equation is an expression representing the principle of conservation of energy for liquids. It is expressed as :

$$\frac{P}{\rho} + g\,h + \frac{v^2}{2} = \text{constant}$$

- The property by virtue of which a liquid opposes relative motion between its different layers is called viscosity. It is similar to the frictional force experienced by solids in motion.
- The property of matter to regain its natural shape and size or to oppose any attempt to deform it is called elasticity.
- The internal restoring force, arising due to an external deforming force applied on a body, and acting per unit area of cross-section of the body is called stress. The stress may be longitudinal, compressional and shearing.
- Strain is defined as the change in dimension of a body per unit dimension. Strains are of three types, namely

  (i) linear strain,

  (ii) volume strain, and

  (iii) shearing strain.
- Hooke's law states that, within elastic limit, the stress is directly proportional to the corresponding strain.

1.7 ANSWERS TO SAQs

SAQ 1

According to the problem, we have :

$$h = 0.76 \text{ m};\ \rho = 13.6 \times 10^3 \text{ kg m}^{-3};\ \text{and } g = 9.8 \text{ ms}^{-2}$$

From Eq. (1.5), the atmospheric pressure, P, is given as :

$$P = h\rho g$$

$$= (0.76 \text{ m}) \times (13.6 \times 10^3 \text{ kg m}^{-3}) \times (9.8 \text{ ms}^{-2})$$

$$= 1.013 \times 10^5 \text{ Nm}^{-2}$$

SAQ 2

(a) Let V and ρ be the volume and density of the solid respectively and ρ' be the density of water. Thus, from the problem, we have :

$$\rho' = 10^3 \text{ kg m}^{-3}$$

Now, we can write :

Weight of the body = $V\rho g$

Volume of solid body outside water $= \dfrac{V}{4}$

Volume of solid body inside water $= V - \dfrac{V}{4} = \dfrac{3V}{4}$

Weight of water displaced by solid $= \dfrac{3V}{4} \times 10^3 \times g$

As solid body is floating, the following condition must be satisfied :

Weight of body = Weight of water displaced by it.

$$V\rho g = \frac{3V}{4} \times 10^3 \; g$$

or, $$\rho = \frac{3}{4} \times 1000$$

$$= 750 \text{ kg m}^{-3}$$

(b) According to the Archimedes' principle, we have :

The apparent loss of weight of the copper tube in water = Weight of water displaced by the upper cube.

If V be the volume of the cube, then above condition can be written as :

$$(0.50 - 0.40) \text{ kg} \times g = V \times (\text{Density of water}) \times g$$

or, $$(0.10 \times g) \text{ kg} = \text{V} \times 10^3 \times g \text{ kg m}^{-3}$$

or, $$V = \frac{0.1}{10^3} \text{ m}^3$$

$$= 10^{-4} \text{ m}^3$$

To know whether the cube is hollow or solid, we can calculate the density of the cube.

$$\text{Density of the copper cube} = \frac{m}{V} = \frac{0.50 \text{ kg}}{10^{-4} \text{ m}^3} = 5 \times 10^3 \text{ kg m}^{-3}$$

Since the density of the copper cube is less than that of pure copper, we conclude that the cube must be hollow.

(c) Let V be the total volume of the piece of ice and V_{sub} be the volume of the portion to the piece of ice submerged in the inside water. ρ_i be the density of ice, ρ_w be the density of water. Then, according to Eq. (1.8), we have :

$$\frac{V_{sub}}{V} = \frac{\rho_i}{\rho_w}$$

or, $$1 - \frac{V_{sub}}{V} = 1 - \frac{\rho_i}{\rho_w}$$

or, $$\frac{V - V_{sub}}{V} = \frac{\rho_w - \rho_i}{\rho_w}$$

or, $$\frac{V - V_{sub}}{V} = \frac{(1 - 0.92) \times 10^3 \text{ kg m}^{-3}}{10^3 \text{ kg m}^{-3}}$$

$$= 0.08 \ (\approx 0.1)$$

Therefore, approximately one-tenth of the piece of ice is above the water surface.

SAQ 3

(a) You know that the tension in the wire is developed due to the weight of the load suspended by it. Thus, we can write the force, F, causing tension in the wire as :

$$F = M g$$

$$= (4.0 \text{ kg}) \times (3.1\ \pi \text{ ms}^{-2})$$

Further, the area of cross-section in the wire,

$$A = \pi r^2$$

$$= \pi\ (2.0 \times 10^{-3})^2 \text{ m}^2$$

$$= 4.0 \times 10^{-6}\ \pi \text{ m}^2$$

The tensile stress developed in the wire is :

$$= \frac{F}{A}$$

$$= \frac{(4.0 \text{ kg}) \times (3.1\ \pi \text{ ms}^{-2})}{4.0 \times 10^{-6}\ \pi \text{ m}^2}$$

$$= 3.1 \times 10^6 \text{ N m}^{-2}$$

(b) Let L be the maximum length of wire suspended without breaking. If ρ be the density and A be the area of cross-section of the wire, then weight W of the wire is

$$W = m g$$

$$W = A L \rho g$$

(Because $m = \rho V$ and the volume, V is equal to the product of length (L) of the wire and its area of cross-section (A); that is, $V = AL$. Thus, we have written $m = A\,L\,\rho$ above.)

And the stress developed in wire due to its own weight W is given as

$$\text{Stress} = \frac{W}{A}$$

$$= L\,\rho\,g$$

The value of stress developed in the wire must not exceed the breaking stress. Thus, we may write :

$$L\rho g = 7.9 \times 10^6\ \text{N m}^{-2}$$

or
$$L = \frac{7.9 \times 10^6\ \text{Nm}^{-2}}{(7.9 \times 10^3\ \text{kg m}^{-3}) \times (9.8\ \text{ms}^{-2})}$$

$$= 10^2\ \text{m}$$

So, the length of the wire must be less than 100 m

SAQ 4

(a) We can write longitudinal stress $= \dfrac{F}{A}$

$$= \frac{M\,g}{\pi\,r^2}$$

$$= \frac{(4.0\ \text{kg}) \times (3.1\ \pi\ \text{ms}^{-2})}{\pi\ (2 \times 10^{-3}\ \text{m})^2}$$

$$= 3.1 \times 10^6\ \text{N m}^{-2}$$

And, longitudinal strain $= \dfrac{\Delta l}{l}$

$$= \frac{0.031 \times 10^{-3}\ \text{m}}{2.0\ \text{m}}$$

$$= 0.0155 \times 10^{-3}$$

Thus, Young's modulus, $Y = \dfrac{\text{Longitudinal Stress}}{\text{Longitudinal Strain}}$

$$= \frac{3.1 \times 10^6}{0.0155 \times 10^{-3}}\ \text{N m}^{-2}$$

$$= 2.0 \times 10^{11}\ \text{N m}^{-2}$$

(b) (i) Longitudinal Stress $= \dfrac{F}{A}$

$$= \frac{4.8 \times 10^3\ \text{N}}{1.2 \times 10^{-4}\ \text{m}^2}$$

$$= 4.0 \times 10^7\ \text{N m}^{-2}$$

(ii) Longitudinal Strain $= \dfrac{\text{Stress}}{Y}$

$$= \frac{4.0 \times 10^{7}\ \text{Nm}^{-2}}{1.2 \times 10^{11}\ \text{Nm}^{-2}}$$

$$= 3.3 \times 10^{-4}$$

(iii) Longitudinal Strain $= \Delta l \,/\, l$

or, Δl = Longitudinal Strain $\times\, l$

$$= (3.3 \times 10^{-4}) \times (4\ \text{m})$$

$$= 1.32 \times 10^{-3}\ \text{m}$$

(c) When the rubber ball is taken from the surface of the lake water to the bottom of the lake, the increase in pressure on the ball,

$$p = h\,\rho\,g$$

$$= (360\ \text{m}) \times (10^{3}\ \text{kg m}^{-3}) \times (10\ \text{ms}^{-2})$$

$$= 3.6 \times 10^{6}\ \text{N m}^{-2}$$

The volume strain of the ball at the bottom of the lake is

$$= \frac{\Delta V}{V}$$

$$= \frac{0.0012}{100}$$

$$= 1.2 \times 10^{-5}$$

And the bulk modulus B, is defined as

$$B = \frac{p\,V}{\Delta V}$$

Substituting the values of p and $\left(\dfrac{\Delta V}{V}\right)$ from above, we get

$$B = \frac{p\,V}{\Delta V}$$

$$B = \frac{3.6 \times 10^{6}\ \text{Nm}^{-2}}{1.2 \times 10^{-5}}$$

$$= 3.0 \times 10^{11}\ \text{Nm}^{-2}$$