UNIT 3 FRICTION

Structure

3.1 INTRODUCTION

In the preceding units, it is assumed that the surfaces in contact are smooth and the forces between the bodies act normally to the surface of contact. However, in practice, it is impossible to have perfectly smooth surface. There always exists microscopic roughness which tends to prevent any possible sliding motion between the two bodies. In this unit, you are going to learn the laws governing dry friction formulated by Coulomb and their applications in different situations. You will study the relation between the normal and frictional forces at a point of contact between two non-lubricated rigid bodies. You will also study the frictional forces in various devices such as inclined plane, wedges, screws, belts and rope drives. The study of frictional forces is essential to solve the practical problems in Engineering Mechanics.

Objectives

After studying this unit, you should be able to

- understand the laws of dry friction,
- determine the magnitudes of frictional forces in different situations,
- work out friction-related quantities such as coefficient of angle of friction, angle of repose to solve problems involving dry friction, and
- apply laws of friction to various devices like inclined plane, wedges, screws, belts and rope drives.

3.2 LAWS OF FRICTION

As a result of studies carried out by Coulomb in 1781, one can work out friction-related quantities such as coefficient of friction, angle of friction, angle of repose to solve problems involving dry friction, and he noted that :

(i) The total amount of friction that can be developed is independent of the magnitude of the area of contact.

(ii) The total frictional force that can be developed is proportional to the normal force transmitted across the surface of contact.

(iii) For low velocities, the total amount of friction that can be developed is practically independent of velocity. However, it is less than the frictional force corresponding to impending motion

Consider a body of weight W resting on a floor. Let P be the force applied to it as shown in Figure 3.1. As forces P and W are concurrent, there must be a third force (provided by the floor) equal in magnitude and opposite in nature to the resultant of P and W to keep the body in equilibrium. Let it be denoted by R. The normal and frictional (i.e., parallel to the floor) components of R are represented by N and F, respectively.

As P increases, F will also increase corresponding to the limiting condition of impending motion. The maximum value of F that can be developed is called **limiting static friction** and is proportional to the normal reaction N.

Figure 3.1

Mathematically $F \propto N$

$\therefore$ $F = \mu N$.

where, μ is a constant and is called the **coefficient of static friction.** The angle between the normal reaction N and the resultant reaction R is called the angle of friction. If it is denoted by ϕ, then we get

$$\tan \phi = \frac{F}{N} = \mu$$

Therefore, the tangent of angle of friction is equal to the *coefficient of friction.*

It is our common experience that when the body begins to move, there is a decrease in frictional effect from the limiting static friction. An idealized plot of this action as a function of time is shown in Figure 3.2.

This shows that there is a drop from the limiting frictional effect to a frictional effect that is constant with time. It is independent of the velocity of the object.

Generally, the coefficient of friction for dynamic condition are about 25 percent less.

Figure 3.2

Table 3.1 gives the values of static coefficients of friction for various material-on-material combination.

Table 3.1

Surface of Contact	μ
Steel on cast iron	0.40
Copper on steel	0.36
Hard steel on hard steel	0.42
Mild steel on mild steel	0.57
Rope on wood	0.70
Wood on wood	0.20-0.75

(**Source :** F. P. Bowden and D. Tabor (1950), *The Friction and Lubrication of Solids*, Oxford University Press, New York.)

3.3 PROBLEMS INVOLVING DRY FRICTION

Just before the conditions of impending motion, the bodies are in equilibrium. Using the equations of equilibrium, you can work out the unknown frictional forces and hence determine the various friction-related quantities. The most important thing is to ascertain the direction of frictional forces which always oppose possible, impending or actual relative motion at the contact surfaces.

Example 3.1

Find the force *P* needed to start block B as shown in Figure 3.3 moving to the right if the coefficient of friction is 0.3 for all surfaces of contact. Block A weighs 80N and Block B weighs 160 N.

(a) **(b)**

Figure 3.3

Solution

Figure 3.3(a) shows the free-body diagrams for block A and block B. As the possible motion of the block B is towards right, the direction of frictional forces, F_A and F_B, are shown acting towards left, for block B. When block B moves towards right the relative motion of block A will be towards left and hence the frictional force, F_A, for block A is acting towards right as shown in figure.

Now the conditions of equilibrium can be applied to determine the force P.

For Block A

$\Sigma F_x = 0$

$$\therefore \quad F_A = T \cos 30^{\circ} = 0.866\, T \qquad \ldots (3.1)$$

$\Sigma F_y = 0$

$$\therefore \quad N_A = W_A - T \sin 30^{\circ}$$

$$= 80 - 0.5\, T \qquad \ldots (3.2)$$

Also $\quad F_A = \mu\, N_A \qquad \mu = 0.3$

$$\therefore \quad 0.866\, T = 0.3\,(80 - 0.5\, T)$$

$$T = \frac{24}{1.016} = 23.62 \text{ N}$$

$$\therefore \quad F_A = 0.866 \times 23.62 = 20.45 \text{ N} \qquad \ldots (3.3)$$

$$\therefore \quad N_A = 80 - 0.5 \times 23.62$$

$$= 68.19 \text{ N}$$

For Block B

$\Sigma F_y = 0$

$$\therefore \quad N_B = N_A + W_B$$

$N_B = 68.19$ N and $W_B = 160$ N

$$= 68.19 + 160$$

$$= 228.19 \text{ N} \qquad \ldots (3.4)$$

$$F_B = \mu\, N_B$$

$\mu = 0.3$ and $N_B = 228.19$

$$F_B = 0.3 \times 228.19$$

$$= 68.46 \text{ N} \qquad \ldots (3.5)$$

$\Sigma F_x = 0$

$$\therefore \quad P = F_A + F_B$$

from Eqs. (3.3) and (3.5), $F_A = 20.45$ N and $F_B = 68.48$ N

$$\therefore \quad P = 20.45 + 68.46$$

$$= 88.91 \text{ N} \qquad \ldots (3.6)$$

Therefore, a force of 88.91 N is needed to move the block B to the right. The required magnitude of the force to move the block B towards right is 88.91 N.

Example 3.2

A force of 200 N inclined at 60° to the horizontal is applied to the block A weighing 400 N. Determine whether block A moves if the coefficient of friction is 0.5. If not, then find the maximum value of the coefficient of friction when it is just on the point of moving (Figure 3.4).

Solution

There is a possibility of movement of block A towards right. Therefore, the direction of frictional force, F_A, will be towards left.

$$N_A = W - P \sin 60^{\circ}$$

where, $W = 400$ N and $P = 200$ N

$$\therefore \quad N_A = 400 - 200 \sin 60^{\circ}$$

$$= 226.795 \text{ N}$$

$$F_{A(\max)} = \mu N_A$$

$$\mu = 0.5 \quad \text{and} \quad N_A = 226.795 \text{ N}$$

$$\therefore \quad F_{A\,(\max)} = 0.5 \times 226.795$$

$$= 113.398 \text{ N}$$

Figure 3.4

This is the limiting static friction that is developed between the surfaces of contact. As the horizontal component of P (i.e 200 cos 60° = 100 N) is less than 113.398 N, the block A will not move if the coefficient of friction is 0.5.

But, if the limiting static friction is less than the horizontal component of P then the block A will move.

For this case,

$$\therefore \quad F_A \leq 100 \text{ N}$$

$$\therefore \quad F_{A(\max)} = 100$$

$$\therefore \quad \mu N_A = 100$$

$$\therefore \quad \mu_{(\max)} = \frac{100}{N_A} = \frac{100}{226.795}$$

= 0.4409

Therefore, if the coefficient of friction is 0.4409 or less, then the block A will move under the given conditions.

SAQ 1

(a) Mark the directions of frictional force and normal reaction at the surface of contact for block A shown in Figure 3.5. Determine the developed frictional force. What will be the value of limiting frictional force if the coefficient of friction is 0.3?

Figure 3.5

(b) A ladder weighing 80 N rests at a corner as shown in Figure 3.6. What is the minimum angle (with the horizontal) possible before the slip occurs? The coefficient of static friction at A is 0.2 and at B is 0.3.

Figure 3.6

(c) Two blocks – A weighing 30 N and B weighing 50 N – are on a rough horizontal surface as shown in Figure 3.7. Find the minimum value of *P* just sufficient to move the system. If the coefficient of friction between block A and the ground is 0.28 and that between block B and the ground is 0.22, find the tension in the string.

Figure 3.7

(d) For the system shown in Figure 3.8, find the value of load W so that the blocks A and B are just on the point of sliding. The coefficient of friction between the blocks and the ground is 0.25. The weight of the blocks are 800 N each.

Figure 3.8

(e) A force P is applied at an angle α to the packing crate measuring 0.5 m × 0.8 m as shown in Figure 3.9. If the coefficient of friction is 0.3, determine the largest allowable value of the angle α and the corresponding value of P if it moves to the left without tipping.

Figure 3.9

3.4 INCLINED PLANE

One of the simplest engineering devices of lifting loads to higher altitudes is the inclined plane. The component of the weight of body along the inclined plane opposes the movement if the body has to move up or helps the action if it has to move down.

Consider a body resting on an inclined plane which is just on the point of moving down. The maximum angle of inclination at which this happens is called the **angle of repose.**

As the body is just on the point of moving down, the direction of frictional force will be acting up the inclined plane.

Resolving the forces along and the normal to the inclined plane and applying the conditions of static equilibrium (Figure 3.10(a)), we get

Figure 3.10

$\Sigma F_x = 0$

$\therefore \qquad F_A = W \sin \alpha$

$\Sigma F_y = 0$

$\therefore \qquad N_A = W \cos \alpha$

But $\qquad \mu = \dfrac{F_A}{N_A} = \dfrac{W \sin \alpha}{W \cos \alpha} = \tan \alpha$

This shows that the tangent of the angle of repose is equal to the coefficient of friction.

Also, we know, $\mu = \tan \phi$

where, $\qquad \phi$ = angle of friction

Therefore, angle of repose is equal to the angle of friction.

If the angle of inclination α of the plane is less than the angle of repose ϕ, the body will be at rest and an external force will be required to move it. If $\alpha > \phi$ then the body will run down the plane and an external force will be required to prevent the body from running down.

Example 3.3

A body weighing 500 N is resting on an inclined plane making an angle of 30^o with the horizontal. The coefficient of friction is 0.3. A force *P* is applied parallel to and up the inclined plane. Determine the least value of *P* when the body is just on the point of movement.

(i) Case I : moving down, and

(ii) Case II : moving up.

Solution

Here, the angle of friction $\phi = \tan^{-1} (0.3) = 16.699^o$

$= 16^o\, 41'\, 57''$

$\therefore \qquad \alpha > \phi$

Case I

When the body is just on the point of moving down, the frictional force will be acting upwards along the plane.

Figure 3.11(a)

Resolving all forces parallel to the inclined plane, we get

$$P_1 + F_1 - W \sin 30^o = 0$$

$\therefore \qquad P_1 = 500 \sin 30^o - F_1$

$= 250 - F_1 \qquad \text{. . . (3.7)}$

Resolving all the forces normal to the inclined plane, we get

$$N_1 - W\cos 30^o = 0$$

$\therefore$ $N_1 = 500 \cos 30^o = 433$ N . . . (3.8)

$$F_1 = \mu\, N_1 = 0.3 \times 433 = 129.9 \text{ N}$$

Putting this in Eq. (3.7), we get,

$$P_1 = 250 - 129.9 = 120.1 \text{ N}$$

Case II

When the body is just on the point of moving up, the frictional force will be acting downwards.

Figure 3.11(b)

Resolving all the forces parallel to the inclined plane, we get

$$P_2 - F_2 - W \sin 30^o = 0$$

$\therefore$ $P_2 = 500 \sin 30° + F_2 = 250 + F_2$. . . . (3.9)

Resolving all the forces normal to the inclined plane, we get

$$N_2 - W \cos 30^o = 0$$

$\therefore$ $N_2 = 500 \cos 30° = 433$ N . . . (3.10)

$\therefore$ $F_2 = \mu\, N_2 = 0.3 \times 433 = 129.9$ N

Putting the value of F_2 in Eq. (3.9), we get

$$P_2 = 250 + 129.9 = 379.9 \text{ N}$$

Thus, it is seen that when the applied force is 379.9 N or more the body will move up and when it is 120.1 N or less it will move down. When the force applied is between 120.1 N and 379.9 N, the body will neither move upwards nor downwards, experiencing variable friction varying from 129.9 N acting upwards decreasing to zero and then increasing up to 129.9 N acting downwards.

The mistake most frequently made in the solution of a problem involving friction is to write the friction force in the form $F = \mu$ N. It is to be remembered that only in case of impending or actual sliding motion of bodies with respect to one another the frictional force will be maximum, i.e. $F_{max} = \mu$ N. In all other cases, the frictional force acting on a body is found by solving the equations of static equilibrium of the body.

SAQ 2

(a) A crate weighing 5 kN is kept on an inclined plane making an angle of 30^{o} with the horizontal.

(i) Determine the value of *P* (the horizontal force) required to move the crate up the plane.

(ii) What is the minimum value of *P* required to keep the crate from sliding down the plane?

(iii) For what range of value of *P* will the crate remain in equilibrium position shown in Figure 3.12.

Figure 3.12

(iv) If *P* = 5.2 kN, find the magnitude and direction of the frictional force acting on the crate.

(b) If the weight of block Q is 1.2 kN, find the minimum value of weight of block P to maintain the equilibrium as shown in Figure 3.13.

Figure 3.13

(c) Blocks M and N rest on an inclined plane as shown in Figure 3.14. The coefficient of friction between block M and block N is 0.4 and that between block N and the plane is 0.5. If the weight of block M is 600 N and that of N is 800 N, for what value of θ, the motion of one or both of the blocks is impending? If $\theta = 15^{o}$, find frictional forces between M and N and between N and the plane.

Figure 3.14

3.5 WEDGE FRICTION AND SCREW FRICTION

3.5.1 Wedge Friction

Wedges are generally used to move heavy loads by applying a force which is considerably smaller than the weight of the load. Owing to the friction existing between the surfaces in contact, the wedge remains in place after being forced under the load. Therefore, for small adjustments in the position of heavy pieces of machinery wedges are extensively used.

The problems involving wedges can be solved by applying the friction laws to the various parts of the device. The example given below will illustrate the procedure to solve problems involving wedges.

Example 3.4

A block weighing 800 N is raised up with the help of two wedges – 6^o wedge B and C of negligible weights as shown in Figure 3.15. If the coefficient of static friction is 0.25 for all surfaces of contact, determine the smallest force *P* to be applied to raise the block A.

Figure 3.15

Solution

Let us first draw the free-body diagrams for block A and wedges B and C. Block A is in contact with the vertical wall and the horizontal surface of block B. Therefore, the normal reactions N_1 and N_2 will be perpendicular to the respective surfaces of contact as shown in Figure 3.16. The direction of movement of block A with respect to the wall being vertically upwards, the direction of frictional force will be vertically downwards as friction opposes the motion. Similarly, the motion of block A with respect to wedge B being towards right, the direction of frictional force at the surface of contact of block A and the wedge will be towards left as shown in the free body diagram of block A.

In the limiting condition, we know $F_1 = \mu\, N_1$ and $F_2 = \mu\, N_2$. Therefore, there are only two unknowns : N_1 and N_2 and two equations of equilibrium being available, we can find the values of N_1 and N_2.

$\Sigma\, F_x = 0$

$\therefore \qquad N_1 - F_2 = 0$

$\therefore \qquad N_1 = F_2 = 0.25\, N_2 \qquad \ldots (3.11)$

$\Sigma\, F_y = 0$

$\therefore \qquad N_2 - W - F_1 = 0$

$$\therefore \quad N_2 - 800 - 0.25\ N_1 = 0$$

$$\therefore \quad N_2 - 800 - 0.25\ (0.25\ N_2) = 0$$

$$\therefore \quad N_2\ (1 - 0.0625) = 800$$

$$\therefore \quad N_2 = \frac{800}{0.9375} = 853.33\ \text{N} \qquad \text{. . . (3.12)}$$

Figure 3.16 : Free-body Diagram of Block A

Now, let us draw the free-body diagram of wedge B. Keeping in mind that the frictional forces oppose the motion and the normal reactions, as the name suggests, are perpendicular to the surfaces of contact, the various forces acting on wedge B will be as indicated in the free-body diagram of wedge B shown in Figure 3.17.

Figure 3.17 : Free-body Diagram of Wedge B

Applying the equations of equilibrium, we get

$\Sigma\ F_y = 0$

$$\therefore \quad N_3 \cos 6^o - N_2 - F_3 \sin 6^o = 0$$

$$\therefore \quad N_3 \cos 6^o - N_2 - 0.25\ N_3 \sin 6^o = 0 \text{ (as } F_3 = 0.25\ N_3)$$

$$\therefore \quad 0.9945\ N_3 - 853.33 - 0.0261\ N_3 = 0$$

$$\therefore \quad N_3 = \frac{853.33}{0.9684} = 881.18\ \text{N} \qquad \text{. . . (3.13)}$$

$\Sigma\ F_x = 0$

$$\therefore \quad F_2 + F_3 \cos 6^o + N_3 \sin 6^o - P = 0$$

$$\therefore \quad P = F_2 + F_3 \cos 6^o + N_3 \sin 6^o$$

$$F_2 = 0.25 N_2, \text{ and } F_3 = 0.25 N_3$$

$$\therefore \quad P = 0.25\ N_2 + 0.25\ N_3 \cos 6^o + N_3 \sin 6^o$$

Putting the values of N_2 and N_3, we get

$$0 = 0.25 \times 853.33 + 0.25 \times 881.18 \cos 6^o + 881.18 \sin 6^o$$

$$= 213.33 + 219.09 + 92.11$$

$$= 524.53 \text{ N} \qquad \ldots (3.14)$$

Therefore, a force of 524.53 N is required to raise the block A.

You can solve the problem by constructing the triangle of forces R_1, R_2 and W for block A, where R_1 is the resultant reaction of N_1 and F_1 and R_2 is the resultant reaction of N_2 and F_2 respectively.

As the three forces keep the block in equilibrium, the forces must the concurrent. This can be solved graphically or by using Lami's theorem for three concurrent forces (Figure 3.18).

Figure 3.18

By applying Lami's Theorem, we get

$$\frac{W}{\sin (90^o + 2\phi)} = \frac{R_1}{\sin (180^o - \phi)} = \frac{R_2}{\sin (90^o - \phi)}$$

where, ϕ = angle of friction $= \tan^{-1} \mu = \tan^{-1} (0.25)$

$$= 14.036^o$$

$$\therefore \qquad R_2 = \frac{W \cos \phi}{\cos 2\phi} = \frac{W \cos 14.036^o}{\cos 28.072^o}$$

$$= \frac{800 \times 0.9701}{0.8823} = 879.61 \text{ N}$$

Similarly, for wedge B, there are three forces acting : R_2, R_3 and P

where R_2 = resultant reaction of N_2 and F_2

R_3 = resultant reaction of N_3 and F_3

The three forces acting are as shown in Figure 3.19.

By applying Lami's Theorem, we get

$$\frac{R_2}{\sin (90^o + \phi + 6^o)} = \frac{P}{\sin (180^o - 2\phi - 6^{o)}} = \frac{R_3}{\sin (90^o + \phi)}$$

$$\therefore \qquad P = \frac{R_2 \sin (2\phi + 6^o)}{\cos (\phi + 6^o)}$$

$$= \frac{879.61 \sin 34.072^o}{\cos 20.036^o} = 524.53 \text{ N}$$

Figure 3.19

3.5.2 Screw Friction

A screwjack is a device used for lifting or lowering heavy loads by applying comparatively smaller efforts at the end of the lever. The thread of a screw jack may be considered as inclined plane wound round a cylinder and the principles used in solving problems on inclined plane can be applied to solve problems involving screw friction. If α is the angle of the inclined plane and ϕ is the angle of friction, we know that the horizontal force required to pull the load up is given by

$$P = W \tan(\alpha + \phi)$$

This force P which drags the load along the inclined plane is related to the force P_1 applied at the end of the lever of the screwjack. This can be found out by taking moments of the forces about the center line of the cylinder. If L is the length of the lever and r is the mean radius of the screw, we get

$$P_1 \times l = P \times r \qquad \ldots (3.16)$$

$$\therefore \qquad P_1 = \frac{r}{l}\, p$$

$$\therefore \qquad P_1 = \frac{r}{l}\, W \tan(\alpha + \phi)$$

$$= \frac{r}{l}\, W \, \frac{(\tan\alpha + \tan\phi)}{(1 - \tan\alpha \tan\phi)}$$

when there is one complete revolution, the load is raised through one pitch, i.e. centre to centre distance between two consecutive threads.

$$\therefore \qquad \tan\alpha = \frac{p}{2\pi r}$$

and $\qquad \tan\phi = \mu$

Using these relations we can work out the horizontal effort P_1 required to raise the load up.

If the load remains in position even after removal of the effort P_1 the screwjack is said to be self-locking. It does not work in reverse direction because the angle of inclination, in such cases, will be less than the angle of friction

$$\alpha < \phi \;\therefore\; \tan\alpha < \tan\phi$$

$$\therefore \qquad \tan\alpha < \mu \text{ i.e. } \mu > \tan\alpha$$

Therefore, if the coefficient of friction is greater than $\frac{P}{2\pi r}$, the screwjack will be self-locking. To lower the load, the effort P at the thread required will be $W \tan(\phi - \alpha)$ hence the effort at the end of the lever will be given by

$$P_2 = \frac{r}{t} \times W \times \tan(\phi - \alpha)$$

SAQ 3

(a) Two 8^o wedges are used to push a block horizontally as shown in Figure 3.20. If coefficient of friction is 0.25 for all surfaces of contact, determine the minimum load P required to push the block weighing 6 kN.

Figure 3.20

(b) Two wedges lift a heavy block of 8 kN as shown in Figure 3.21. If the angle of wedges is 10^o and the coefficient of friction is 0.3 for all surfaces of contact, find the value of P required to drive the wedges under the load.

Figure 3.21

(c) The pitch of the thread of a screwjack is 5 mm and its mean diameter is 60 mm. The coefficient of friction is 0.08. Find the force that should be applied at the end of the lever 200 mm long measured from the axis of the screw (i) to raise a load of 20 kN, and (ii) to lower the same load.

(d) The pitch of a square threaded screwjack is 8 mm and the mean diameter is 50 mm. The length of the lever is 400 mm. If a load of 2 kN is to be lifted, what force at the end of the lever will be required? Take $\mu = 0.2$. State with reasons whether the screw is self-locking or not.

3.6 SIMPLE MACHINES

A machine is a tool by means of which, a given force can be tackled by applying another force of suitable magnitude, direction, sense or line of action or a combination of these. In lifting machines, which are used to lift heavy weights, the former (i.e. the force to be overcome) is called the 'load' and has a large magnitude while the latter (i.e. the force to be applied) is called 'effort' and must be smaller so that hoisting, holding or lowering of heavy weights is facilitated.

3.6.1 Some Basic Terms

Let 'W' denotes the load and 'P' the effort, which is necessary to lift the load 'W'. Let 'x' and 'y' be the displacements of the points of application of W and P, respectively as measured in their respective directions. The ratio $\frac{W}{P} = M$ is called the 'mechanical advantage' (M. A.) of the machine, while the ratio $\frac{y}{x} = V$ or

(V. R.) is called its 'velocity ratio'. The work done $(W \times x)$, is called the output of the machine while the work put in by the effort which is $= P \times y$ in achieving the same, is called the input to the machine. For an ideal or frictionless machine (by virtue of the principle of conservation of energy), there is no loss of energy, hence we have :

Output from the machine = Input into it

i.e. $$W x = P y$$

or $$\frac{W}{P} = \frac{y}{x}, \qquad \text{i.e.} \qquad M = V$$

i.e. the Mechanical Advantage = Velocity Ratio for an ideal machine. If the ideal load is W_1 and the actually applied effort = P, we shall have

$$W_I = P \cdot \frac{y}{x} = PV \qquad \ldots (3.17(a))$$

or Ideal Load = Actual Effort × Velocity Ratio, and similarly, if we call ideal effort P to lift a given load W (actual load), we shall have

$$P_I = \frac{W}{y/x} = \frac{W}{V} \qquad \ldots (3.17(b))$$

or Ideal Effort =Actual load ÷ Velocity Ratio.

It is clear that for lifting a given load W through a given height, the smaller the effort, the greater will be the displacement. This applies much more to an actual machine because howsoever high the standard of its workmanship and lubrication may be, a certain amount of friction is always present in it, with the result that out of the work put into it, a portion is used up against friction and hence not available for raising the load, i.e.

$W.x$ is always $< P.y$ in an actual machine.

Since $$Wx < Py,$$

$$\frac{W}{P} < \frac{y}{x} \qquad \text{or} \qquad M < V \qquad \ldots (3.18(a))$$

In other words, Mechanical Advantage is always less than Velocity Ratio in an actual machine.

Now the efficiency (η) of a machine is defined as

$$\eta = \frac{\text{Output}}{\text{Input}} = \frac{Wx}{Py} = \frac{M}{V}. \qquad \ldots (3.18(b))$$

in words, $\text{Efficiency} = \dfrac{\text{Mechanical Advantage}}{\text{Velocity Ratio}}$

and is generally expressed as a percentage, which naturally is always less than 100%.

Now since $W_1 = PV = \dfrac{Py}{x}$

We have, $Py = W_1 x$.

From $$\eta = \frac{Wx}{W_1 x} = \frac{W}{W_1} \qquad \ldots (3.18(c))$$

In words,

$$\text{Efficiency} = \frac{\text{Actual load}}{\text{Ideal load}},$$

and the ideal load will always be greater than the actual load.

Also since $P_1 = \dfrac{W}{V} = \dfrac{W}{\frac{y}{x}}$ from Eq. (3.17(b))

We have $Wx = P_1 y$

or $$\eta = \frac{P_1 y}{Py} = \frac{P_1}{P} \qquad \ldots (3.18(d))$$

In words, $\text{Efficiency} = \dfrac{\text{Ideal Effort}}{\text{Actual Effort}}$

And ideal effort is always less than the actual effort, which is obviously due to the friction in the machine. Hence we may say that additional effort due to friction

$$= (P - P_1) = \left(P - \frac{W}{V}\right),$$

or effort lost in friction $= \left(P - \dfrac{W}{V}\right)$ . . . (3.19(a))

and similarly, additional load created by friction

$$= (W_1 - \text{W}) = (PV - W) \qquad \ldots (3.19(b))$$

where P and W are values of actual effort and load respectively. For any machine, its velocity ratio is solely dependent on the geometry of its components and has nothing to do with the condition of its lubrication or maintenance, which means that the given drawing or details of a machine, we can determine its velocity ratio and its value remains the same for any load. The mechanical advantage, however, is dependent in addition on the workmanship that went into the fabrication of the machine, its standard of maintenance, conditions regarding lubrication etc. at the time of the test. The M. A. also varies with the load since friction is dependent on load. It is of course true that the better the workmanship, maintenance and lubrication of a machine the closer its mechanical advantage approaches the velocity ratio and its efficiency, unity.

3.6.2 Some Basic Machines

Now we will work out the velocity ratios for few basic machines.

(a) Lever

Consider the lever shown in Figure 3.22 with its fulcrum at O, and arms '*a*' and '*b*' at the tips of which the load and the effort are applied respectively.

Figure 3.22

For a small rotational displacement of the lever '$d\theta$' when the load is lifted through a height dx we have $dx = a\, d\,\theta$ and $dy = b\, d\theta$

$$\therefore \qquad V = \frac{dy}{dx} = \frac{bd\,\theta}{ad\,\theta} = \frac{b}{a}, \qquad \text{. . . (3.20)}$$

$$\text{i.e.} \qquad V = \frac{\text{Length of effort arm}}{\text{Length of load arm}}.$$

(b) Simple Pulley

Consider the pulley shown in Figure 3.23 in which A and B denote the initial positions of the load and effort respectively and AA′ and BB′ the corresponding final ones. Evidently in this case, considering the rope to be inextensible, $y = x$ from which we have

$$V = \frac{y}{x} = 1 \qquad \text{. . . (3.21)}$$

Figure 3.23 : A Simple Pulley

(c) Inclined Plane

Consider an inclined plane whose inclination to the horizontal is θ, on which a block (weight W) is placed. It is moved by effort P applied in the horizontal direction (Figure 3.24).

If the block is moved a distance AA′ ('s') along the plane, we have $x = s \sin\theta$ which is the component of the displacement of W in its direction and $y = s \cos \theta$, which is the component of the displacement of P in its direction.

Figure 3.24 : Inclined Plane

Here $\therefore \quad V = \frac{y}{x} = \frac{s \cos\theta}{s \sin\theta} = \cot\theta$ . . . (3.22)

(d) Simple Gearing

Consider a pair of meshing gear-wheels A and B with number of teeth N_A and N_B, respectively, as in Figure 3.25. It is obvious that for the smooth meshing of the wheels, '*p*', the centre-to-centre distance between any two adjacent teeth (measured along their pitch-circles – this term is explained later) must be the same for both the wheels. This is called the *pitch of the teeth*.

Figure 3.25 : A Pair of Gear Wheels

It is easy to see that if A is rotated clockwise so that its tooth No. 1 moves exactly into the position occupied by tooth No. 2 (Figure 3.25), it will push B anticlockwise so that Bs slot 1′ will occupy the position occupied by slot 2′. If this is repeated N_A times, it is clear that A will have made exactly one revolution clockwise while N_A slots of B will have crossed the line of centres C_1 C_2, i.e. B will have rotated $\left[\dfrac{N_A}{N_B}\right]$ revolutions anticlockwise.

Let us replace the two toothed wheels by circular discs of radii

$r_A = \left[\dfrac{N_A}{(N_A + N_B)} \times C_1 C_2\right]$ and $r_B = \left[\dfrac{N_B}{(N_A + N_B)} \times C_1 C_2\right]$ respectively,

If A was made to drive B by friction without slip, they would have had the same rotation-ratio as written above (i.e. $\dfrac{N_A}{N_B}$). Circles with the above radii are called the pitch circles of two wheels. To obtain 'p', the pitch which is the distance between the centre-lines of two adjacent teeth. It is measured along the pitch-circle circumference of either wheel, as stated earlier.

Clearly $p_A = \dfrac{2\pi r_A}{N_A}$, and $p_B = \dfrac{2\pi r_B}{N_B}$ where, p_A and p_B are the pitches of A and B, respectively. Further, $p_A = p_B$ if the wheels have to mesh. This relation shows that $\dfrac{r_A}{r_B} = \dfrac{N_A}{N_B}$ or the pitch circle radii of two meshing wheels are proportional to their number of teeth.

When a system of meshing toothed wheels is used to lift a machine, the velocity ratio will depend on the other details of the same. Suppose as in Figure 3.25 we have a concentric load-drum of radius R rigidly fixed to the toothed wheel B, over which the rope carrying the load W is wrapped and a lever (length L) rigidly fixed to the toothed wheel A, for applying the effort P, which is done always at right angles to it (lever). If A makes one revolution, y, the distance moved by the effort = $2\pi L$. The wheel B will then rotate $\dfrac{N_A}{N_B}$ revolutions and hence x , the distance moved by W,

$= \left(\dfrac{N_A}{N_B}\right) 2\pi R$.

and $\quad \therefore \quad V.R. = \dfrac{y}{x} = \dfrac{2\pi L}{\left[\dfrac{N_A}{N_B}\right] 2\pi R} = \dfrac{L}{R}\left(\dfrac{N_B}{N_A}\right).$ . . . (3.23)

(e) Screw-jack

Consider a screw-jack held with the axis of its screw vertical and with the bottom of the screw fixed to a base plate. Its nut carries a load platform on which a load W is placed. A rigid lever (length L) is welded to the nut, at the end of which the effort P may be applied in horizontal plane at right angles to the lever to rotate the nut so that as it rotates; it moves axially and the load W is raised or lowered depending on the direction of P (Figure 3.16).

Suppose we apply P continuously as above and rotate the nut so as to raise the load W. If we make one complete revolution of the nut, $y = 2\pi L$ and the nut will move axially by a distance = l , the lead of the screw, where l =

np where *p* is the pitch of the screw-threads and $n = 1, 2, 3$ etc. depending on whether the screw is single, double or triple-threaded, etc.

Figure 3.26 : Screw Jack

Obviously x is now = 1,

Then $$V.R. = \frac{y}{x} = \frac{2\pi\pi}{l} = \frac{2\pi L}{np} \quad \text{. . . (3.24)}$$

(f) Worm and Worm-wheel

Figure 3.27 shows the outline of a worm and worm-wheel.

Figure 3.27 : Worm and Worm-wheel

This machine consists of a toothed wheel (the worm-wheel) rotating in a pair of bearing shown at B (Figure 3.27) and a screw (called the worm) with its axis arranged in the middle plane of the wheel. The worm rotates in bearings B_1 and B_2 which are so positioned that the imaginary cylindrical surface midway between the roots and tips of its threads is tangential to the pitch-circle of the wheel. Also the pitch of the worm-screw threads and the circular pitch of the teeth of the worm-wheel is same, so that the former mesh properly with the slots in the wheel-teeth (Figure 3.27).

It is evident from the arrangement that with one complete rotation (shown by arrow '*a*') of the worm, the tooth of the worm-wheel which is next to the engaging thread of the worm, will be pushed in the direction of the apparent advance of the worm by a distance equal to its pitch (which is equal to the pitch of the wheel). The previous tooth of the wheel is engaged in the worm-screw-threads and will be ready in its turn for onward displacement.

In short, one complete rotation of the worm corresponds to the rotation of the wheel by an angle θ given by

$$\theta = \frac{1}{N} \times 2\pi \text{ radians} = \frac{2\pi}{N} \text{ radians,}$$

where N = the number of teeth of the worm-wheel.

If effort P is applied at the end of a lever (length = L) attached to the worm at right angles to its length and the load W, hung by rope wound round a drum (radius R) rigidly fixed to the worm-wheel, we shall have, corresponding to one full rotation of the worm,

$$y = 2\pi L \qquad \text{and} \qquad x = \theta \times R = \frac{2\pi}{N} \times R$$

$$V.\,R. = \frac{L}{R} \cdot N. \qquad \text{. . . (3.25)}$$

(g) Differential Axle and Wheel

In this machine a stepped axle is used in an ingenious manner to obtain a large mechanical advantage which is necessary when hoisting a very heavy loads (Figure 3.28).

Figure 3.28 : Differential Axle and Wheel

As shown in the figure, a stepped axle (called the differential axle) with diameters d_1 and d_2 is provided with bearings B_1 and B_2 in which it can rotate about its own axis. The rope supporting the load has its one end tied to the smaller stepped part at K, wound clockwise (as seen in the direction of the arrow '*a*') several times round it, then taken downwards and around a floating pulley X, then up again to be wound clockwise round the bigger stepped part and finally fixed to it at L. The floating pulley X carries the load W. For applying the effort, we have the effort-pulley E (the 'wheel') rigidly fixed to the axle over which another rope is wound clockwise at the end of which effort P is applied. The diameter of E is D. The direction of winding of the load and effort-ropes should be carefully noted.

It is now easy to see that if one complete clockwise rotation is given to the axle, E will rotate through 360^{o} and the displacement 'y' of the point of application of the effort P will be equal to πD. At the same time, the length of the load-rope which will be wound round the bigger stepped part will be 'πd_1' while that unwound and hence paid out from the smaller stepped part will be πd_2. The net result will be that the load W will be raised by a distance.

$$x = \frac{\pi (d_1 - d_2)}{2}$$

The V. R. of the machine, therefore, will be

$$= \frac{y}{x} = \frac{\pi D}{\frac{\pi (d_1 - d_2)}{2}} = \frac{2D}{(d_1 - d_2)} \qquad \ldots (3.26)$$

This shows that by either increasing D or more effectively by reducing the difference between d_1 and d_2, the velocity ratio can be increased by any desired extent.

3.6.3 Law of Lifting Machine

For any lifting machine, the relation between the values of the effort P required to lift gives load W, is called **the law of that machine**. Naturally the law will depend entirely on its velocity-ratio, the friction in its various moving parts which will in turn depend on their weights, its state of lubrication and maintenance, etc. Hence, for a given machine, the law can be established only after experimentation on it, observing the values of the effort actually required to lift specific loads. It has been experimentally found that the law is of the form

$$P = mW + C \qquad \ldots (3.27)$$

where, m and C are constant for all loads, i.e. the relation is a linear one (Figure 3.29).

Figure 3.29 : Law of Lifting Machine

It is seen from the actual $P - W$ graph that with $W = 0$, an effort = C is required to move the load-hook alone with no load there. Or 'C' is the effort required to move only the components of the machine, against friction therein. If the velocity-ratio of the machine is V, then it is obvious that in ideal case (i.e. frictionless), P_1, the ideal effort would be $= \frac{W}{V}$. The graph of this effort P_1 is also plotted in Figure 3.29.

Drawing the ordinate LMN at abscissa, W, it has seen that the actual effort P required to lift the load W is given by $LN = m\, W + C$. Since the ideal effort for lifting the load is given by $P_1 = \frac{W}{V} = MN$ (see the figure), P_F, the effort required to overcome the friction in the machine at the load $W = P - P_1 = LN - MN = LM$, which is the intercept between the two lines. It is given by

$$P_F = (mW + C) - \frac{W}{V} \qquad \text{. . . (3.28)}$$

Normally, the slope of the actual P/W graph = α = ($\tan^{-1}$ m) is greater than $\beta = \tan^{-1} \frac{1}{V}$, the slope of the ideal P/W graph, and hence P_F value increases with load.

Efficiency of a Lifting Machine – Its Maximum Limit

Since the efficiency

$$\eta = \frac{M.A.}{V.R.} = \frac{W}{P} \times \frac{1}{V}$$

and, since $P = mW + C$, we have

$$\eta = \frac{W}{(mW + C)} \times \frac{1}{V}$$

i.e.,

$$\eta = \frac{1}{\left[m + \frac{C}{W}\right] V} \qquad \text{. . . (3.29(a))}$$

This shows that when $W = 0$, $\eta = 0$, and as W increases, η also increases. Now as $W \rightarrow \infty$, we can see that

$$\eta \rightarrow \frac{1}{mV}, \text{ or } \eta_{max} = \frac{1}{mV} \qquad \text{. . . (3.29(b))}$$

A typical graph of η vs. W is given in Figure 3.30.

Figure 3.30 : Typical Efficiency Curve of a Lifting Machine

Example 3.5

For a lifting machine, it is observed that for loads of 500 N and 1500 N, efforts of magnitudes 50 N and 120 N respectively are required.

(a) Determine the law of the machine.

(b) If velocity ratio is 20, calculate the efficiency for a load of 3 kN.

(c) Has the machine been ideal, what effort would have been required for the load in (b)? Hence, calculate the effort lost in friction at that load.

(d) Calculate the maximum possible efficiency.

Solution

(a) Assuming the law to be $P = mW + C$

We have $50 = m \times 500 + C$ . . . (i)

and $120 = m \times 1500 + C$ . . . (ii)

Solving Eqs. (ii) – (i) gives : $70 = 1000\, m$, $\therefore\; m = 0.07$

Substituting this in Eq. (i), we get

$$C = 50 - 35 = 15$$

$\therefore$ The law of the machine is

$$P = 0.07\, W + 15 \quad \text{. . . (iii)}$$

(and, obviously, P and W to be in N units).

(b) P for 3 kN load (i.e., W = 3000 N),

We have from Eq. (iii) above

$P_3 = 0.07 \times 3000 + 15 = 210 + 15 = 225$ N.

$$\eta_3 = \frac{M.A.}{V.R.} = \left(\frac{3000}{225}\right) \times \frac{1}{20} = \frac{2}{3} = 66.67\%$$

(c) $P_{ideal} = \dfrac{3000}{20} = 150N.$

Effort lost in Friction = 225 – 150 = 75 N.

(d) Max. Possible efficiency $= \dfrac{1}{mV} = \dfrac{1}{0.07 \times 20} \times 100.$

$= 71.4$ %.

Example 3.6

In a differential pulley block two pulleys have diameters of 20 cm and 16 cm, respectively.

(a) Calculate the velocity ratio.

(b) Given that the efficiency at 30kN load being 60%, calculate the effort required.

Solution

$d_1 = 20$ cm

$d_2 = 16$ cm

(a) $V.R = \dfrac{2d_1}{(d_1 - d_2)} = \dfrac{2 \times 20}{(20 - 16)} = 10.$

(b) $M.\,A. = V.\,R \times \eta$

$\therefore$ M. A. at 30 kN load, = 10 × 0.6 = 6.

$\therefore$ Effort to lift 30 kN load $= \frac{30}{6} = 5$ kN.

Example 3.7

For a given value of the coefficient of friction μ (or angle of friction ϕ), calculate the pitch of the threads of a single-threaded screw jack (screw-diameter = d) which will have the maximum efficiency and calculate the value of this efficiency.

Solution

We have for a screw-jack, $P = W \tan(\theta + \phi)$ and $V.R. = \frac{1}{\tan\theta}$, where θ = the angle of the equivalent inclined plane.

$$\therefore \quad \eta = \frac{M.A}{V.R} = \frac{W}{P} \div \frac{1}{\tan\theta} = \frac{\tan\theta}{\tan(\theta+\phi)} \quad \ldots (I)$$

For η_{max}, $\frac{d\eta}{d\theta} = 0$ (treating θ as a variable).

$$\therefore \quad \tan(\theta+\phi)\sec^2\theta - \tan\theta\sec^2(\theta+\phi) = 0$$

i.e. $$\frac{\sin(\theta+\phi)}{\cos(\theta+\phi)} \times \frac{1}{\cos^2\theta} = \frac{\sin\theta}{\cos\theta}\,\frac{1}{\cos^2(\theta+\phi)}$$

i.e. $$\frac{\sin(\theta+\phi)}{\cos\theta} = \frac{\sin\theta}{\cos(\theta+\phi)}$$

or $\sin(\theta+\phi) \times \cos(\theta+\phi) = \sin\theta\cos\theta$

i.e. $$\frac{1}{2}\sin 2(\theta+\phi) = \frac{1}{2}\sin 2\theta$$

$\therefore$ Either $\theta + \phi = \theta$ or $2(\theta+\phi) = 180 - 2\theta$.

First alternative is not acceptable because it gives $\phi = 0$.

$\therefore$ We have $\theta + \phi = 90^o - \theta$ or $\theta = 45^o - \frac{\phi}{2}$

Since $\tan\theta = \frac{p}{\pi d}$, where p = the pitch of the screw

$$\therefore \quad p = \pi d\tan\theta = \pi d\tan\left(45^o - \frac{\phi}{2}\right)$$

$$= \frac{\pi \cdot \left[1 - \tan\frac{\phi}{2}\right]}{\left[1 + \tan\frac{\phi}{2}\right]} . d.$$

This gives the required pitch.

Putting value of θ in (I),

$$\eta_{max} = \frac{\tan\left[45 - \frac{\phi}{2}\right]}{\tan\left[45 + \frac{\phi}{2}\right]}.$$

$$\eta_{max} = \frac{\left[1 - \tan\frac{\phi}{2}\right]}{\left[1 + \tan\frac{\phi}{2}\right]} \times \frac{\left[1 - \tan\frac{\phi}{2}\right]}{\left[1 + \tan\frac{\phi}{2}\right]}$$

$$\eta_{max} = \left[\frac{\left[1 - \tan\frac{\phi}{2}\right]}{\left[1 + \tan\frac{\phi}{2}\right]}\right]^2$$

3.7 SUMMARY

In this unit, you have studied laws of friction and problems involving dry friction. The relative sliding motion of one body on another body is resisted by forces called frictional forces. The sense of these frictional forces is such as to oppose the impending or actual sliding motion. When there is no impending motion, the frictional forces can be found by using the equations of static equilibrium. The limiting static friction is reached when relative sliding motion of the surfaces is impending and is given by :

$$F_{(max)} = \mu N$$

where, μ is the coefficient of static friction and N is the normal reaction on the surface concerned.

When sliding motion occurs, the retarding friction force has the magnitude $\mu_K N$ where, μ_K is the coefficient of kinetic friction.

The angle between the normal reaction, N, and the resultant reaction, R, is called the angle of friction when sliding motion of the surfaces is impending. This angle ϕ is related to the coefficient of friction by :

$$\tan\phi = \mu$$

The maximum angle of inclination of the inclined plane when the body kept on it is just on the point of moving down the plane, is called the angle of repose.

The angle of repose is equal to the angle of friction.

You have also studied in this unit, the engineering applications where dry friction plays an important role, e.g., in wedges used to lift heavy loads and screw jacks frequently used in presses and other mechanisms. By drawing free-body diagrams indicating correct sense of friction forces and applying equations of equilibrium, you can analyse such engineering problems.

3.8 ANSWERS TO SAQs

SAQ 1

(a) The direction of frictional force, F, and normal reaction, N, are marked in Figure for Answer to SAQ 1(a). N is acting vertically upwards and F is horizontally acting towards left, i.e. opposite to the direction of tending motion.

Figure for Answer to SAQ 1(a)

The horizontal force which tends to move the block A is 60 cos 45^o acting towards right :

$$60 \cos 45^o = 42.426 \text{ N}$$

$\therefore$ The frictional force developed will also be 42.426 N but acting towards left, if the motion is not impending.

If the coefficient of friction is 0.3, the limiting frictional force will be 0.3 × normal reaction = 0.3 (150 – 60 sin 45^o) = 32.272 N.

As the actual frictional force is greater than limiting frictional force, the motion is impending.

(b) If the slip is impending, the various forces and reactions will be acting in the directions as shown in Figure for Answers to SAQ 1(b).

Figure for Answer to SAQ 1(b)

Taking moments of all the forces about C, we get (with $AB = l$)

$$N_A \frac{l}{2} \sin\alpha + F_A \frac{l}{2} \cos\alpha + F_B \frac{l}{2} \sin\alpha - N_B \frac{l}{2} \cos\alpha = 0 \qquad \ldots \text{(I)}$$

Also, $\Sigma F_x = 0$

Also, $\therefore N_A = F_B$

and, F_B = limiting frictional force at B

$= 0.3\ N_B$

F_A = limiting frictional force at A

$= 0.2\ N_A$

Putting these values in Eq. (I), we get

$$N_A \sin\alpha + 0.2\ N_A \cos\alpha + N_A \sin\alpha - \frac{N_A}{0.3} \cos\alpha = 0$$

$\therefore \quad 2\sin\alpha = (3.333 - 0.2)\ \cos\alpha$

$\therefore \quad \tan\ \alpha = 1.5666$

$\therefore \quad \alpha = 57^{o}\ 26'\ 56''$

(c) Applying equations of equilibrium to block A, we get $N_A = W_A = 30$ N

and, $T = F_A = 0.28\ N_A = 0.28 \times 30 = 8.4$ N

$\therefore$ Tension in the string = 8.4 N

Now applying the equation of equilibrium to block B also, we get :

$$N_B + P\ \sin 30^{o} - W_B = 0$$

$\therefore \quad N_B = W_B - 0.5P$

$= 50 - 0.5P$

Figure for Answer to SAQ 1(c)

and $\quad P\cos\ 30^{o} - F_B - T = 0 \qquad (F_B = 0.22\ N_B)$

$\therefore \quad 0.866P - 0.22\ (50 - 0.5P) - 8.4 = 0$

$\therefore \quad 0.866P - 11 + 0.11P - 8.4 = 0$

$\therefore \quad 0.976P = 11 + 8.4 = 19.4$

$\therefore \quad P = \dfrac{19.4}{0.976} = 19.877$ N

$\therefore$ The minimum value of P just to move the system is 19.877 N.

(d) The free-body diagram for the hinge, rigid rods and blocks A and B are shown in Figure for Answers to SAQ 1(d).

Applying equations of equilibrium to the whole set-up, we get

$$C_A \cos 60^{o} - C_B \cos 60^{o} = 0$$

$\therefore \qquad C_A = C_B = C \quad \text{(say)}$

and $\qquad C_A \sin 60^o + C_B \sin 60^o - W = 0$

$$C = \frac{W}{2 \sin 60^o} = 0.5774 \text{ W}$$

Figure for Answer to SAQ 1(d)

Applying equations of equilibrium to block A, we get

$$N_A - W_A - C_A \sin 60^o = 0$$

but, $\qquad C_A = 0.5774$ W

$\therefore \qquad N_A = W_A + 0.5774\, W \sin 60^o$

$$= 800 + 0.5 \text{ W}$$

and $\qquad F_A - C_A \cos 60^o = 0; \quad \text{and, } F_A = 0.25\ N_A$

$\therefore \qquad 0.25\ (800 + 0.5W) - 0.5774\, W \cos 60^o = 0$

$\therefore \qquad 200 + 0.125W - 0.2887W = 0$

$\therefore \qquad W = \dfrac{200}{0.1637} = 1221.75 \text{ N}$

(Alternately, considering block B also, we will again get W = 1221.75 N.)

Therefore, the value of load, W, when the block are just on the point of sliding is 1221.75 N.

(e) The overturning moment due to force P about the left corner is $P \cos \alpha\ (0.5) + P \sin \alpha\ (0.8)$ and the stabilising moment due to weight, W, about the left corner is 0.4 W.

As the block does not overturn, we have :

or $\qquad W \geq \dfrac{P}{0.4}\ (0.5 \cos \alpha + 0.8 \sin \alpha)$

$\therefore \qquad W \geq (1.25 \cos \alpha + 2 \sin \alpha)\ P \qquad \ldots \text{(II)}$

Applying the equations of equilibrium when the block is just on the point of sliding, we get

$$N + P \sin \alpha - W = 0 \qquad \therefore \; N = W - P \sin \alpha$$

and $\quad F - P \cos \alpha = 0$

But $\quad F = 0.3$ N, $\qquad$ (N = normal reaction)

$\therefore \quad 0.3\,(W - P \sin \alpha) = P \cos \alpha$

$\therefore \quad 0.3\,W = P(\cos \alpha + 0.3 \sin \alpha)$

$\therefore \quad W = (3.333 \cos \alpha + \sin \alpha)\,P \qquad \ldots \text{(III)}$

Figure for Answer to SAQ 1(e)

Comparing Eqs. (II) and (III), we get

$$1.25 \cos \alpha + 2 \sin \alpha = 3.333 \cos \alpha + \sin \alpha$$

$\therefore \quad \sin \alpha = 2.083 \cos \alpha$

$\therefore \quad \tan \alpha = 2.083$

$\alpha = 64^{\circ}\, 21'\, 20''$ and correspondingly $W = (1.442 + 0.9014)\, P$

or, $\quad P = 0.4266$ W

SAQ 2

(a) (i) When the crate is about to move up, applying equations of equilibrium after resolving the forces along the normal to the inclined plane, we get

$$P \cos 30^{\circ} - W \sin 30^{\circ} - F = 0$$

$\therefore \quad 0.866\, P = 5 \times 0.5 + F = 2.5 + 0.25$ N
. . . (IV)

Figure for Answer to SAQ 2(a)

and $N - P \sin 30^{\circ} - W \cos 30^{\circ} = 0$, where $W = 5$ kN $\qquad \ldots$ (V)

$\therefore \quad N = 0.5\, P + 4.33$

Putting this value in Eq. (IV), we get

$$0.866\,P = 2.5 + 0.25\,(0.5\,P + 4.33)$$

$$\therefore \quad 0.741P = 3.5825$$

$$\therefore \quad P = 4.835 \text{ kN}$$

This is the minimum value of P to move the crate in upward direction.

(ii) When the crate is just on the point of moving down, the direction of friction force, $F = 0.25$ kN, will be acting upwards. Working on similar lines, we get,

$$0.866\,P = 2.5 - 0.25\,(0.5\,P + 4.33)$$

$$\therefore \quad 0.991\,P = 1.4175$$

$$\therefore \quad P = 1.430 \text{ kN}$$

This is the minimum value of P to keep the crate sliding down the plane.

(iii) Considering the above two results, it can be concluded that the crate will be in equilibrium for the range of values of P between 1.430 N and 4.835 N.

(iv) If $P = 5.2$ kN (assuming F acting upwards)

$$P \cos 30^{\circ} - W \sin 30^{\circ} + F = 0$$

$$\therefore \quad F = W \sin 30^{\circ} - P \cos 30^{\circ}$$

$$= 5 \times 0.5 - 5.2 \times 0.866 = -\,2.00 \text{ kN}$$

The negative sign indicates downward direction of the frictional force (i.e. our assumption was wrong).

(b) Consider forces acting on block Q. Applying equations of equilibrium after resolving the forces normal and parallel to the inclined plane, we get

$$N_Q - W \cos 35^{\circ} = 0$$

$$\therefore \quad N_Q = 1.2 \times 0.819 = 0.983 \text{ kN}$$

Figure for Answer to SAQ 2(b)

and $\quad T - W \sin 35^{\circ} + F_Q = 0$

$\therefore \quad T - 1.2 \sin 35^{o} + 0.2 \times 0.983 = 0$

$\therefore \quad T = 0.492$ kN

Now, consider forces acting on block P,

Here $\quad T - F_P = 0$

$\therefore \quad F_P = 0.492$ kN

$\therefore \quad 0.25\ N_P = 0.492$

$\therefore \quad N_P = 1.968$ kN

$\therefore \quad W_P = N_P = 1.968$ kN

$\therefore$ To maintain the equilibrium, the minimum value of weight of block P is 1.968 kN.

(c) If only block M is on the point of moving down then applying the equations of equilibrium, we get

$$N_M = W_M \cos\theta = 600 \cos\theta$$

$$F_M = W_M \sin\theta = 600 \sin\theta$$

But $\quad F_M = 0.4\ N_M$

$\therefore \quad 600 \sin\theta = 0.4 \times 600 \cos\theta$

$\therefore \quad \tan\theta = 0.4$

$\therefore \quad \theta = 21^{o}\ 48'\ 5''$

If both blocks together are on the point of sliding down the inclined plane, then working on similar lines and noting $\mu = 0.5$, we get

$$1400 \sin\theta = 0.5 \times 1400 \cos\theta$$

$\therefore \quad \tan\theta = 0.5$

$\therefore \quad \theta = 26^{o}\ 33'\ 54''$

As this angle is greater than $21^{o}\ 48'\ 5''$, the block M will slide first.

If $\theta = 15^{o}$, the motion is not impending. Frictional forces can be worked out from equilibrium equations

$$F_M = 600 \sin 15^{o}$$

$$= 155.29 \text{ N}$$

Figure for Answer to SAQ 2(c)

This is the frictional force between block M and block N. (Note : This is less than limiting frictional force which is $0.4 \times 600 \cos 15^{o}$ $= 231.82$ N).

Similarly,

$$F_N = 1400 \sin 15^{\circ} \;\; . \; . \; . \; (\because \; W = W_M + W_N = 600 + 800 = 1400 \text{ N})$$

$$= 362.35 \text{ N}$$

$\therefore$ The frictional force between the block N and the inclined plane is 362.35 N.

SAQ 3

(a) Here, the coefficient of friction is 0.25

$\therefore$ The angle of friction $\phi = \tan^{-1} (0.25) = 14.036°$ for all surfaces.

$\therefore$ R_A, R_B, R_C and R_D are making an angle of 14.036° to N_A, N_B, N_C and N_D, respectively when motion is impending. The suffixes A, B, C and D refer to wedge A, wedge B, block C and ground D, respectively. As wedge B is moving down, the frictional forces F_A and F_C will act upwards.

Figure for Answer to SAQ 3(a)

Three forces, R_B, R_D and W, are acting on block C (Refer to Figure for Answer to SAQ 3(a)) and apply the Lami's theorem, we get

$$\frac{W}{\sin (90^{\circ} + 14.036^{\circ} + 14.036^{\circ})} = \frac{R_B}{\sin (180^{\circ} - 14.036^{\circ})}$$

$$\therefore \qquad R_B = \frac{W \sin 165.964^{\circ}}{\sin 118.072^{\circ}}, \; . \; . \; . \; (W = 6 \text{ kN})$$

$$= \frac{6 \times 0.2425}{0.8824}$$

$$= 1.649 \text{ kN}$$

At the contact surface of wedge B and block C, the reaction offered by B is equal to reaction offered by C

$\therefore \qquad R_B = R_C = 1.649$ kN

Now, consider wedge B. Three forces R_A, R_C and P are acting as shown in figure. Note that contact surface between wedge A and B is inclined at 8° to the vertical.

Applying Lami's theorem, we get

$$\frac{P}{\sin (180^\circ - 14.036^\circ - 22.036^\circ)} = \frac{R_C}{\sin (90^\circ + 22.036^\circ)}$$

$$\therefore \qquad P = \frac{R_C \sin 143.928^\circ}{\sin 112.036^\circ} \text{ and } R_C = 1.649 \text{ kN}$$

$$\therefore \qquad P = \frac{1.649 \times 0.5888}{0.9269}$$

$$= 1.0475 \text{ kN}$$

$\therefore$ The minimum load, P, required to push the block horizontally is 1.0475 kN.

(b) The coefficient of friction of all surfaces of contact is 0.3. Therefore, the angle of friction will be $\tan^{-1}(0.3)$, i.e. $16^\circ\ 41'\ 57''$ for all surfaces. Three forces R_A, R_C and W act on the heavy block as shown in Figure for Answer to SAQ 3(b).

(**Note :** A and C denote wedges and D ground.)

Applying Lami's theorem, we get

$$\frac{R_A \text{ (or } R_C)}{\sin (180^\circ - 16^\circ\ 41'\ 57'' - 10^\circ)} = \frac{W}{\sin 2\,(16^\circ\ 41'\ 57'' + 10^\circ)}$$

$$\therefore \qquad R_A = R_C = \frac{8 \sin 153.3^\circ}{\sin 53.398^\circ}$$

$$= 4.478 \text{ kN}$$

Consider left hand side wedge. Three forces, R_A, R_D and P, act on it as shown in Figure for Answers to SAQ 3(b), we get

Figure for Answer to SAQ 3(b)

$$\frac{P}{\sin (180^\circ - 2\phi - 10^\circ)} = \frac{R_A}{\sin (90^\circ + \phi)}$$

$$P = \frac{R_A \sin(2\phi + 10^o)}{\cos\phi}$$

$$= \frac{4.478 \sin 43.398^o}{\cos 16.699^o}$$

$$= 3.212 \text{ kN}$$

$\therefore$ The value of P to drive the wedges under the load is 3.212 kN.

(c) (i) The force P_1 to be applied at the end of the lever to raise the load up is given by

$$P_1 = \frac{r}{1} W \tan(\alpha + \phi)$$

where $\alpha = \tan^{-1}\left(\frac{p}{2\pi r}\right)$

and $\phi = \tan^{-1}\mu$

Here P = pitch of the thread = 5 mm

d = mean diameter

$\therefore$ $\alpha = \tan^{-1}\left(\frac{5}{60\pi}\right) = 1^o\ 31'\ 10''$

and $\phi = \tan^{-1} 0.08 = 4^o\ 34'\ 26''$

r = mean radius = 30 mm

l = Length of lever = 200 mm

W = 20 kN

$\therefore$ $P_1 = \frac{30}{200} \times \frac{20}{1} \times \tan(1^o\ 31'\ 10'' + 4^o\ 34'\ 26'')$

$= 3 \tan(6.093^o)$

$= 0.32$ kN = 320 N

(ii) To lower the load, P_2 required is given by

$$P_2 = \frac{r}{t} \text{W} \tan(\phi - \alpha)$$

$$= \frac{30}{200} \times 20 \tan(4^o\ 34'\ 26'' - 1^o\ 31'\ 10'')$$

$$= 3 \tan(3.054^o)$$

$$= 0.160 \text{ kN} = 160 \text{ N}$$

(c) To lift a load, the force required at the end of the lever is given by

$$P_1 = \frac{r}{l} \text{W} \tan(\alpha + \phi)$$

$$= \frac{25}{400} \times \frac{2}{1} \times \tan(\alpha + \phi)$$

$$= 0.125 \tan(\alpha + \phi)$$

$$\alpha = \tan^{-1} \frac{8}{50\pi} = 2^{o}\ 54'\ 56''$$

$$\phi = \tan^{-1} 0.2 = 11^{o}\ 18'\ 35''$$

Putting the values of α and ϕ, we get

$$P_1 = 0.125 \tan 14.225^{o}$$

$$= 0.03169 \text{ kN} = 31.69 \text{ N}$$

As $\frac{P}{\pi D} < \mu \ (0.051 < 0.2),$ the screw jack is self locking. It cannot work in reverse direction as the angle of inclination is less than the angle of friction.