UNIT 4 LIGHT

Structure

4.1 INTRODUCTION

In the previous unit, you studied about sound which is responsible for our contact with the surroundings through one of the sense organs (ear). In the present unit, you will study about light which is necessary for the functioning of yet another sense organ called eye. Light enables us to see and appreciate the beauty of nature and admire man-made wonders of the world. Be it rainbow in the sky, or vast diversity of hues in nature, or the Taj Mahal, only light makes us to see and appreciate them all. Did you ever ask yourself : *What is light*? *How does it enable us to see*? *What are the principles of image formation*? *How are we able to see very small objects and objects at a very, very far off distances*? These are some of the questions we will address in this unit.

Most of the phenomena related to light such as reflection, refraction, interference and diffraction can be understood on the basis of the wave theory of light. However, in the present unit, we shall confine ourselves to the **geometrical optics** which was developed before the wave theory was proposed. Geometrical optics successfully explains the phenomena of reflection, refraction and image formation due to these phenomena. *The basic assumption of the geometrical optics is that light travels in a straight line*. It is important to mention here that this is a simplifying assumption and is valid only when the wavelength of light is very small compared to the dimensions of the object light encounters. *In fact, according to the wave theory, light do bend around objects* and the bending is perceptible only when dimensions of the objects are of the order of the wavelength of light.

In Section 4.2, you will learn the laws of reflection and the laws of refraction obeyed by light. On the basis of these laws, the phenomena of image formation by plane and curved mirrors can be understood and you will learn it in Section 4.3. In Section 4.4, you will learn the principles of image formation by refracting surfaces such as lens. You will also learn the phenomena of total internal reflection which is of great importance for the modern day optical communication through optical fibres. In Section 4.5, we discuss the principles of operation of some optical instruments which enable us to see very small objects and objects at very long distances. And, lastly in Section 4.6, you will learn the basics of photometry.

Objectives

After studying this unit, you should be able to

- state the laws of reflection and the laws of refraction,
- draw ray diagrams for image formation by plane mirror, spherical mirrors and lenses on the basis of ray tracing rules,
- derive the mirror formula and lense formula,
- define the power of a lens,
- explain the concept of total internal reflection and the various phenomena associated with it,
- establish a relation between refractive index, angle of minimum deviation and angle of prism for a prism,
- derive the relation connecting focal length, object-distance and image-distance and magnification for lens,
- discuss the working of a simple microscope, a compound microscope and a telescope, and
- define the basic parameters associated with photometry.

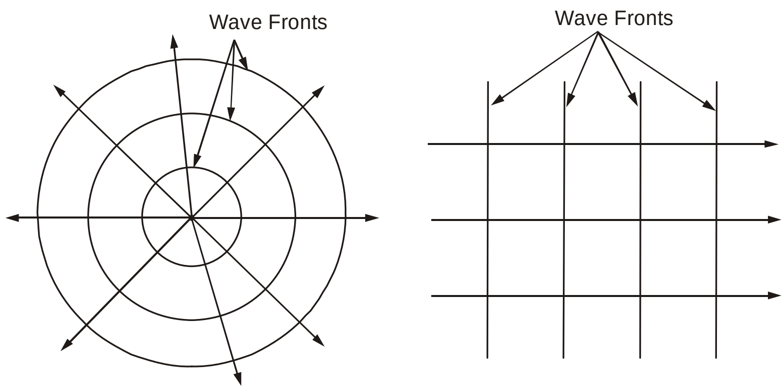
4.2 LAWS OF REFLECTION AND LAWS OF REFRACTION

We are able to 'see' the things around us because light forms image on the retina of our eyes. Image formation involves the processes of reflection and refraction of light. Would not you like to know the laws governing the reflection and refraction of light? These laws, to be discussed in the following paragraphs, are given in terms of propagation of light as rays. However, according to the wave theory, light is a wave. It is, therefore, necessary to know how the propagation of light waves is represented as rays of light.

Rays of Light

According to Huygens wave theory, propagation of light can be considered as the propagation of wave fronts. To understand the concept of wave front, you may recall that when two waves have maximum amplitudes at the same instant of time, they are said to be in phase. *Wave fronts are surfaces on which waves at every point are in phase*. These wave fronts travel outward from the source of the wave. A point source of light emits spherical waves and the corresponding wave fronts are spherical. A two-dimensional representation of the spherical wave fronts as concentric circles are shown in Figure 4.1(a). Similarly, a wave which travels in only one direction has plane wave fronts as shown in Figure 4.1(b). Waves with plane wave fronts are called plane waves.

A line perpendicular to the wave fronts points in the direction of propagation of waves and it is called a **ray**. Thus, a ray of light points along the direction of propagation of the light denoted by arrow in Figures 4.1(a) and (b). And a beam of light is a group of rays traveling along the same direction.



(a) **(b)**

Figure 4.1 : (a) Two-dimensional Representation of Spherical; and (b) Plane Wave Fronts

When a beam of light is incident on a surface separating two media, it can be absorbed; and/or reflected and/or refracted depending upon the nature of the surface. If the surface is rough or uneven (such that the dimensions of the unevenness is large compared to the wavelength of light), it is reflected in many directions. Such reflections are called **diffuse reflections**. On the other hand, if the surface on which light incident is smooth such as a glass plate or a polished metal surface, it is reflected in a particular direction only. Such reflections are called **specular reflections**. *It is important to mention here that the laws of reflection and the laws of refraction of light are valid for specular reflections. Also, the validity of these laws holds only in the domain of geometrical optics, that is, when the dimensions of the physical systems through which light propagates are very large compared to the wavelength of light*.

With this background knowledge, you are now ready to learn the laws of reflection and the laws of refraction.

Laws of Reflection

These laws are as follows :

(i) The incident ray, the reflected ray and the normal to the reflecting surface are all in the same plane and meet each other at the same point.

(ii) The angle of incidence (i) is equal to the angle of reflection (r).

Figure 4.2 shows the ray diagram for the reflection of a beam of light as per the laws of reflection.

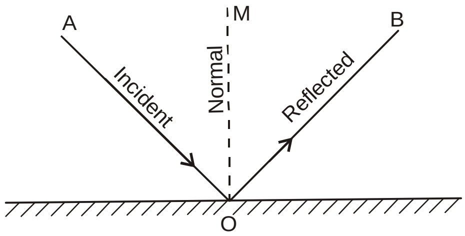


Figure 4.2 : Reflection of Light from a Plane Surface

Laws of Refraction

When a beam of light is incident on a surface separating two transparent media, a part of it (light) is transmitted from one medium to another. The transmitted light is refracted, that is, its direction of propagation changes with respect to its direction in the first media.

On the basis of experiments, Snell discovered the following two laws of refraction :

(a) The incident ray, the refracted ray and the normal to the surface separating the two media lie in the same plane.

(b) The ratio of the sine of the angle of incidence (i) to the sine of the angle of refraction (r) is constant for the two given media; that is :

$$n_1 \sin i = n_2 \sin r$$

or,
$$\frac{n_2}{n_1} = \frac{\sin i}{\sin r} \qquad \ldots (4.1)$$

where $\frac{n_2}{n_1}$ is called the refractive index of medium 2 with respect to medium 1.

Thus, according to Snell's law, when a ray of light passes from rarer medium (low refractive index) to a denser medium (larger refractive index), it bends towards the normal (Figure 4.3(a)) and *vice-versa* (Figure 4.3(b)). Here *PO* and *OQ* are the incident ray and refracted ray respectively. The angle between the normal and the incident ray is called the **angle of incidence** (i) and the angle between the normal and the refracted ray is called **angle of refraction** (r) as shown in Figure 4.3.

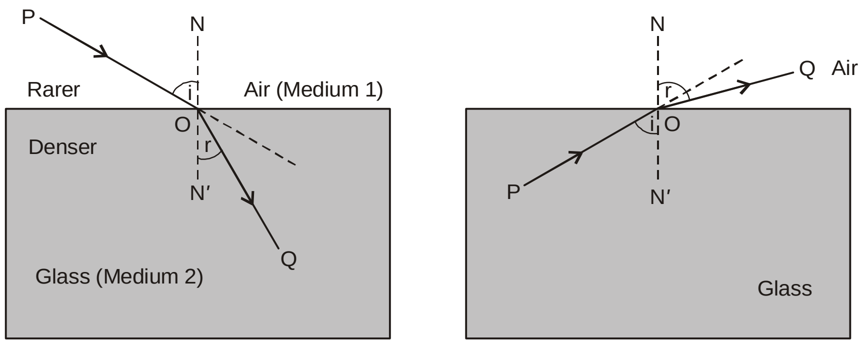


Figure 4.3 : Refraction of Light Propagating from (a) Rarer Medium to Denser Medium; and (b) Denser Medium to Rarer Medium

At this stage, you may ask : What is refractive index? Refractive index is a characteristic of the medium which indicates the extent of refraction of light while it propagates in that medium. Let us briefly discuss this concept.

Refractive Index

The refraction of light is caused due to change in the velocity of light when it passes from one medium to another. To know the extent of change in the velocity of light, we define a parameter called index of refraction or refractive index. Since velocity of light (an electromagnetic wave) is maximum in vacuum, the index of refraction of a medium is a measure of the decrease in the velocity of light in a given medium with respect to its velocity in vacuum. Written mathematically :

$$\text{Refractive Index, } n = \frac{\text{Velocity of light in vacuum}}{\text{Velocity of light in medium}}$$

$$= \frac{c}{v} \qquad \text{. . . (4.2)}$$

Since v can never exceed c, the value of n cannot be less than 1. Further, we know that the velocity of a wave is given as :

$$v = (\text{Frequency, } f) \times (\text{Wavelength, } \lambda)$$

The frequency of a wave including light wave is determined by the source of wave; it is independent of the medium in which the wave propagates. Therefore, change in velocity of a wave implies change in its wavelength. Further, a medium with larger value of the refractive index is called an optically denser medium and *vice-versa*. Also, if air is replaced by some other medium, then we define *relative refractive index*. If light travels from medium 1 to medium 2, the relative refractive index of medium 2 with respect to medium 1 is defined as :

$$\frac{n_2}{n_1} = \frac{\text{Velocity of light in medium 1}}{\text{Velocity of light in medium 2}}.$$

4.3 IMAGE FORMATION BY REFLECTING SURFACES

As mentioned earlier, image formation involves reflection and refraction of light. The laws of reflection help us understand the images formed by reflecting surfaces – plane mirrors and spherical mirrors. Let us now discuss the image formation by a plane mirror and by spherical mirrors.

4.3.1 Plane Mirror

Refer to Figure 4.4. When an object *O* is placed in front of a plane mirror *MM '*, the image *I* is formed by the mirror. The position of the image *I* is located on the basis of the law of reflection. Rays *OA* and *OB* are reflected by the mirror along *AC* and *BD* respectively. When these reflected rays are extended behind the mirror, they meet each other at point *I*, which is the location of the image. *The image I appears to be formed behind the mirror*. It is of the same size as that of the object and the distance of the image from the mirror is same as the distance of the object from mirror. The image *I* is **virtual, erect** and **laterally inverted** if the object is of finite size.

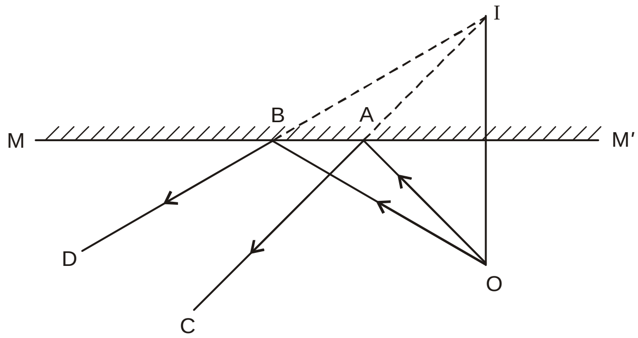


Figure 4.4 : Image *I* of an Object *O* Formed by a Plane Mirror *MM′*

Before you proceed further, you may like to know the difference between the **real image** and the **virtual image**. **Real images** are characterised by the following features :

(a) It can be obtained on the screen, and

(b) The rays of light actually pass through the location of the image.

On the other hand, a **virtual image** (as formed by the plane mirror as in Figure 4.4) cannot be obtained on the screen because the rays of light actually do not pass through the location of the image; they only *appear* to meet at the image point when they are extended backwards.

Images are also formed by curved reflecting surfaces. In fact, in real life situations, most of the time we need to obtain enlarged or smaller images of objects such as in the rear-view mirror of an automobile. For this purpose, spherical mirrors are used. We now discuss the spherical mirrors and images formed by them.

4.3.2 Spherical Mirrors

There are two types of spherical mirrors :

(a) Concave mirror, and

(b) Convex mirror.

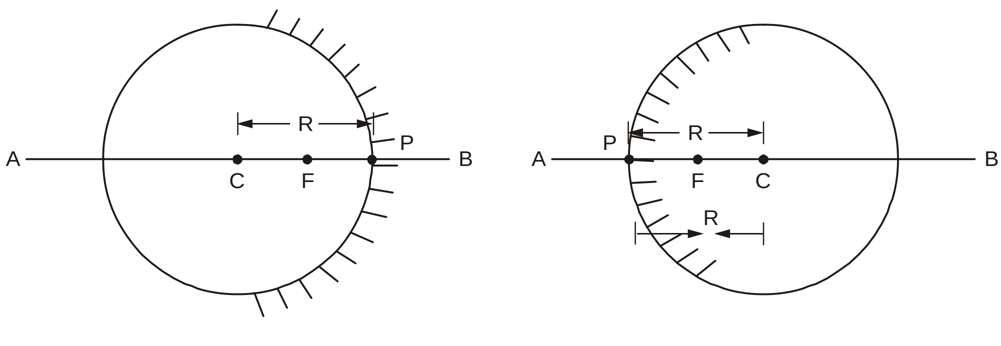


Figure 4.5 : (a) Concave Mirror; and (b) Convex Mirror

Spherical mirrors – concave as well as convex – are obtained by cutting pieces from a hollow glass sphere and coating its surfaces with shiny metal. To study the image formed by these mirrors, you must recapitulate some of important terms and definitions associated with them. Refer to Figure 4.5. For both types of mirrors, the line *AB* passing through the centre of curvature *C* and the pole *P* (centre of the mirror) is called **principal axis**. The distance *CP* is called **radius of curvature** (*R*) of the mirror, *F*, the mid point of *CP*, is the **focus** of the mirror, and FP, the distance between the focus and the pole, is the **focal length** f. Obviously, therefore, we have :

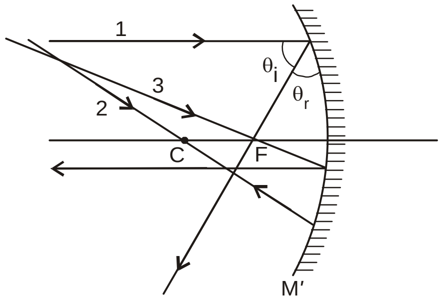
$$f = \frac{R}{2} \qquad \text{. . . (4.3)}$$

The position and the nature of images formed by concave and convex mirrors can be determined by following certain geometrical rules followed by the rays of light incident on these mirrors. These rules, called **ray tracing rules**, are given below :

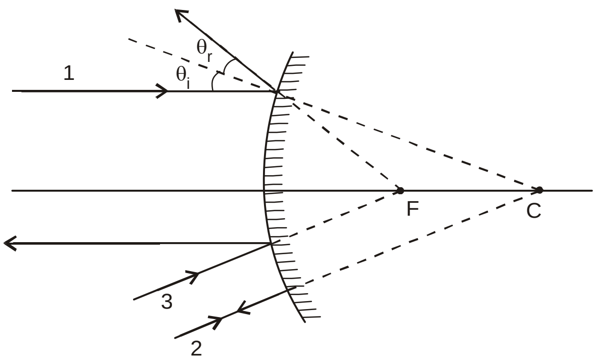
(1) The incident ray parallel to the principal axis of a spherical mirror, after reflection, passes through the focus of a concave mirror or appears to pass through the principal focus of a convex mirror (ray 1 in Figures 4.6(a) and (b)).

(2) The incident ray passing or appear to pass through the centre of curvature of spherical mirror, after reflection, returns through the same path (ray 2 in Figures 4.6(a) and (b)).

(3) The incident ray passing or appear to pass through the focus of a spherical mirror, after reflection, returns parallel to the principal axis (ray 3 in Figures 4.6(a) and (b)).



(a)



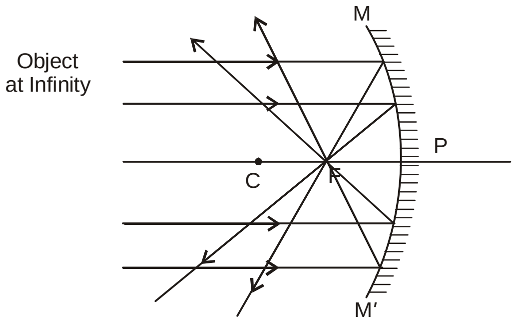
(b)

Figure 4.6 : Ray Tracing Rules for (a) Concave; and (b) Convex Mirror

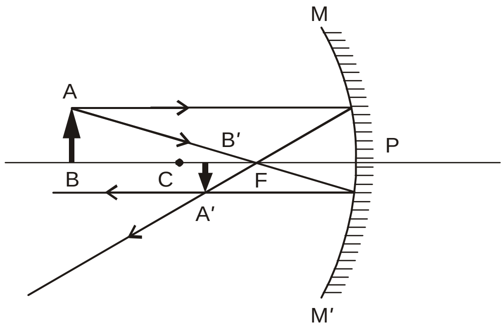
Now, let us apply these rules for locating images formed by spherical mirrors when the object is kept at different positions in front of the mirrors.

Concave Mirror

(a) **The Object is Kept at Infinity (that is, it is at a very large distance from the mirror) :** The incident rays are parallel to the principal axis and the image is formed at the focus, F, of the mirror (Figure 4.7(a)). It is a real, point image.



(a)



(b)

Figure 4.7 : Images Formed by a Concave Mirror when (a) Object is at Infinity; and (b) Object is Placed Beyond the Centre of Curvature

(b) **The Object is Kept Beyond the Centre of Curvature *C* :** The image is formed between *F* and *C*, and it is a *real*, *inverted* and *diminished* (smaller in size) image (Figure 4.7(b)).

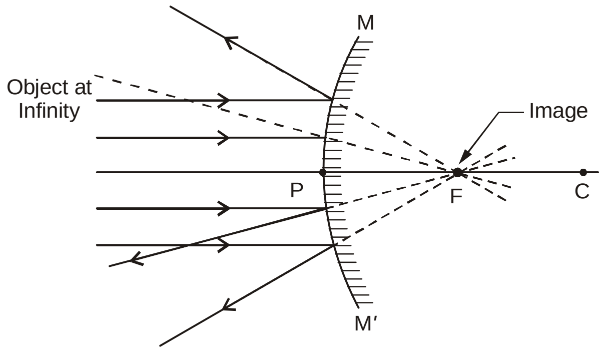
Thus, following the ray tracing rules we can determine the location and nature of the images formed by a concave mirror.

Convex Mirror

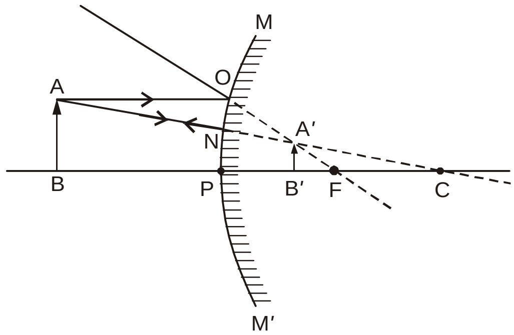
From the geometry of the convex mirror (Figure 4.5), you know that its centre of curvature *C* and the focus *F* lie behind the reflecting surface of the mirror. Let us apply the ray tracing rules to determine the nature and position of image formed by a convex mirror.

(a) **The Object is Placed at Infinity :** Refer to Figure 4.8(a). The parallel rays from the object are incident on the mirror surface. Since all these rays are parallel to the principal axis, after reflection from the mirror, they all *appear to pass* through the focus *F* of the mirror. Thus, the image is formed at *F* and it is virtual and a point image.

(b) **The Object is Kept at any Position between Infinity and the Pole of the Mirror :** Let us consider ray *AO* which is parallel to the principal axis (Figure 4.8(b)). According to the ray tracing rule, after reflection, this ray will appear to pass through the focus *F*. Consider another ray *AN* which, when extended behind the mirror, appears to pass through *C*, the centre of convature. Thus, this ray will return along the same path after reflection. These two rays originating from point *A* appear to intersect each other at point *A*′. Thus, *A*′ is the image of *A* and *A*′*B*′ is the image of the object *AB*. You may note that the image is formed between *P* and *F*, and it is a **diminished**, **virtual** and **erect image**.



(a)



(b)

Figure 4.8 : Images Formed by a Convex Mirror of an Object Placed at (a) Infinity; and (b) At any Point in Front of the Mirror (Except at Infinity)

Till now, you learnt to locate the images formed by spherical mirrors using purly geometrical method following ray tracing rules. Images formed by spherical mirrors can also be located using mathematical formula called mirror formula. You will learn it now.

Mirror Formula

Mirror formula is a mathematical relation between the object-distance (u), the image-distance (v) and focal length (f) of the mirror. The question is : What is the reference point for measuring these distances? To measure these distances, we follow a sign convention called **Cartesian sign conventions**. It helps us to determine the location and characteristics of the image by assigning appropriate signs to distances and heights of the objects and their images. It is valid for concave mirror as well as convex mirror. Let us learn it now before deriving the mirror formula.

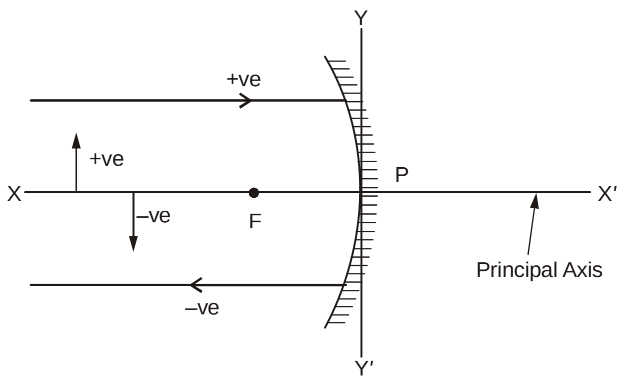


Figure 4.9 : Signs Assigned to Distances of the Object and Image as per the Cartesian Sign Convention

Cartesian Sign Conventions

Refer to Figure 4.9 which depicts the sign conventions given below :

(a) All the distances are measured from the pole of the mirror.

(b) The incident ray travels from left to right which means that the object is kept on the left hand side of the mirror.

(c) The distances in the direction of the incident ray are positive, whereas the distances in the direction opposite to the incident ray are considered negative.

(d) The height of the object or the image in the upward direction from the principal-axis of the mirror is taken as positive and *vice-versa*.

Now, to derive the mirror formula, refer to Figure 4.10, which shows an object *AB* placed in front of a concave mirror *MM ′*. The image *A′B′*, obtained by ray tracing method, is a real, inverted and located between *F* and *C* of the mirror. Let the height of the object is "*h*". If we assume that the curvature of the mirror is small, we can approximate that curved length *KP* as a straight line perpendicular to the principal axis.

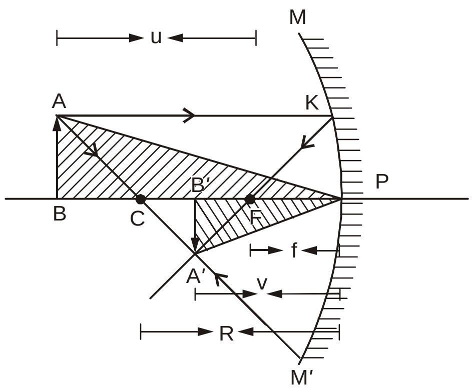


Figure 4.10 : Image *A′B′* of an Object *AB* Formed by a Concave Mirror

Now consider the right angled triangles, $\Delta\, ABP$ and $\Delta\, A'B'P$ (both are shaded in figure). Geometry of the figure shows that these two triangles are similar. Thus, we can write :

$$\frac{A'B'}{AB} = \frac{PB'}{PB} \qquad \ldots (4.4)$$

Further, according to the Cartesian sign conventions listed above, we can write :

$$PB = -\,u;\ PB' = -\,v;\ PF = -\,f;\ PC = -\,R$$

Therefore, Eq. (4.4) can be written as :

$$\frac{A'B'}{AB} = \frac{-\,v}{-\,u}$$

$$= \frac{v}{u} \qquad \ldots (4.5)$$

Similarly, considering the two similar right angled triangles, $\Delta\, ABC$ and $\Delta\, A'B'C$, we can write :

$$\frac{A'B'}{AB} = \frac{CB'}{CB} \qquad \text{. . . (4.6)}$$

From the geometry of the figure, we can write :

$$CB' = PC - PB'$$

$$CB = PB - PC$$

Substituting these in Eq. (4.6), we get :

$$\frac{A'B'}{AB} = \frac{PC - PB'}{PB - PC}$$

$$= \frac{(-R) - (-v)}{(-u) - (-R)} = \frac{-R + v}{-u + R} \qquad \text{. . . (4.7)}$$

Comparing Eqs. (4.5) and (4.7), we can write :

$$\frac{-R + v}{-u + R} = \frac{v}{u}$$

or, $\quad -Ru + vu = -vu + Rv$

or, $\quad 2vu = Rv + Ru$

or, $\quad 2vu = R\,(v + u)$

$$2vu = 2f\,(v + u)$$

using Eq. (4.3). Dividing both sides by *vuf*, we get :

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \qquad \text{. . . (4.8)}$$

Eq. (4.8) is called **mirror formula** and is valid for all spherical mirrors.

You may be aware that one of the important properties of mirrors is its ability of magnify or diminish the size of the image with respect to the size of the object. This property is know as magnification. You will learn it now.

Magnification

Magnification (m) produced by a mirror is defined as the ratio of the height of the image (h') to the height of the object (h). Written mathematically (Figure 4.10) :

$$m = \frac{\text{Size of the image}}{\text{Size of the object}}$$

$$= \frac{A'B'}{AB}$$

$$= -\frac{h'}{h} \qquad \text{. . . (4.9)}$$

As the object is placed above the principal axis, the size of the object is considered +ve. But h', size of the image, is positive for the errect image, and negative for inverted image. Therefore, magnification m of a spherical mirror is positive for erect images and negative for inverted images. You know that the image formed by a convex mirror is always real and inverted. **Therefore, magnification (m) of a convex mirror is always negative**. *However, in case of* **concave mirror**, *we can have positive as well as negative magnifications.*

Now, the question is : *Can we express the magnification of a mirror in terms of object and image distances from the mirror*? It can indeed be done because, the size of the image depends on the position of the object in front of the mirror. The relation between m, u and v can be obtained by comparing Eqs. (4.5) and (4.9). We can, therefore, write :

$$m = \frac{v}{u} \qquad \text{. . . (4.10)}$$

Another group of image forming optical components are collectively known as refracting surfaces. In these components such as lens and prism, image formation takes place due to the refraction of light. You will learn about them now. Before proceeding further, try to solve an SAQ.

SAQ 1

Draw the ray diagrams to locate the position of the image formed by a concave mirror when the object is placed at

(a) a point between the focus and centre of curvature,

(b) the focus, and

(c) a point between focus and the pole of the mirror.

4.4 IMAGE FORMATION BY REFRACTING SURFACES

You know that both the rear-view mirror in automobiles and binoculars form images of objects. The question is : Is the image formation in both the cases based on the same principle/law? It is not so; images formed by mirrors are due to reflection of light whereas images formed by binocular (lenses) are due to refraction of light. The most widely used refracting surfaces in the imaging devices is called lens. You will learn about it now.

4.4.1 Lenses

A lens is a contribution of two transparent spherical surfaces. Like spherical mirrors, lenses are also divided in two categories namely the convex lens and concave lens. The convex lens is also known as **converging lens** (because it converges the rays of light incident on it) and the concave lens is also known as **diverging lenses** (because it diverges the incident rays of light). Both the types of lenses are shown in Figure 4.11. Note that the convex lens is thicker in the middle compared to its edges, whereas the concave lens is thinner in the middle compared to its edges. Because a lens is made of two spherical surfaces, it has two centres of curvature and the foci; one on either side. The point c at the centre of the lens on the principal axis through which a ray goes undeviated (that is, the ray is refracted) is called the *optical centre of lens*.

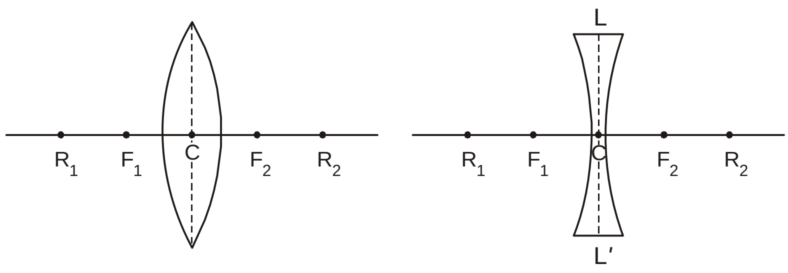


Figure 4.11 : (a) Convex (or Converging) Lens; and (b) Concave (or Diverging) Lens

As in the case of spherical mirrors, the position and the nature of the images formed by convex and concave lenses can be determined using certain geometrical rules followed by the rays of light incident on these lenses. The **ray tracing rules** for lenses are given below.

(1) The incident ray parallel to the principal axis, after refraction, converges (for convex lens) or *appears to converge* (for concave lens) at the focus.

(2) The incident ray passing through the optical centre of the lens remains undeviated.

(3) The incident ray passing through the focus (for convex lens) or *directed towards the focus* (*for concave lens*), after refraction, becomes parallel to the principal axis.

Applying these rules, we can determine the nature and location of images formed by a lens.

Let us first do it for the convex lens.

Convex Lens

As in the case of mirrors, the location and nature of image depends on the location of the object in front of the lens. Let us discuss some cases.

(a) **The Object is placed at Infinity :** Refer to Figure 4.12. The ray *AP* from the object at infinity is parallel to the principal axis. Thus, after refraction from the lens *L′*, it passes through *F*, the focus of the lens. Similarly, the ray *A′P′* being parallel to the principal axis, passes through *F* after refraction. Thus, the image is formed at the focus (*F*) of the lens. It is a real, and point image.

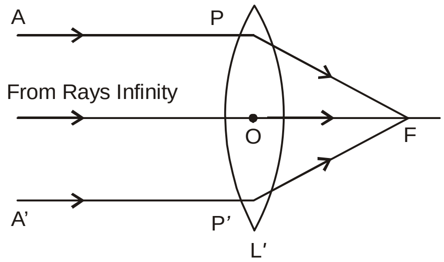


Figure 4.12 : Image Formed by a Convex Lens of an Object Kept at Infinity (that is, at a Very Very Large Distance) from the Lens

(b) **The Object is Placed beyond 2*F* :** Refer to Figure 4.13. The distance of object *AB* from the lens is more than twice of its focal length. The ray *BP*, which is parallel to the principal axis of the lens, passes through *F*. Ray *BQ* which passes through the optic centre *O* of the lens goes undeviated and intersects the ray *PK* at *B*′. Thus, *B*′ is the image point corresponding to the object point *B*. And, *A*′*B*′ is the image of *AB*. Note that it is a **real**, **inverted** image and is **smaller in size** compared to the object.

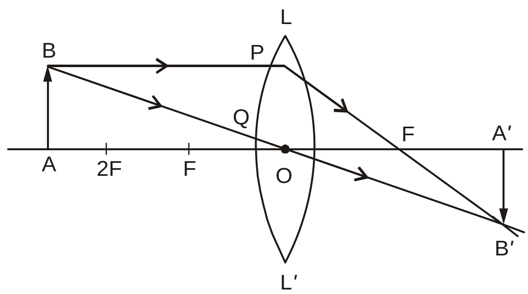


Figure 4.13 : Image Obtained by a Convex Lens of an Object Placed at a Distance Greater than 2*F* of the Lens

Concave Lens

Let us now apply the ray tracing rules to ascertain the position and nature of images formed by concave lens.

Refer to Figure 4.14, which shows **an object *AB*, placed beyond *F***, in front of a concave lens *LL*′. Ray *BP*, parallel to the principal axis, appears to diverge from the focus *F* of the lens. Ray *BP*′, which passes through the optical centre *C* of the lens remains undeviated and intersects the intrapolated ray *PM* at point *B*′. Thus, *B*′ is the image point of *B* and *A*′*B*′ is the image of *AB*. Note that the image *A*′*B*′ is **smaller in size**, **virtual** and **erect**.

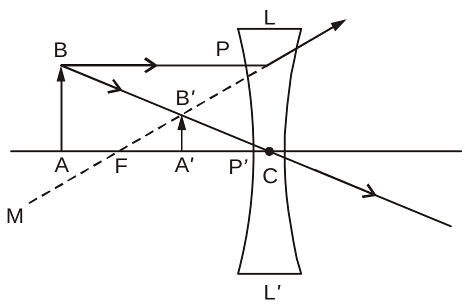


Figure 4.14 : Image *A*′*B*′ Formed by a Concave Lens of an Object *AB* Placed at a Distance Greater than the Focal Length of the Lens

It is important to mention here that the images formed by concave lens are always smaller in size compared to the object, virtual and erect. You should convince yourself of this statement by locating image position corresponding to different positions of the object from the lens.

An alternative method to determine the location and nature of the images formed by lenses is provided by the lens formula. You will learn it now.

Lens Formula

The lens formula is a mathematical relation among the object distance (*u*), image distance (*v*) and the focal length (*f*) of the lens. Before deriving the relation, you must know the cartesian sign convention followed by the lenses.

Cartesian Sign Conventions

The cartesian sign conventions used for lenses are similar to those for spherical mirrors. The sign conventions are given below :

(1) All the distances are measured from the optical centre *C* of the lens.

(2) All the distances measured in the direction of incident ray are taken as positive and *vice-versa*.

(3) The height of the object or the image in the upward direction from the principal axis is taken as positive and *vice-versa*.

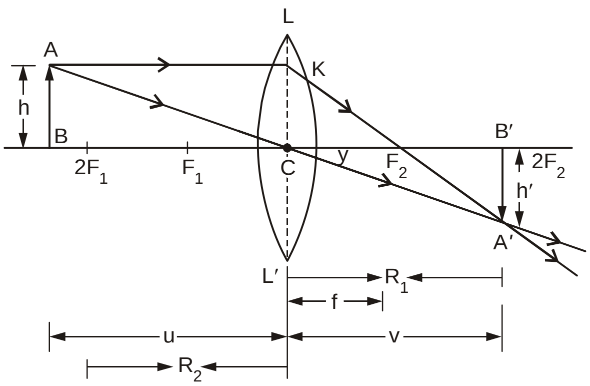


Figure 4.15 : Image *A′B′* of an Object *AB* Formed by a Convex Lens

Now, to obtain the lens formula, let us consider a thin convex lens of small aperture (Figure 4.15). Let an object *AB* is placed beyond $2F_1$ of the lens and *A′B′* is the image formed. Image *A′B′* is smaller in size (h') than the size of the object (h). Consider the right angled triangles, $\Delta\, ABC$ and $\Delta\, A'B'C$. These triangles are similar. Therefore, we can write, using the sign conventions :

$$\frac{A'B'}{AB} = \frac{CB'}{CB} = \frac{+\,v}{-\,u} \qquad \ldots (4.11)$$

where *u* and *v* respectively are the distance of the object and image from the optical centre of the lens. The positive and negative signs before *v* and *u* respectively are due to the sign convention mentioned above. You must conceive yourself about its correctness before proceeding further.

Further, the right angled triangles, $\Delta CK\,F_2$ and $\Delta\,A'B'\,F_2$ are similar triangles and, therefore, we can write :

$$\frac{A'B'}{CK} = \frac{F_2B'}{CF_2} \qquad \ldots (4.12)$$

But, $\qquad AB = CK \qquad \ldots (4.13)$

Substituting Eq. (4.13) in Eq. (4.12), we get :

$$\frac{A'B'}{AB} = \frac{CB' - CF_2}{CF_2}$$

$$= \frac{+ v - f}{f} \qquad \text{. . . (4.14)}$$

Comparing Eqs. (4.11) and (4.14), we get :

$$\frac{v}{-u} = \frac{v - f}{f}$$

or, $$vf = -vu + uf$$

Dividing both sides by *uvf*, we get :

$$\frac{1}{u} = -\frac{1}{f} + \frac{1}{v}$$

or, $$\boxed{\frac{1}{f} = \frac{1}{v} - \frac{1}{u}} \qquad \text{. . . (4.15)}$$

Eq. (4.15) is called the **lens formula**. It is very useful in designing lenses of known focal lengths. However, for this purpose, we also need to relate the focal length of the lens with material and geometric properties such as the refractive index of the lens material (glass) and the radius of curvature of the spherical surfaces of which the lens is made. We give the relation below without deriving it :

$$\frac{1}{f} = (n - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{. . . (4.16)}$$

Eq. (4.16) is yet another (practical) form of the lens formula used for designing lenses of given focal lengths. This formula holds true for both the convex and concave lenses.

Magnification

Magnification (m) is defined as the ratio of the size of the image (h') produced by a lens to the size of the object (h). It is expressed as :

$$m = \frac{\text{Size of the image}}{\text{Size of the object}}$$

$$= \frac{h'}{h} \qquad \text{. . . (4.17)}$$

From Figure 4.15, we can write the magnification for a **convex lens**, (producing a real image) as :

$$m = \frac{A'B'}{AB}$$

$$= \frac{-h'}{h} \qquad \text{. . . (4.18)}$$

From Eqs. (4.11) and (4.18), we get :

$$m = \frac{v}{-u} \qquad \text{. . . (4.19)}$$

And the magnification of a **convex lens** or a **concave lens** producing **virtual** image is given by :

$$m = \frac{h'}{h} = \frac{v}{u} \qquad \text{. . . (4.20)}$$

From Eqs. (4.19) and (4.20), it is obvious that the magnification is **positive** when image formed is **virtual** and it is **negative** when image formed is **real**. *The magnification is always **positive** for a **concave lens** because it always produces a **virtual image**.*

Power of a Lens : Diopter

The ability of a lens to converge or diverge incident rays of light is expressed in terms of its power. The power (P) of a lens is defined as the reciprocal of its focal length (f); the focal length is measured in **metre**. That is :

$$P = \frac{1}{f} \qquad \text{. . . (4.21)}$$

and the SI unit of power is diopter (D). Thus, when the focal length of a lens is of one metre, its power is said to be **one dioptre**. That is :

$$P = \frac{1}{1\text{m}} = 1\text{D}$$

Eq. (4.21) implies that *shorter the focal length of a lens, it is able to converge or diverge light more strongly, that is, more powerful the lens is.*

For converging lens, the power is positive and power of a diverging lens is negative because, by convention, the focal length of a convex lens is taken as positive and that of a concave (diverging) lens, it is taken as negative.

The relation between the power of a lens and its material and geometric parameters can be written by combining Eqs. (4.16) and (4.21) :

$$P = \frac{1}{f}$$

$$= (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

where R_1 and R_2 are the radii of curvatures of the lens in metres.

You may ask : **If we place two lenses in contact with each other, what is the power of this combination**? The powers of lenses are additive. Refer to Figure 4.16 which shows two lenses L_1 and L_2 of focal length f_1 and f_2 and their optical centres are C_1 and C_2. The power of the equivalent lens (combination of these two lenses) is given by :

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

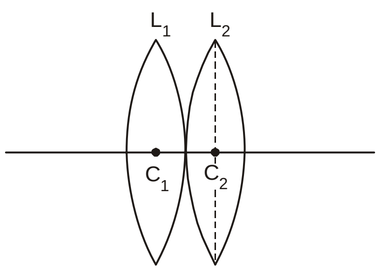


Figure 4.16 : A Combination of Two Convex Lenses

If f_1 and f_2 are in metre, we can write :

$$P = P_1 + P_2$$

In general

$$P = P_1 + P_2 + P_3 + \ldots$$

where $P_1, P_2, P_3, \ldots$ are powers of individual lenses constituting the equivalent lens.

The magnification produced by the combination of lenses is equal to

$$m = m_1 \times m_2 \times m_3 \times \ldots$$

where m_1, m_2, m_3, . . . etc. are the magnifications of individual lenses.

Suppose a source of light is immersed in water and facing the water-air interface. *Can you say with certainty that the light will always emerge out of the water into the air*? Under certain conditions, light will not emerge out! This is because of a phenomenon called total internal reflection. You will learn it now.

Now, to fix your understanding of the ray tracing rules for lens, you should answer the following SAQ.

SAQ 2

(a) Determine the location and nature of the images formed by a convex lens when the object is placed at

(i) $2F$,

(ii) between F and $2F$,

(iii) F, and

(iv) between F and the centre of the lens.

(b) Find the focal length of the convex lens used for obtaining a magnified image of an object on the screen placed at a distance of 10 m from the lens. Take the desired magnification to be 19.

4.4.2 Total Internal Reflection

The phenomenon of total internal reflection is caused due to the laws of refraction. In this phenomenon, light incident from a denser medium at the interface separating the denser medium from a rarer medium is completely reflected back into the denser medium, provided the angle of incidence is greater than the critical angle for that pair of media. To understand how the total internal reflection of light can be explained on the basis of the Snell's laws, refer to Figure 4.17.

The rays of light (ON, ON_1, ON_2, ON_3) from the object O in a denser medium say, water, is incident at different angles on the surface, PQ, separating the two media. These incident rays get refracted into the rarer medium. Note that the angles of incidence for rays ON to ON_3 increases progressively. Thus, the angles of refraction should also increase progressively. At a particular angle of incidence (such as for the ray N_2), the angle of refraction becomes 90°. At this angle of incidence, the refracted ray travels along the surface *PQ*; that is, light is not refracted into the air medium. The angle of incidence for which the angle of refraction, *r*, is equal to 90° is called **critical angle**.

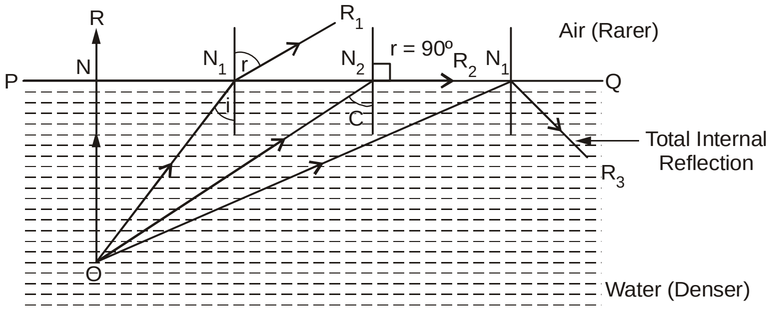


Figure 4.17 : Total Internal Reflection of Light

Further, if the light is incident in the denser medium at an angle of incidence greater than the critical angle (ray ON_3 in the figure), it is totally reflected back into the denser medium and is not refracted at all into the rarer medium. Therefore, we may write the necessary conditions for observing the phenomenon of total internal reflection as :

- The incident ray should travel from a denser medium to a rarer medium.
- For a given pair of media, the angle of incidence should be greater than critical angle.

The value of the critical angle is a function of the refractive indices of the two media. To find an expression for the critical angle, recall that, according to Snell's law (Eq. (4.1)) :

$$\frac{n_2}{n_1} = \frac{\sin i}{\sin r} \qquad \text{. . . (4.1)}$$

where n_1 and n_2 are the refractive indices of the media of incidence and refraction respectively. When the ray of light is incident at critical angle, we have :

$$i = C \text{ and } r = 90^{\circ}$$

Substituting these values of *i* and *r* in Eq. (4.1), we get :

$$\frac{n_2}{n_1} = \frac{\sin C}{\sin 90^{\circ}}$$

$$= \sin C \qquad \text{. . . (4.22)}$$

Phenomena Based on Total Internal Reflection

Mirage

It is an optical illusion whereby we "see" an inverted image of an object, which gives the impression as if there is some reflecting (such as water) surface near the object. Mirage is frequently observed in deserts and coal-tar roads in hot summer days and it is caused due to the phenomenon of total internal reflection of light.

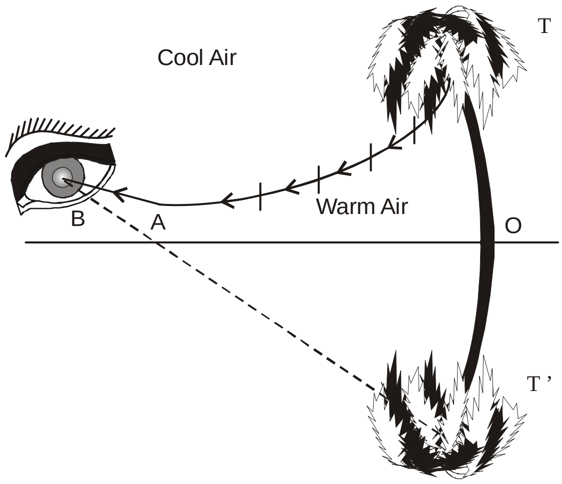


Figure 4.18 : The Inverted Image of a Tree due to Total Internal Reflection of Light by Layers of Atmosphere

To understand the role of total internal reflection in mirage, you may recall that in a hot summer days, the surface of earth becomes hot and the temperature of atmosphere rises near the surface of earth. There is a gradual decrease in the temperatures of the layers of atmosphere as one goes up from the surface of the earth. Therefore, density as well as refractive index of the layers of air above the earth's surface increases.

Now, refer to Figure 4.18, which shows a tree at *O* and an observer at *B* in a desert. A ray of light traveling from point *T* of the tree, passes through air of gradually decreasing refractive index. *That is, for any two consecutive layers of air, the ray of light is incident from a denser to a rarer medium*. As a result, the ray of light is refracted more and more away from the normal and accordingly the angle of incidence goes on increasing. When the angle of incidence becomes greater than the critical angle at some air layer, total internal reflection takes place. Then, the ray of light starts traversing layers of increasing refracting indices and goes on bending gradually more and more towards the normal. Ultimately, when the ray reaches the eye of the observer, it appears to be coming from the point T'. Hence, point T' is the inverted image of point *T* tree.

This inverted image gives the impression that there is a pool of water near the tree.

Looming

Looming is another type of mirage. It is observed when warmer air lies above the cooler air. In Arctic regions, rays of light from a distant object (such as a ship) are refracted downward.

In every cold regions, sometimes an errect, virtual image of a distant object is seen above the object. For example, a ship moving in the sea (Figure 4.19) is observed to be hanging in mid-air, above the ship itself. Sometimes, the curvature (bending) of the light rays may tend to bring into view objects normally below the horizon.

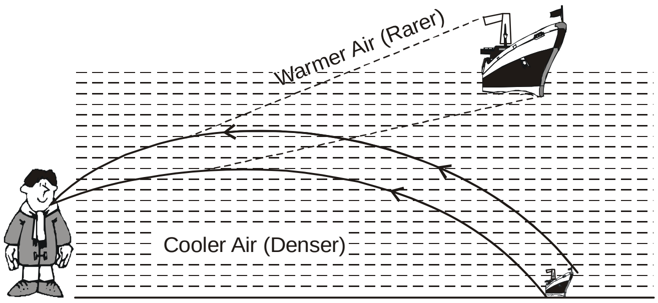


Figure 4.19 : Looming is Caused by Atmospheric Refraction Downward

Optical Fibre Communication

Optical fibres are used to transmit light from one place to the other. The light travels through the fibre as it suffers a series of total internal reflections. The optical fibres consist of thousands of strands of glass or quartz of refractive index about 1.7. The thickness of a strand is about 10^{-6} cm. The strands are coated with a layer of some material of lower refractive index (about 1.5). The ends of the strands are polished and clamped firmly after aligning them carefully. When light is incident at a small angle at one end, it is refracted into the strands (or fibres) and is incident on the interface of the fibres and the coating. The angle of incidence being greater than the critical angle, the rays of light undergoes total internal reflections. It suffers repeated internal reflections till the angle of incident remains greater than the critical angle for the fibre material with respect to the coating material. *Thus, for light, each fibre acts as a pipe and such a bundle of fibres can be used to transmit images along paths of any shape.*

One of the important optical component which works on the principles of refraction is called prism. You will learn about it now. But, before that, answer the following SAQ.

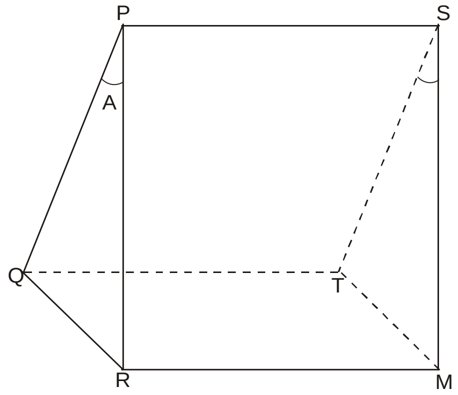
SAQ 3

(a) Calculate the critical angle for a glass-water interface if the refractive indices of glass and water are 3/2 and 4/3 respectively.

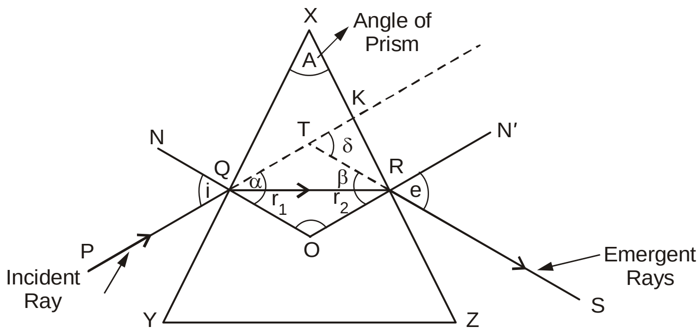
(b) A ray of light is incident from glass on the interface separating it from air at an angle of 40^o and is deviated through 15^o. Calculate the critical angle for the glass-air surface.

4.4.3 Prism

A triangular prism has two refractive surfaces *PQTS* and *PRMS* as shown in Figure 4.20(a). In two dimensions, its principal section is represented by an equilateral triangle *XYZ*, where *XY* and *XZ* are the refracting edges (Figure 4.20(b)). When a ray of monochromatic light is incident on one of the refracting faces *XY*, it undergoes refraction two times at faces *XY* and *XZ* and emerges from the another face of the prism as an emergent ray. As a result, the direction of the emergent ray changes with respect to the incident ray.



(a)



(b)

Figure 4.20 : (a) A Triangular Prism; and (b) Refraction of a Ray of Light by a Prism

You may ask : **How much deviation is suffered by a ray of light when it passes through a prism**? The deviation depends on the geometry and characteristics of the material of the prism. Let us consider a ray of light *PQ* incident on the refracting surface *XY* of the prism at point *Q* (Figure 4.20(b)). The angle of incidence is $\angle PQN$ (= i). The refracted ray (*QR*) makes angles r_1 and r_2 with the normals *NO* and *N ′O* respectively. The emergent ray, *RS*, makes an angle $\angle N'RS = e$ at point *R*. When the incident ray and the emergent ray are extended backwards, they meet at point *T*. **The angle ∠ KTS between these two rays is called angle of deviation, δ (pronounced as delta)**.

Now, from Figure 4.20(b), we can write :

$$i = \alpha + r_1 \qquad \text{(vertically opposite angles),}$$

Similarly, $$e = \beta + r_2 \qquad \text{(vertically opposite angles),}$$

Rearranging the above equations, we get :

$$\alpha = i - r_1; \quad \beta = e - r_2 \qquad \text{. . . (4.23)}$$

Again the geometry of the Figure 4.20(b) suggests that

$$\delta = \angle\alpha + \angle\beta$$

(Because sum of the two internal opposite angles of a triangle is equal to the external angle.)

Substituting the values of α and β from Eq. (4.23), we get :

$$\delta = (i - r_1) + (e - r_2) = i + e - (r_1 + r_2) \qquad \text{. . . (4.24)}$$

Now, let us consider the triangle, $\Delta\ QOR$. Since the sum of three angles of a triangle is 180°, we can write :

$$r_1 + r_2 + \angle QOR = 180^o$$

or $$\angle QOR = 180^o - (r_1 + r_2) \qquad ...(4.25)$$

Further, since the sum of the two opposite angles of a quadrilateral is 180°, we can write :

$$\angle QAR + \angle QOR = 180^o$$

$$\angle QOR = 180^o - \angle QAR \qquad ...(4.26)$$

Thus, from Eqs. (4.25) and (4.26), we get :

$$A = r_1 + r_2 \qquad ...(4.27)$$

Substituting Eq. (4.27) in Eq. (4.24), we get

$$\boxed{i + e = A + \delta} \qquad ...(4.28)$$

Eq. (4.28) indicates that the **sum of the angle of incidence (*i*) and the angle of emergence (*e*) is equal to the sum of the angle of prism (*A*) and the angle of deviation (δ)**. Refer to Figure 4.21 which depicts the plot between the angle of incidence and the angle of deviation for a prism. Note that as the angle of incidence is increased gradually, the angle of deviation (δ) first decreases and attains a minimum value, called **angle or minimum deviation, (δ_m)**, and then starts increasing.

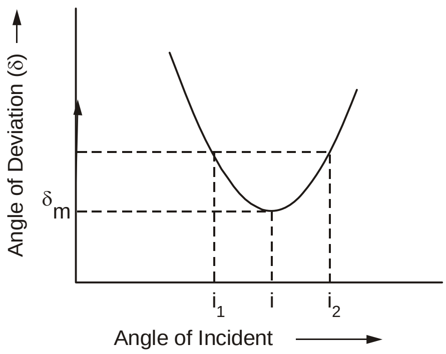


Figure 4.21 : A Plot between Angle of Deviation and the Angle of Incidence for a Prism

The angle of minimum deviation, δ_m, is characterised by the fact that in this case, the angle of incidence is equal to the angle of emergence. This is possible only when the ray of light passes through the prism parallel to the base of the prism. In other words, we say that the light passes through the prism symmetrically and the path of rays can be reversed.

The angle of minimum deviation, δ_m, of a prism is a characteristic properties of a prism and is dependent of the angle of the prism and the refractive index of the material of the prism.

To obtain the relation between these parameters, note that, at δ_m, the angle of emergence, *e*, is equal to the angle of incidence, *i*. Thus, from Eq. (4.24), we have $r_1 = r_2 = r$ (say). Therefore, we can write :

$$r_1 + r_2 = 2r$$

or $$r = \frac{A}{2} \qquad \text{. . . (4.29)}$$

from Eq. (4.27). Further, for $\delta = \delta_m$, $i = e$ and Eq. (4.28) becomes :

$$A + \delta = i + i$$

or, $$i = \frac{A + \delta}{2} \qquad \text{. . . (4.30)}$$

Now using Snell's law of refraction at the face *XY* of the prism, we can write the refractive index of the prism material (with respect to air) as :

$$n = \frac{\sin\, i}{\sin\, r} \qquad \text{. . . (4.31)}$$

Substituting for *i* and *r* respectively Eqs. (4.30) and (4.29) in Eq. (4.31), we get

$$n = \frac{\sin\left(\dfrac{A + \delta_m}{2}\right)}{\sin\left(\dfrac{A}{2}\right)} \qquad \text{. . . (4.32)}$$

Eq. (4.32) gives the relation between the angle of minimum deviation, angle of the prism and its refractive index.

So far, you have learnt about images formed by mirrors and lenses. The image forming ability of these optical components have been used to manufacture a variety of optical instruments. In the following, we discuss some of the most widely used optical instruments namely microscopes, compound microscope and telescopes.

4.5 OPTICAL INSTRUMENTS

The ability of human eye to see is limited. It cannot see very small objects as well as objects at great distances. Optical instruments enhance our ability and facilitate us to see very tiny objects/distant objects. The optical instruments like simple microscope and compound microscope are used to obtain magnified image of an object. In your daily life, you must have come across with the simple microscope or compound microscope. Simple microscope is used by jewellers and watchmakers to see the minute parts. Compound microscope is used for biological investigations such as to see the presence of bacteria, cells, parasites etc. On the other hand, to view distant objects with clarity, we use telescope. You must have used binocular which works on the same principle as the telescope. The telescope is used to explore the finer details of the distant objects like stars.

4.5.1 Simple Microscope

It is an optical instrument consisting of a convex lens of short focal length. Refer to Figure 4.22, which depicts the ray diagram for the image formation by a simple microscope. An object (*AB*) is placed between the optical centre (*C*) and focus (*F*) of the lens. The image (*A′B′*) is **virtual** and **erect** and **magnified** as compared to the object.

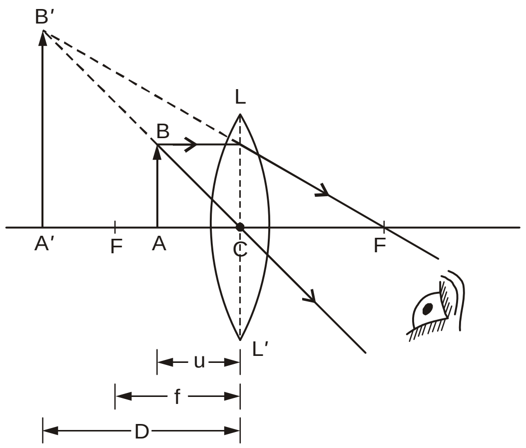


Figure 4.22 : Ray Diagram for the Image Formed by a Simple Microscope

To see the magnified image distinctly, the distance of the object from the lens is adjusted, keeping the lens close to the eye, in such a manner that the image is formed at the **least distance of distinct vision (*D*)** from the eye ($D \approx 25$ cm for normal human eye). At *D*, the image is distinctly visible. The **magnifying power** of a simple microscope is given by :

$$\therefore \qquad m = 1 + \frac{D}{f} \qquad \ldots (4.33)$$

where *f* is the focal length of the lens. Eq. (4.33) shows that *if the focal length of the lens is small, the value of the magnifying power is large*. It is, however, not possible to enhance the magnification beyond a certain value because serious distortions in the image are introduced when the lens is of very small focal length.

4.5.2 Compound Microscope

Compound microscope is used to view extremely small objects which cannot be seen clearly by a simple microscope.

Refer to Figure 4.23, which depicts the ray diagram for the image formation by a compound microscope. Note that, unlike the simple microscope, it consists of two convex lenses called **object lens** and **eye lens**. The lens towards the object is called the objective and the other lens system towards the eyes is called the eyepiece. The objective is of shorter-focal length than the eyepiece. However, the eyepiece lens (L_1L_1') has a large aperture than the object lens (LL') which implies that the former has larger light gathering capacity. The two lenses are mounted coaxially at the two ends of a tube.

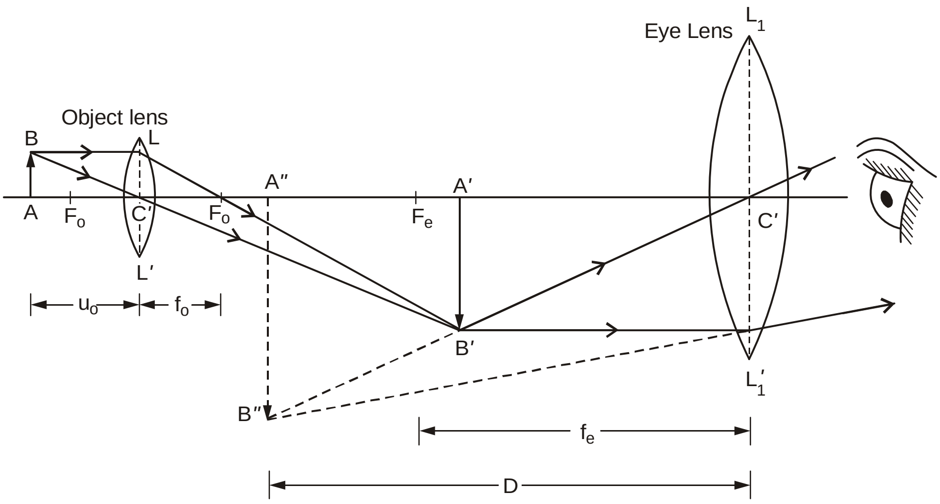


Figure 4.23 : The Ray Diagram for the Image Formed by a Compound Microscope

The magnification of the image obtained by a compound microscope takes place in two stages. When the object is placed at a distance u_0, slightly greater than the focal length (f_o) of the objective lens LL', a real, inverted and magnified image $A'B'$ is formed on the other side of the objective lens. The eyepiece lens $L_1L'_1$ is so adjusted that the image $A'B'$, formed by the objective, is at its focal length (f_e) and acts as an object for the eye lens. The distance of object from the object lens is so adjusted that the final image $A''B''$, which is virtual and magnified, is formed at the least distance of distinct vision (D) from the eye lens (Figure 4.23). In this condition, the compound microscope is said to be in normal use.

The magnifying power of a compound microscope is given by :

$$m = m_o \times m_e$$

where m_o and m_e are the magnification produced by the objective and the eye lenses respectively. We can also write :

$$m = \frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right) \qquad \text{. . . (4.34)}$$

If $u_0 \approx f_0$, the image $A'B'$ must lie between optical centre and focus of the eye lens, so that the final image is formed at least distance of distinct vision from the eye. Since the focal length of the eye lens is also small, the distance of the image $A'B'$ from the object lens is nearly equal to the length of the microscope tube, that is, $v_0 \approx L$. Now, using the sign conventions, we can write :

$$u_o \approx -\ f_o,$$

and $$v_o \approx L$$

Substituting for u_0 and v_0 in Eq. (4.34), we get :

$$m = -\ \frac{L}{f_o}\left(1 + \frac{D}{f_e}\right) \qquad \text{. . . (4.35)}$$

From Eq. (4.35), it is obvious that compound microscope will have large magnifying power if both the f_0 and f_e are small. The negative sign in the expression indicates that the image formed in compound microscope is inverted.

4.5.3 Telescope

You might have used binocular for looking at a distant object or during a cricket match to see clearly and distinctly. It forms enlarged image of a distant object which is not visible distinctly by naked eyes. *A binocular is essentially a telescope which works on the principle of image formation by lens*.

Telescopes are classified into two categories : refracting telescopes and reflecting telescopes. **A refracting telescope uses a pair of lenses and a reflecting telescope uses a combination of lens and a mirror**. The refracting telescopes are further divided into two categories : **astronomical telescope** and **terrestrial telescope**. The telescopes which are used to see heavenly bodies like stars, planets are called **astronomical telescope** whereas the telescopes which are used to see distant terrestrial objects are called **terrestrial telescopes**.

As most of the heavenly bodies are spherical in shape, the formation of inverted image does not affect the observation. But, in case of terrestrial telescope, the objects are on the earth and need not be spherical in shape and it would be necessary to obtain an erect image. Therefore, the terrestrial telescopes have an additional convex lens placed between the objective and the eyepiece which enables us to obtain an erect final image. In the following, we shall confine ourselves to the astronomical telescope only.

Astronomical Telescope

Refer to Figure 4.24 which depicts the ray diagram for image formation by a refracting astronomical telescope. It consists of two convex-lens system. The lens OO' facing the object is called **objective lens** and the other lens EE' is called **eyepiece**. Both the lens are mounted coaxially as in a compound microscope. The objective has a large aperture so that it has a large light gathering power and is of large focal length (f_o). On the other hand, the eyepiece lens has small aperture and is of short focal length (f_e).

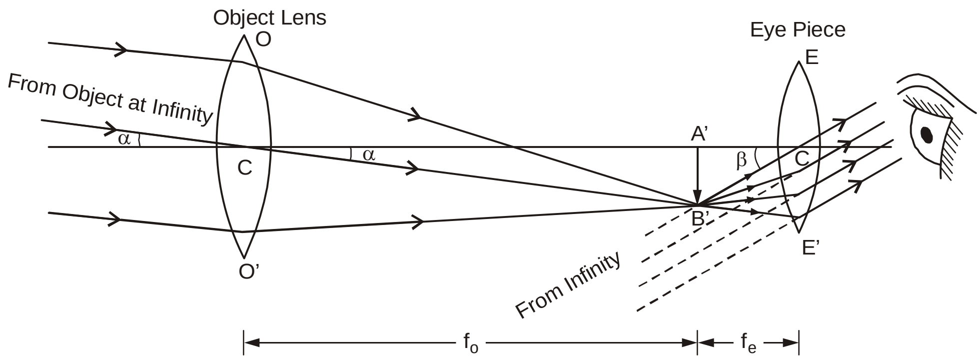


Figure 4.24 : Ray Diagram for the Image Formed by an Astronomical Telescope

As the heavenly objects are located at very far-off distance, the rays of light from them enter the objective lens as a beam of parallel rays. These parallel rays, after refraction through the objective lens, form the real and inverted image $A'B'$ at its focal plane on the other side of the objective. Now, the eyepiece lens EE' is so adjusted that the image $A'B'$ is located at its focal plane. For such an arrangement, the final enlarged image is formed at infinity. Also, for this arrangement of the image $A'B'$, the telescope is said to be in **normal adjustment**.

The magnifying power of a telescope in its normal adjustment is given by :

$$m = -\frac{f_o}{f_e} \qquad \text{. . . (4.36)}$$

The negative sign in Eq. (4.36) indicates that the final image is **real** and **inverted**. Note that the value of the magnifying power (m) will be large if the eyepiece is of short focal length and objective is of large focal length. Also, in normal adjustment, the distance between the two lenses is equal to sum of their focal lengths, $(f_0 + f_e)$.

You know that the visible light constitutes a very small part of the electromagnetic refraction. However, for the purpose of 'seeing', the visible light is very important for us. For a variety of purposes, such as landing and take-off of aircraft during foggy whether, we need to know visibility and other parameters related to the visible light. This is the subject matter of photometry which you will learn now.

4.6 PHOTOMETRY

Photometry is a branch of optics which deals with the measurement of the visible light. To do so, several photometric quantities have been defined. Let us know them now.

Luminous Flux (Φ)

It is defined as the electromagnetic energy emitted per second by a source **over visible wavelength**. The luminous flux is measured in **lumen**. Its symbol is lm. Since lumen is energy per unit time, it is similar to power.

The luminous flux within unit solid angle (one steradian) by a point source having a uniform intensity of one candela. It is found experimentally that about 621 lumens of green light of wavelength 5.540×10^{10} m is equal to 1 watt.

Luminous Intensity (*I*)

A source of light such as a lamp radiates luminous flux in all directions. The question is : **How much of the luminous flux is reaching a point at some distance from the source**? To know this, we define a quantity called luminous intensity.

Consider a point source of light *S* (Figure 4.25) and we are interested to know the luminous flux at point *A* at distance *r* from the source. The luminous intensity (*I*) at point *A* is defined as luminous flux Φ radiated in a small cone of 'solid angle' ω drawn round *SA* with *S* at the apex as shown in Figure 4.25. Mathematically, it is written as :

$$I = \frac{\Phi}{\omega} \qquad \text{. . . (4.37)}$$

The solid angle ω is measured in **steradian** (symhol sr). Therefore, the unit of luminous intensity is lm.sr^{-1}.

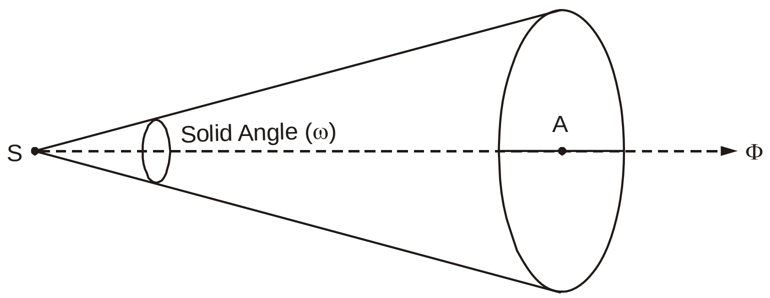


Figure 4.25 : Luminous Flux Radiated by a Point Source *S* in the Solid Angle ω

For practical purposes, luminous intensity is expressed in a unit called **candela** (symbol cd) defined as the luminous intensity of

$\frac{1}{600000}$ m^2 $\left(\frac{1}{60}\text{cm}^2\right)$ of the surface of a black body at the temperature of freezing platinum at pressure 101325 Nm^{-2}. Candela is the SI unit for luminous intensity and its standard is maintained. Thus, 1 lm = 1 cd sr.

Illuminance (*E*)

Illuminance also known as illumination (*E*) and is defined as the power per unit area of an illuminated surface. In other words, *illuminance is luminous flux per unit area*.

You must distinguish this quantity from luminous flux because the later refers to the power emitted by a source of light. The unit of illuminance is **lux** (lx) and is defined as lumen per square meter. Illuminance is practically a very important quantity because this helps us determine whether or not an area is lighted sufficiently for "seeing", that is, for reading and other activities.

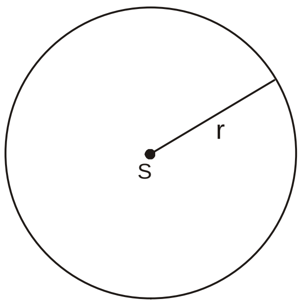


Figure 4.26 : A Spherical Surface of Radius *r* Illuminated by a Source *S*

To find out how illuminance varies with distance from the source, let us imagine concentric spheres of different radii *r* drawn round a small lamp *S* as centre. The total luminous flux from *S* will be incident on the surface areas equal to $4 \pi r^2$ (Figure 4.26). So, according to the definition of illuminance, we have

$$E = \frac{\phi}{4\pi r^2}$$

or

$$E \propto \frac{1}{r^2} \qquad \ldots (4.38)$$

Eq. (4.38) shows that the illuminance varies inversely as the square of the distance from the source.

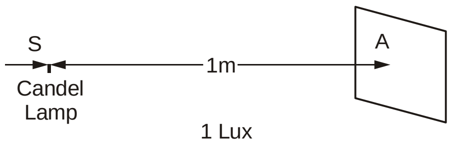


Figure 4.27 : A Plane Surface (*A*) Kept at a Distance of 1 m from Lamp *S*

One lux is the illuminance of a surface *A* one metre away from a lamp *S* of 1 cd when the light falls normally on A (Figure (4.27)).

Luminance (*L*)

Luminance (*L*) of a surface is the luminous flux per unit area coming from that surface. Note that the luminance is with reference to an illuminated surface whereas luminous flux refers to a source of light. To further appreciate the difference between illuminance and luminance, consider something written on a blackboard with chalk. Illuminances of the written words with chalk and the surface of the blackboard are the same. However, the luminance of the chalk is very much higher than that of surface of blackboard since the chalk reflects much more light compared to blackboard.

SAQ 4

(a) An object of size 3.0 cm is placed at a distance of 14 cm in front of a concave lens of focal length 28 cm. Calculate the distance of the image formed. What type of image will it be?

(b) An object is at a distance of 6 m from a convex mirror of focal length 12 cm? Where is the image formed? What is its magnification?

(c) Two thin lenses are in contact and the focal length of the combination is 100 cm. If the focal length of one lens is 20 cm, calculate the power of the other lens.

4.7 SUMMARY

- Image formed by a plane mirror is of the same size as of the object, virtual, erect and laterally inverted.
- In real images, the rays actually pass through the image points whereas in virtual images, the rays of light appear to pass through the image points.
- The refractive index (*n*) of a medium is the ratio of the velocity of light in vaccum to the velocity of light in the medium.
- The mirror formula is

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

where *u* and *v* are the object and image distances from the pole of the mirror, and *f* is the focal length of the mirror.

- The expression for the magnification (*m*) by a spherical mirror is

$$m = \frac{\text{Height of the image}}{\text{Height of the object}} = -\frac{v}{u}$$

In convex mirror, magnification is always positive whereas in concave mirror, magnification for the real image is negative and that for virtual image positive.

- The lens formula is

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

where *f* is the focal length, *v* is the image distance and *u* is the object distance.

- The expression for the magnification by a lens is :

$$m = \frac{v}{u}$$

Magnification (m) is always positive for a concave lens but in a convex lens, m is positive when the image is virtual and is negative when the image is real.

- The power (P) of a lens is the reciprocal of its focal-length (f),

$$P = \frac{1}{f}$$

f is measured in metre. The SI unit of power is diopter (D).

- The relation between refractive index and critical angle is,

$$\frac{n_2}{n_1} = \frac{1}{\sin C},$$

where n_1 and n_2 are the refractive indices of medium 1 (medium of incidence), medium 2 (medium of refraction) respectively and C is the critical angle.

- The expression for the refractive index of the material of the prism is

$$n = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

where the symbols have their usual meaning.

- The magnification produced by a simple microscope (convex lens) is given by :

$$m = 1 + \frac{D}{f}$$

where D is the least distance of distinct vision and f is the focal length.

- The magnification of a compound microscope is

$$m = m_o \times m_e$$

$$= -\frac{L}{f_o}\left(1 + \frac{D}{f_e}\right)$$

where L is the length of the microscope tube, f_o and f_e are the focal lengths of the objective and the eyepiece respectively and D is the least distance of distinct vision.

- The magnification of an astronomical telescope is given by :

$$m = -\frac{f_o}{f_e}$$

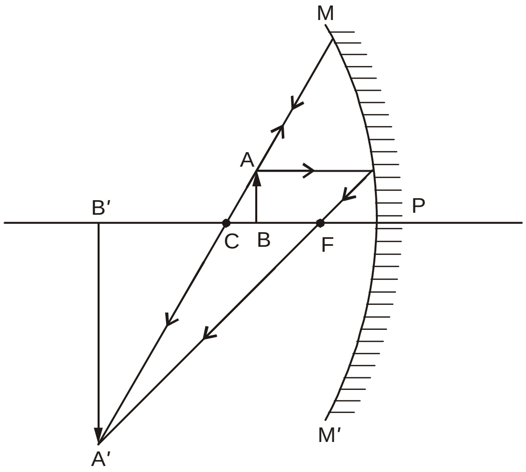
- Luminous flux is defined as the luminous energy emitted per second. It is measured in lumen (lm).

- Luminous intensity of a lamp is defined as the luminous flux per unit solid angle. It is measured in lm sr^{-1}. Practical unit of luminous intensity is candela.

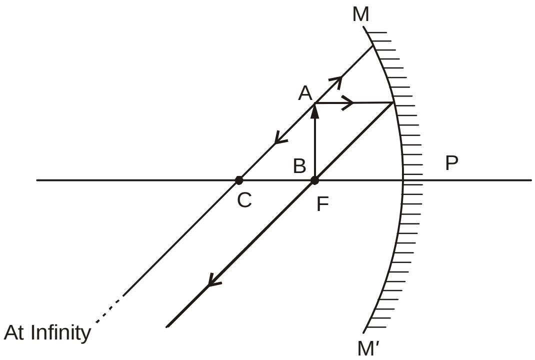
4.8 ANSWERS TO SAQs

SAQ 1

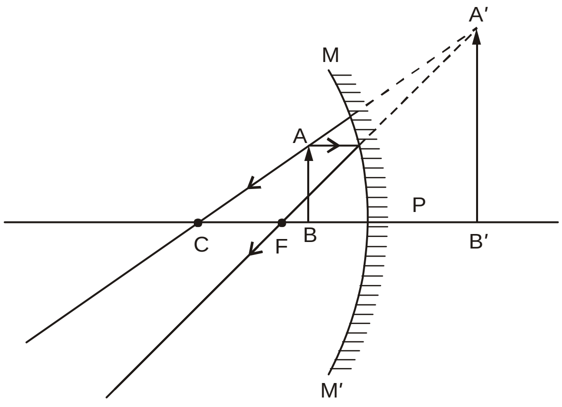
(a)



(b)

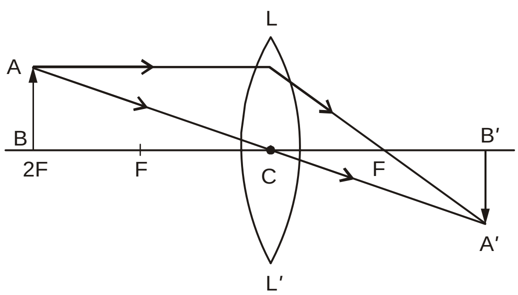


(c)

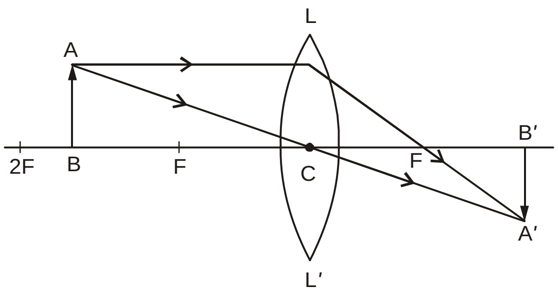


SAQ 2

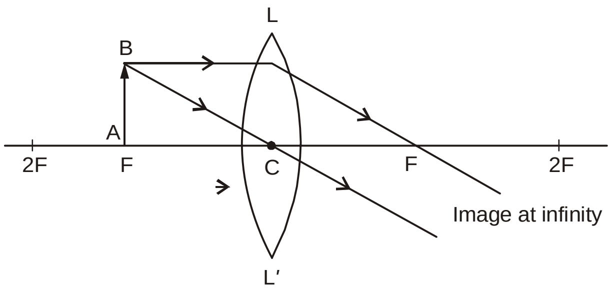
(a) (i)



(ii)

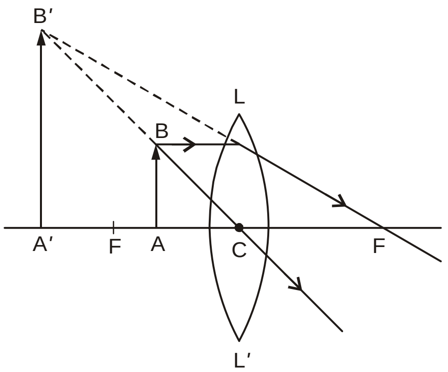


(iii)



4.2

(iv)



(b) The convex lens forms real and inverted image. Thus, $m = -19$.

The image lies on the other side of the lens. Thus, $v = +10$ m.

Further, using Eqs. (4.15) and (4.20), we can write :

$$m = \frac{f - v}{f}$$

or, $$-19f = f - (10 \text{ m})$$

or, $$20f = 10 \text{ m}$$

or, $$f = 0.5 \text{ m}$$

SAQ 3

(a) Refractive index of glass = 1.5

Refractive index of water = 1.33

Therefore, the refractive index of glass with respect to water is

$$\frac{1.5}{1.33} = 1.1278 .$$

But, from Eq. (4.22), the ratio of the refractive index of the medium of incidence (glass) to the refractive index of the medium of refraction (water) is given as $\frac{1}{\sin C}$, where C is the critical angle for glass-water interface. Thus, we can write :

$$\frac{1}{\sin C} = 1.1278$$

or, $$C = 62^{\text{o}}\ 27'$$

(b) As per the problem, the angle of refraction $= 40^{\text{o}} - 15^{\text{o}} = 25^{\text{o}}$

Using Snell's law (Eq. (4.21)) :

$$\frac{n_{\text{glass}}}{n_{\text{air}}} = \frac{\sin i}{\sin r}$$

$$= \frac{\sin 40^{\text{o}}}{\sin 25^{\text{o}}}$$

$$= \frac{0.643}{0.423}$$

$$= 1.52$$

If C is the critical angle for glass-air interface, then

$$\frac{n_{\text{glass}}}{n_{\text{air}}} = \frac{1}{\sin C}$$

or, $$\sin C = \frac{1}{1.52}$$

$$= 0.6579$$

$\therefore$ $$C = 41.1^{\text{o}}$$

SAQ 4

(a) As per the problem, we have O (size of the object) = + 3.0 cm; f = – 28 cm; u = – 14 cm.

According to the lens formula :

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

or, $$\frac{1}{v} = \frac{1}{f} + \frac{1}{u}$$

$$= \frac{1}{-28} + \frac{1}{-14}$$

or, $$v = -9.3 \text{ cm.}$$

The negative sign of *v* indicates that the image is on the same side of the object, and therefore, it is virtual. Now, the magnification is given as :

$$m = \frac{v}{u}$$

where *I* is the size of the image.

or, $$I = \frac{v}{u} \times O$$

$$= \frac{-9.3}{-14} \times 3.0$$

$$= 1.99 \text{ cm}$$

Thus, the image is diminished in size.

(b) As per the problem, we have : $u = -6$ m = – 600 cm; $f = +12$ cm.

The mirror formula is :

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$= \frac{1}{12} + \frac{1}{600}$$

$$= \frac{51}{600}$$

or, $$v = 11.76 \text{ cm}$$

or, $$m = \frac{v}{u}$$

$$= \frac{11.76 \text{ cm}}{600 \text{ cm}}$$

(c) Let *F* is the focal length of combination and f_1 and f_2 are the focal length of each lens. Thus, we have :

$$F = 100 \text{ cm} = 1 \text{ m}; \; f_1 = 20 \text{ cm}$$

The power of the combination is

$$P = \frac{1}{F} = 1\text{D}$$

$$P_1 = \frac{1}{f_1} = \frac{1}{0.2 \text{ m}} = 5\text{D}$$

But, the power of the combination is algebraic sum of the powers of individual lenses, that is,

$$P = P_1 + P_2$$

or, $$P_2 = P - P_1$$

$$= 1 - 5$$

$$= -4\text{D}$$