UNIT 2 GEOMETRICAL CONSTRUCTION

Structure

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2.1 INTRODUCTION

Many of the constructions used in engineering drawing are based on plane geometry. In this unit, we will deal with some basic problems in geometric construction which are very often required for the preparation of technical drawings. In many cases, construction can be made more quickly and accurately by the so-called "draftsman's method". It is done using T-square, set-square, scale, etc. at the draftsman's disposal for the solution of problems in geometrical construction which can otherwise be solved by using only a compass and a straight edge. The students, while preparing a technical drawing, should always prefer the draftsman's method for the speedy solution of the problems.

Objectives

After studying this unit, you should be able to

- get familiar with various geometrical constructions that frequently occur in engineering drawings,
- understand the various methods/ways to draw these geometrical construction,
- select appropriate method/technique for drawing these geometrical constructions and related geometrical features, and
- draw tangent and normal to various curves.

2.2 TRIANGLE

A triangle is a plane figure formed by three straight lines containing three angles. The side on which it is supposed to stand is called its *base* and angles at the base are known as the *base angles*. The point where the other two sides meet is called the *vertex* and the angle at the vertex is called the *vertical angle* (Figure 2.1(a)). The line drawn from the vertex and perpendicular to the base or the base produced, if necessary, is called the altitude (Figure 2.1(b)).

(a) An equilateral triangle is that in which all the three sides are equal.

(b) An isosceles triangle is that in which two sides as well as the angles opposite to them are equal.

(c) A scalene triangle has none of its sides and angles equal. The altitude may either be within or outside the triangle as depicted in Figure 2.1(c).

(d) The line joining the angular point of a triangle to the middle point of the opposite side of the angular point is called the median as shown in Figure 2.1(d).

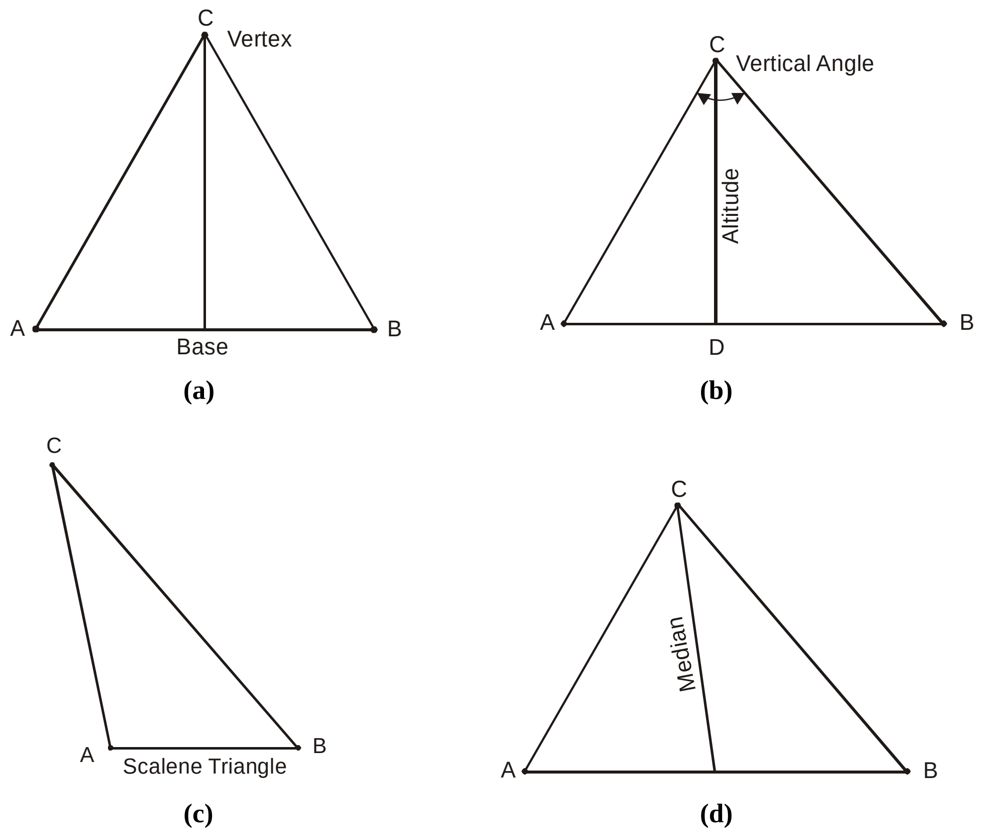


Figure 2.1

2.2.1 To Construct an Equilateral Triangle when the Length of the Side is Given

Use a *T*-square and a set-square only. With a *T*-square, draw a line *AB* of the given length. Using a set-square with angles of 30^o – 60^o and a *T*-square, draw a line through *A* making 60^o angle with *AB*. Similarly, through *B*, draw a line making the same angle (60^o) with AB and intersecting the first line at *C*. *ABC* is the required triangle.

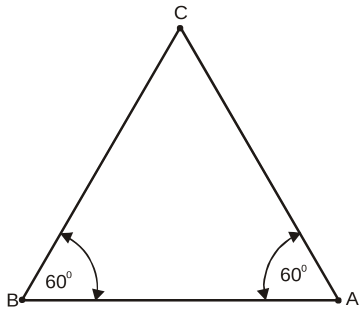


Figure 2.2

To Construct an Equilateral Triangle of a Given Altitude

With a *T*-square, draw a line *AB* of any length. From a point *P* on *AB*, draw, with a set-square, vertical *PQ* equal to the given altitude. With *T*-square and

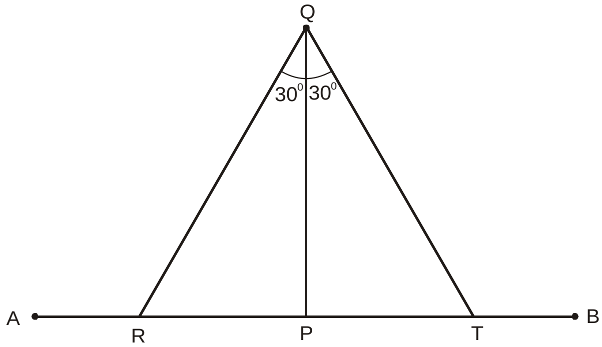


Figure 2.3

30^o – 60^o set square, draw lines through *Q* on both sides, making 30^o angle with *PQ* and cutting *AB* at *R* and *T*. *QRT* is the required triangle.

2.3 SQUARE

A quadrilateral is a plane figure bounded by four intersecting straight lines.

(a) A **square** is a quadrilateral in which all the sides are equal and the angles are at right angles.

(b) A **parallelogram** is a quadrilateral in which the opposite sides are parallel.

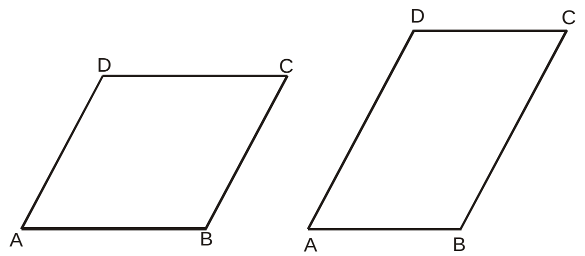
(c) A **rectangle** is a quadrilateral in which the opposite sides are equal and all the angles are at right angles.

(d) A **rhombus** is a quadrilateral in which all the sides are equal and the angles are not at right angles. However in this case, the opposite angles are equal as in Figure 2.4(a).

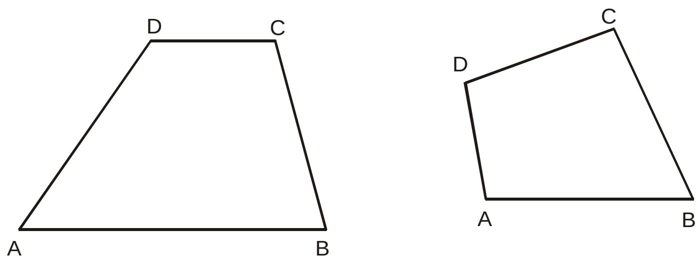
(e) A **rhomboid** is a quadrilateral in which the opposite sides are equal but the angles are not at right angles to each other (Figure 2.4(b)).

(f) A **trapezoid** is a quadrilateral in which only two sides are parallel to each other as depicted in Figure 2.4(c).

(g) A **trapezium** is a quadrilateral in which no side is parallel to each other but may have two pairs of adjacent sides equal. It is also known as kite (Figure 2.4(d)).



(a) Rhombus **(b) Rhomboid**



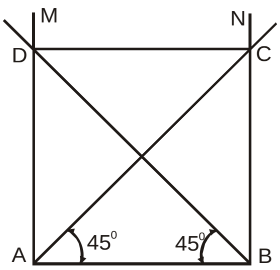
(c) Trapezoid **(d) Trapezium**

Figure 2.4

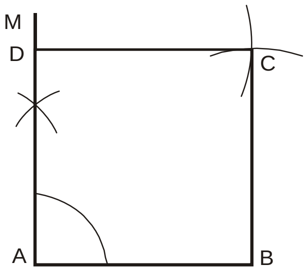
2.3.1 To Construct a Square when Length of One Side is Given

With a *T*-square and a set-square only (Figure 2.5 (a)) and *AB* being the length of one side of a square draw a line *AB*. Through *A* and *B*, draw vertical lines *AM* and *BN*. Draw from points *A* and *B* respectively, lines *AC* and *BD* inclined at 45^o to *AB*, cutting *BN* at *C* and *AM* at *D*. Join *C* with *D*. *ABCD* is the required square.

With the aid of a compass (Figure 2.5(b)) draw a given line *AB*. At *A*, draw a line *AM* perpendicular to *AB*. With *A* as the centre and radius *AB*, draw an arc cutting *AM* at *D*. With *B* and *D* as the centers and having the same radius, i.e. *AB*, draw arcs intersecting each other at *C*. Join *BC* and *CD*. *ABCD* is the required square.



(a)



(b)

Figure 2.5

2.3.2 To Construct a Square when the Length of a Diagonal is Given

Draw the given diagonal *AC* (or *BD*) and bisect it. The bisecting line bisects the diagonal at *O*. With *O* as the center and the radius equal to *OA* (or *OC*), draw a circle to cut the bisecting line at *B* and *D*. Join *AB*, *BC*, *CD* and *DA* to get *ABCD* as the required square.

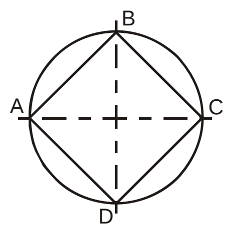


Figure 2.6

2.4 POLYGON

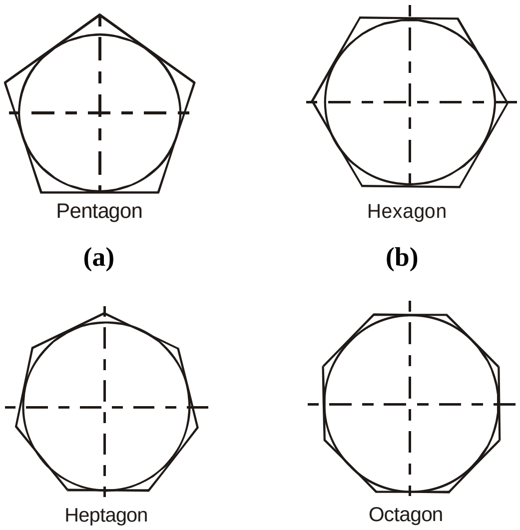
A polygon is a plane figure bounded by more than four straight lines and containing more than four angles. If all the sides and angles of a polygon are equal then it is called a *regular polygon*, but if all the sides and angles are unequal, then it is called an *irregular polygon*. Polygons are named according to the number of their sides and angles.

(a) A pentagon is that which has five equal sides (Figure 2.7(a)).

(b) A hexagon is that which has six equal sides as in Figure 2.7(b).

(c) A heptagon is that which has seven equal sides (Figure 2.7(c)).

(d) An octagon is that which has eight equal sides (Figure 2.7(d)).



(a) **(b)**

(c) **(d)**

Figure 2.7

2.4.1 Constructing Polygons

Outlines of mechanical parts are often polygons. For example, Figure 2.8(a), present a plate with a cut in the shape of an irregular octagon 1-8. By measuring the lengths of an octagon's sides we can transfer or reconstruct it.

(a)

(b) **(c)**

Figure 2.8

Any octagon may be divided into a series of triangles, into which the original octagon is divided. In this example, triangles can be obtained by drawing diagonals 3-7, 3-8 and 2-8 (Figure 2.8(a)). They are reconstructed in the following succession.

Draw a vertical line and transfer to it the measured length 5_1-6_1 of the base 5-6 as shown in Figure 2.8(b).

Draw two horizontal lines through points 5_1 and 6_1 and transfer the lengths 5-4 and 6-7, respectively.

From points 7_1 and 4_1, thus obtained as the centers and with radius equal to the diagonal 7-3 and the side 3-4, respectively, describe two arcs to intersect at point 3_1 which will be one of the vertices of the required octagon.

Proceeding in the same way we find vertex 8_1, then 2_1 and finally 1_1, taking as the base lengths 3_1-7_1, 3_1-8_1, and 2_1-8_1, respectively.

In constructing a polygon, the method of orthogonal coordinates may also be used. In this case, not the lengths of the polygon's sides but the coordinates of its vertices are measured. From the vertices of the octagon (points 1 to 8 in Figure 2.8(a)), draw perpendicular to the horizontal line *AB*. Draw a horizontal line of convenient length (Figure 2.8(c)) and mark off on it the distances between the perpendiculars to this line and transfer to them the corresponding distances between the line *AB* and the vertices (points 1 to 8) of the original octagon shown in Figure 2.8(a).

2.4.2 Special Methods of Drawing Some Regular Polygons, Given the Length of any Number of Sides

Regular polygons can be constructed by the following method :

Draw a line *AB* equal to the given length of a side. On *AB* as the diameter, draw a semi-circle. With either *A* or *B* as center and *AB* as radius, describe an arc on the same side as the semi-circle. Draw a perpendicular bisector of *AB* cutting the semicircle at point 4 and the arc at point 6. Bisect the length 4-6 at point 5. A square of a side equal to *AB* can be inscribed in the circle drawn with center 4 and

Figure 2.9

radius A-4. A regular hexagon of a side equal to *AB* can be inscribed in the circle drawn with center 6 and radius A-6. A regular pentagon of a side equal to *AB* can be inscribed in the circle drawn with center 5 and radius A-5. To obtain point 7, 8, 9, etc, step off a division 6-7, 7-8, 8-9, equal to the division 4-5 or 5-6. Regular polygons of any number of sides can be drawn by this method. Figure 2.9 shows a square, a regular pentagon, a regular hexagon and a regular octagon, all constructed on *AB* as a common side.

2.5 PENTAGON

Method of Drawing a Regular Pentagon

Method I

Using a protractor and a compass, draw *AB* equal to the given length of a side. At *B*, with the protractor, make an external angle of 360/5 = 72^o, as shown in Figure 2.10. Cut *BC* = *BA* with the compass. Draw the right bisects of *AB*. With *C* as the center and a radius equal to *AB*, draw an arc to cut the right bisector at *D*. Then with *A* and *D* as the centers and a radius equal to the given side *AB*, draw an arc to intersect at *E*. Join *BC*, *CD*, *DE*, and *EA*. *ABCDE* is the required pentagon.

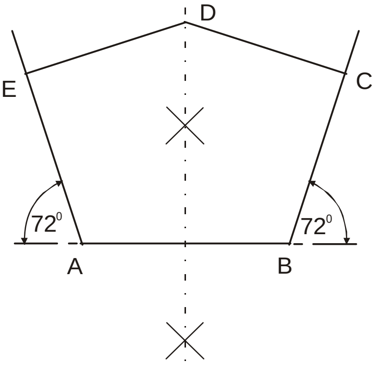


Figure 2.10

Method II

Using a compass only (Figure 2.11), draw a line *AB* equal to the given length of a side. Bisect *AB* at *O*, and *OP* = *AB* on the bisector. Join *A* to *P* and produce it to *Q* such that *PQ* = *AO*. Taking *A* as the center and a radius equal to *AQ*, draw an arc cutting the right bisector at point *D*.

Then *D* is the vertex of the pentagon. Taking *D* as the center and radius *AB*, draw an arc. Again with the centers *A* and *B*, and the same radius, draw arcs to cut the previous arcs at points *E* and *C*. Join *BC*, *CD*, *DE* and *EA*. *ABCDE* is the required pentagon.

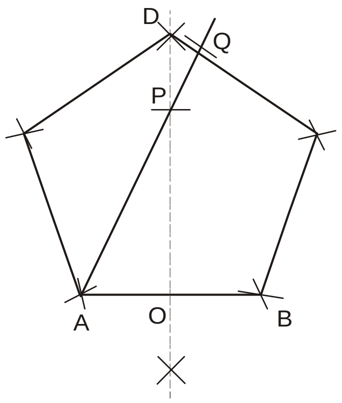


Figure 2.11

2.6 HEXAGON

Method of Constructing a Regular Hexagon

Method I

Using a T-square and a 30° –60° set-square only draw a line *AB* equal to the given length of a side. From point *A*, draw line, A_1 and A_2, making 60° and 120° angles, respectively with *AB*. From *B*, draw lines B_3 and B_4, making 60° and 120° angles, respectively, with *AB* as in Figure 2.12..

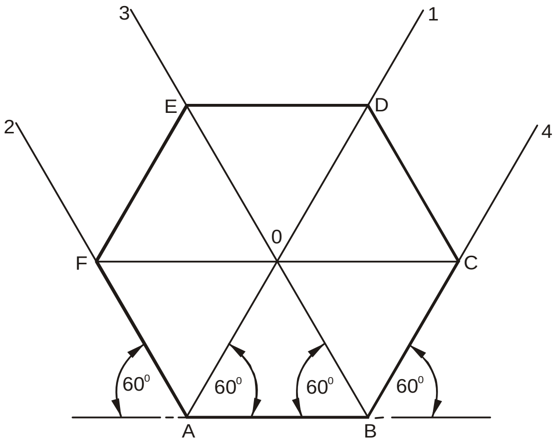


Figure 2.12

From *O*, the point of intersection of A_1 and B_3, draw a line parallel to *AB* and intersecting A_2 at *F* and B_4 at *C*. From *F*, draw a line parallel to *BC* and intersecting B_3 at *E*. From *C*, draw a line parallel to *AF* and intersecting A_1 at *D*. Draw a line joining *E* and *D*. *ABCDEF* is the required hexagon.

Method II

Using a compass, take a point *O* as the center, a radius equal to the given length of a side and draw a circle. With the same radius and starting from any point on the circle, set off six divisions *A*, *B*, *C*, *D*, *E*

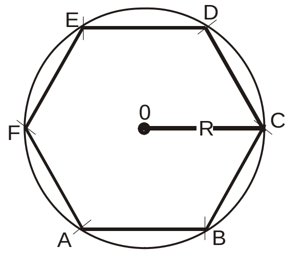
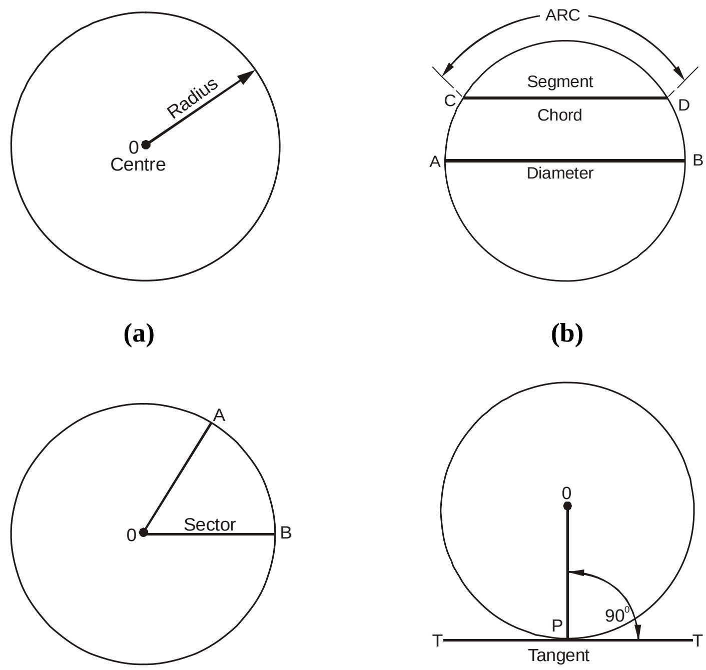


Figure 2.13

and *F* on the circle. Join *AB*, *BC*, *CD*, *DE*, *EF* and *FA*. *ABCDEF* is required hexagon.

2.7 CIRCLE AND CURVES

A circle is a plane figure bounded by one line known as *circumference* which is always equidistant from the fixed point known as center. The fixed distance from the center to the circumference is called *radius* (Figure 2.14(a)). The *diameter* of a circle is a straight line drawn through the center and meeting the circumference at both the ends. The diameter is two times the radius. An *arc* is any part of the circumference of a circle as in Figure 2.14(b). A *chord* is a straight line joining any two points on the circumference (Figure 2.14(b)). A *segment* is a part of a circle which is bounded by an arc and a chord as shown in Figure 2.14(b). A *sector* is a part of a circle which is bounded by an arc and two radii (Figure 2.14(c)). A *tangent* to a circle is a straight line which touches the circumference of the circle but does not cut it as in Figure 2.14(d). *Concentric circles* are those which have the same centre but different radii. *Eccentric* circles are those which have different centers.



(c) (d)

Figure 2.14

2.7.1 Finding the Center of a given Circle or an Arc

In making a drawing of a mechanical part, we often deal with circles and arcs which constitute its outlines, and have to determine their radii and centers. Figure 2.15(a) shows a bracket whose rib has the shape of a concave arc. To determine the center and the radius of this arc, mark three arbitrary points *A*, *B*, and *C* on the arc and join them by two chords *AB* and *BC* (Figure 2.15(b)). Bisect *AB* and *BC* and draw perpendiculars through the middle of each chord *AB* and *BC*. The center of the arc will be at the point *O* which is intersection of these two perpendiculars. *OA* is the radius of the arc *ABC*.

(a) (b)

Figure 2.15

2.7.2 Dividing a Circle into Four or Eight Equal Parts

To draw an outline of the *lid*, shown in Figure 2.16(a), it is necessary to divide the circle into eight equal parts in the following way. First draw two mutually perpendicular axes, horizontal and vertical (Figure 2.16(b)) to intersect at point *O*. From the point *O* as the center, describe a circle on which the eight holes are to be spaced. The point of intersection 1, 3, 5 and 7 of the circle with the horizontal and vertical axes will divide the circle into four equal parts. Points 2, 4, 6 and 8 are obtained with the aid of a compass by bisecting the angle 103, 305, 507 and 701, respectively. A circle can be divided into four and eight equal parts also by means of a $45^{o} – 45^{o}$ triangle (Figure 2.16(c)). When dividing an circle into four or eight equal parts with the aid of a triangle, the lines drawn along its hypotenuse must pass through the center of the circle.

(a)

(b) **(c)**

Figure 2.16

2.7.3 Dividing a Circle into Three, Six and Twelve Equal Parts

The component shown in Figure 2.17(a) has three equi-spaced holes. The circle must be divided into three equal parts. This can be done by means of a compass (Figure 2.17(b)). From *A* as the center and with the radius equal to that of the given circle draw an arc which will intersect the given circle, at points 2 and 3. Points 1, 2 and 3 divide the circle into three equal parts. The same can be obtained by making use of a 30^{o}-60^{o} triangle, as shown in Figure 2.17(c).

Figure 2.17(a)

(b) **(c)**

Figure 2.17

The component shown in Figure 2.18(a) has six equi-spaced recesses and six holes. Here we have to divide a circle into six equal parts. Here we apply the same method as in Figure 2.17, the only difference being that two arcs are described from points 1 and 4 as the centers (Figure 2.18(b)). The identical result can be obtained by making use of a 30^o-60^o triangle (Figure 2.18(c)). Make sure that the dividing lines pass through the center of the circle. To divide a circle into 12 equal parts first we divide the circle, as mentioned above, into six equal parts as shown in Figures 2.18(b) or 2.18(c).

(a)

(b) **(c)**

Figure 2.18

Then divide the circle into 12 equal parts by making use of 30^o-60^o triangle against a ruler, as indicated in Figure 2.19, by dot and dash lines.

2.7.4 To Divide a Circle into Five or Seven Equal Parts

The component shown in Figure 2.20(a) is a threaded die. To draw it, a circle has to be divided into five equal parts as depicted in Figure 2.20(b). First draw a horizontal line and mark on it point *O*. Then, from point *O* as the center, describe a circle of the given diameter. From point *A* as the center and with a radius equal to *AO*, draw an arc to intersect at point *n*. From this point *n*, drop a perpendicular *NC* to the horizontal axis. From point *C* and with a radius equal to *C*1, draw an arc to intersect the horizontal axis at point *m*. Now from point 1 as the center and with a radius equal to *M*1, draw an arc to intersect the circle at point 2. The latter (point 2) is the second required point, point 1 being the first. The rest of the points dividing the circle into five equal parts (points 3, 4 and 5) are found by setting the compass to the length 12 and marking off on the circle.

Figure 2.19

(a) **(b)**

Figure 2.20

The component shown in Figure 2.21(a) has seven equi-spaced holes. To divide a circle into seven equal parts, first draw a horizontal axis as in Figure 2.21(b). From point *O* as the center and with the given radius, draw a circle. Draw a vertical axis intersecting the circle at 1. From point *A* as the center and with the radius *OA*, draw an arc to intersect the circle at point *n*. From point *n*, drop a perpendicular to the horizontal axis to intersect it at point *C*. Set a length *nC* in the compass and from point 1, mark off this length on the circle to obtain the required points 2, 3, 4, 5, 6 and 7. Join 1, 2, 3, 4, 5, 6 and 7 with *O* to obtain seven equal parts.

(a) **(b)**

Figure 2.21

2.8 CONIC SECTIONS

When a right circular cone of revolution is cut by planes at different planes at different angles, four curves of intersection are obtained, they are called *conic sections* (Figure 2.22).

Figure 2.22

When the intersecting plane is perpendicular to the axis, the resulting curve of intersection is a *circle*.

If the plane makes a greater angle with the axis than to the elements, the intersection is an *ellipse*.

If the plane makes the same angle with the axis as the elements, the resulting curve is a *parabola*.

If the plane makes a smaller angle with the axis than do the elements or is parallel to the axis, the curve of intersection is a *hyperbola*.

2.9 ELLIPSE

The ellipse is a curve generated by a point moving so that at any position the sum of its distances from two fix points (called foci) is a constant and is equal to the major diameter.

2.9.1 Ellipse Construction by Foci Method

Draw the major axis AB and minor axis CD and locate the foci F_1 and F_2 by striking arcs centered at C and having a radius OA equal to one half of the major diameter.

Figure 2.23

Determine the number of points needed along the circumference of each quadrant of the ellipse for a relatively accurate layout and mark off this number of division points P, Q, R and S between O and F_1 on the major axis. It may be desirable to use additional points spaced closer together near F_1 in order to form accurately the sharp curvature at the end of the ellipse. With F_1 and F_2 as centers and the distance *AP* and *BP* as radii respectively, strike intersecting arcs to locate *P* on the circumference of the ellipse. Distance *AQ* and *BQ* are radii for locating points *Q*. Locate *R′* and *S′* in a similar manner and complete the ellipse by joining *P′*, *Q′*, *R′*, *S′* and *A* by smooth curve or using a French curve.

2.9.2 Ellipse Construction by Concentric Circle Method

Given the major axis *AB* and the minor axis *CD*. Using the center of the ellipse point *O* as center describe circles having the major and minor axes as diameters. Divide the circles into equal central angles and draw diametrical lines such as P_1P_2. From point P_1 on the circumference of the larger circle, draw a line parallel

Figure 2.24

to *CD*, the minor axis, and from point P'_1, at which the diameter P_1P_2 intersects the inner circle, draw a line parallel to *AB*, the major axis. The point of intersection of these lines, point *E*, is on the required ellipse. At point P_2 and P'_2, repeat the same procedure and locate point *F*. Thus, two points are established by line P_1 P_2. Locate at least five points in each of the four quadrants. The ellipse is completed by drawing a smooth curve through the points. This is one of the most accurate method.

2.9.3 Ellipse Construction by Four Center Method

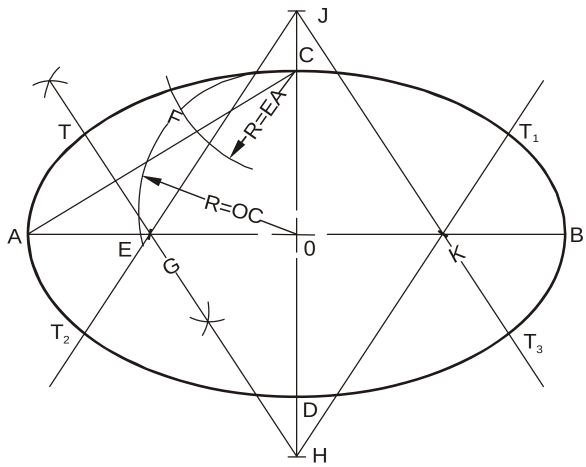


Figure 2.25

Given the major axis *AB* and the minor axis *CD*. Draw *AB* and *CD* and join *A* and *C*. With the center of the ellipse *O* as a center and *OC* as radius strike an arc intersecting *OA* at point *E*. Using *C* as a center and *EA* as radius, strike an arc intersecting the line *AC* at *F*. Draw the perpendicular bisector of the line *AF*. The point *G* and *H* at which the perpendicular bisector intersect the axis *AB* and *CD* (extended) are the centers of two of the arcs forming the ellipse. Locate the other two centers *J* and *K* by laying off *OJ* equal to *OH* and *OK* equal to *OG*. With *G* as center and radius *GA*, draw arc TAT_2, with *K* as center and radius *KB* draw arc $T_1B\ T_3$, with *J* as center and radius *JD*, T_2DT_3, with *H* as center and radius *HC* draw arc TCT_1. The figure, thus, formed by the four circle arcs approximates a true ellipse.

2.9.4 Ellipse Construction by Parallelogram Method

Given the major axis *AB* and the minor axis *CD*. Construct the circumscribing parallelogram. Divide *OA* and *AE* into the same number of equal parts and number the division points from *A*. From *C*, draw a line through point 3 on time *AE*, and from *D*, draw a line through point 3 on line *AO*. The point of intersection

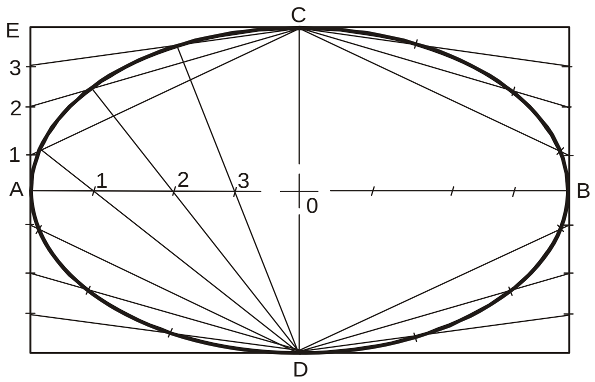


Figure 2.26

of these lines is on the required ellipse. Similarly, the intersections of lines from *C* and *D* through points 1 and 2 are on the ellipse. A similar construction will locate points in the other three quadrants of the ellipse. A smooth curve is to be drawn through these points to draw the ellipse.

2.10 PARABOLA

Parabola is a curve generated by a point moving so that at any position its distance from a fixed point (*the focus*, *F*) is always exactly equal to its distance to a fixed line (*the directrix*, *AB*), as shown in Figure 2.27.

Figure 2.27

To construct a parabola when focus *F* and the directrix *AB* are given, draw the axis of the parabola perpendicular to the directrix *AB*. Through any point on axis, for example point *C*, draw a line parallel to the directrix *AB*. Using *F* as a center and distance *OC* as a radius, strike arcs intersecting the line at points P_4 and P'_4. Repeat this procedure until a sufficient number of additional points have been located to determine a smooth curve. The vertex *V* is located at a point midway between *O* and *F*.

2.10.1 Construction of Parabola by Tangent Method

Given the point *A* and *B* and the distance *CD* from *AB* to the vertex. Extend the axis *CD* and set off *DE* equal to *CD*. *EA* and *EB* are tangents to the parabola at *A* and *B*, respectively. Divide *EA* and *EB* into the same number of equal parts (five

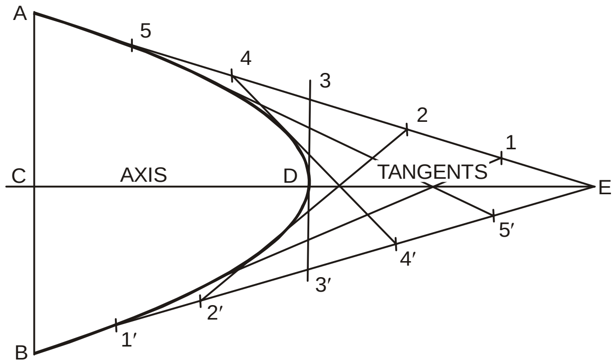
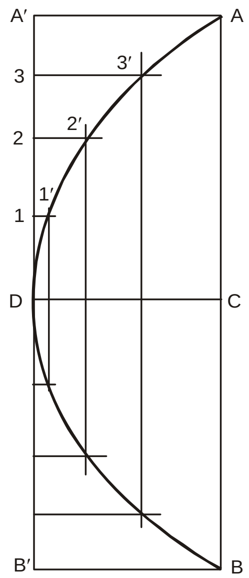


Figure 2.28

here) and number the division points as shown in Figure 2.28. Connect the corresponding points 1 and 1′, 2 and 2′, 3 and 3′ and so forth. These lines, as tangents of the required parabola form its envelope. Draw the tangent curve.

2.10.2 Construction of Parabola by Offset Method

Given the enclosing rectangle *A′ ABB′*. Divide *DA′* into any numbers of equal parts (four here) and draw from the division points perpendiculars parallel to *DC*, along which the offset distances are to be measured off. The offset varies as the square of their distances from *D*. For example, since *D*1 is one fourth of the distance from *A′* to *D*, 11′ will be $(1/4)^2$ or 1/16 of A′ A. Similarly, 2-2′ will be $(1/2)^2$ or ¼ of *A′ A* and 3-3′ will be 9/16 of *A′A*. To complete the parabola, lay off the computed offset values along the perpendicular and draw the parabola with a smooth curve or French curve.



$$3 - 3' = \frac{9}{16} \times A' - A$$

$$2 - 2' = \frac{1}{4} \times A' - A$$

$$1 - 1' = \frac{1}{16} \times A' - A$$

Figure 2.29

This method is preferred by civil engineers for laying out parabolic arches and computing vertical curves for highways.

2.10.3 Construction of Parabola through Two Given Points

Given the points *A* and *B*. Assume a point *C*, and draw line *CA* and *CB*. Divide lines *CA* and *CB* in equal parts and mark them as shown in Figure 2.30. Draw lines joining 1 and 1, 2 and 2, 3 and 3 and so forth. These lines are tangent to the parabolic curve. Join the intersection points as shown in the figure and draw the parabolic curve.

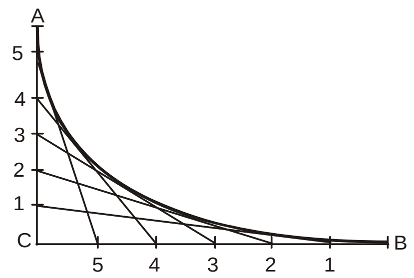
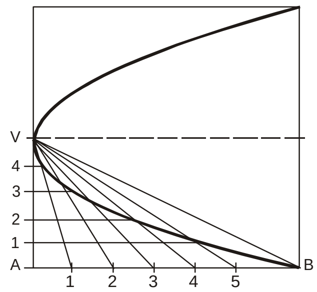


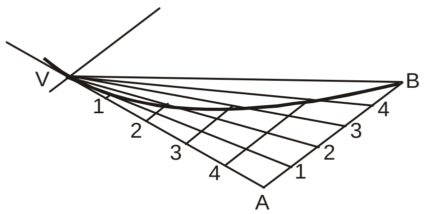
Figure 2.30

2.10.4 Construction of Parabola by Parallelogram Method

Since the dimensions for a parallelogram that will enclose a given parabola are generally known, a parabola may be constructed by the parallelogram method as explained in Figure 2.31. With the enclosing rectangle drawn to the given dimensions, divide *VA* and *AB* into the same number of equal divisions (five here) and number the division points as shown. Draw lines from point *V* to each of the division point along *AB*, i.e.*V*1, *V*2, *V*3 and *V*4. Then draw lines parallel to the axis from points 1, 2, 3 and 4 on *VA*. The intersection of the lines through 1 with *V*1, 2 with *V*2, 3 with *V*3 and 4 with *V*4 are on the parabolic curve. Join them and draw parabolic curve. The method as explained for a parabola enclosed in a rectangle may be applied to a non-rectangular shape as shown in Figure 2.31(b).



(a)



(b)

Figure 2.31

2.11 HYPERBOLA

The *hyperbola* can be described as a curve generated by a point moving so that at any position the difference of its distances from two fixed points (Foci) is a constant and is equal to the transverse axis of the hyperbola. This definition is the basis for the construction shown in Figure 2.32.

To construct a hyperbola draw foci F_1 and F_2 and the transverse axis *AB* as per the given data. Using F_1 and F_2 as centers and any radius R_1 greater than F_1B, strike arcs. With these same centers and a radius equal to $R_1 - AB$, strike arcs intersecting the first arcs at point *P*. Point *P* is on the required hyperbola. Repeat this procedure and locate as many additional points such as P_1, P_2 and so forth, as are required to form the hyperbola. Join these points by smooth curve or with a French curve to draw the hyperbola.

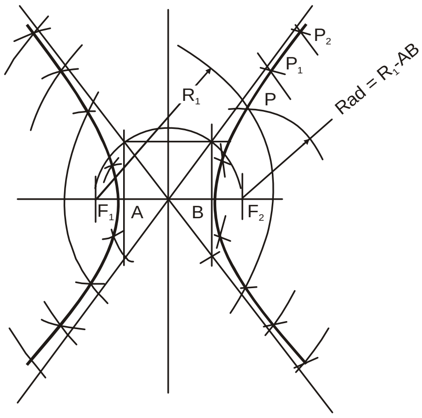


Figure 2.32

2.12 CYCLOID

Cycloids form a group of curve generated by the path of a fixed point on the circumference of a rolling circle. When the circle rolls on the straight line, the path is called a *cycloid* (Figure 2.33).

Figure 2.33

To draw cycloid curve, draw a horizontal line AA_{12} and mark off the length of the rolling circle of diameter D. This length AA_{12} is equal to πD. Then divide the rolling circle and its length, marked off on AA_{12} into say 12 equal parts. Erect perpendiculars at point 1′ through 12′ to intersect the extended horizontal axis of the rolling circle at point O_1 through O_{12}. Now draw horizontal lines passing through the points of division of the rolling circle. From O_1 through O_{12} centers and with a radius equal to $D/2$ describe arcs to intersect the corresponding horizontal lines at the points A_1 to A_{12} of the required cycloid. A smooth curve is then drawn through these points with the aid of a French curve.

2.13 EPICYCLOID

If a circle of diameter D rolls on the outside of another circle of radius R, the path of the point A (on the rolling circle) is called an *epicycloids* (Figure 2.34).

To draw the epicycloids, describe a rolling circle of diameter D and a guiding circle of radius R tangent to it at point 12. Divide the rolling circle into 12 equal parts. With O_o as a center and a radius equal to $R + D/2$ draw an auxiliary arc. From the point A and clockwise layout on the guiding circle mark 12 divisions, each one equal to $1/12\ \pi\ D$. To determine the arc of the guiding circle $AA_{12} = \pi\ D$, use the formula $\alpha = \dfrac{180^{o}\ D}{R}$.

Figure 2.34

The angle α thus obtained is divided into 12 equal parts with the aid of a protractor and the points are denoted as $1'$ through $12'$. From O_o and through these points draw radial lines whose extensions will intersect the auxiliary arc at point O_1 through O_{12}. With O_o as a center describe a series of auxiliary arcs passing through the points of division of the rolling circle.

From O_1 through O_{12} as centers and with a radius equal to $D/2$ describe arcs to intersect the corresponding auxiliary arcs at points A_1 through A_{12}. A smooth curve is then drawn through these points to obtain the required epicycloid.

2.14 HYPOCYCLOID

When a circle of diameter D rolls on the inside of another circle of radius R, the path of the point A (on the rolling circle) is called a *hypocycloid* (Figure 2.35).

Figure 2.35

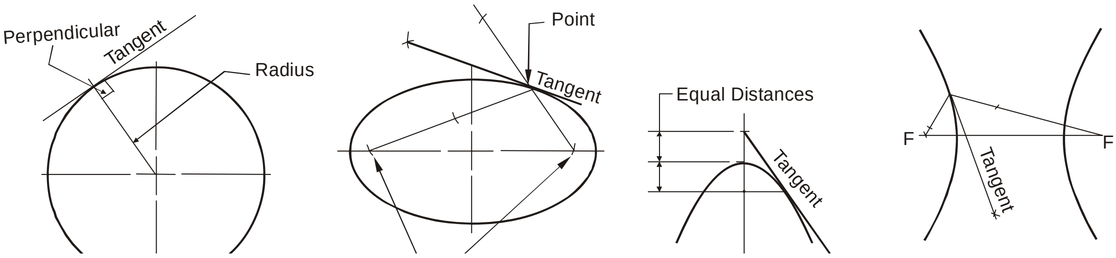
To draw the hypocycloid curve the construction is quite similar to that of the epicycloids.

Describe a rolling circle of diameter D and a guiding circle of radius R tangent to it at point A. Divide the angle α (which is equal to 180 D/R) and the rolling and guiding circles into 12 equal parts. From O_0 and through the points of division on the guiding circle, draw radial lines which will intersect the auxiliary circle of radius $R - \frac{D}{2}$ at point O_1 through O_{12}.

With O_0 as a center, describe auxiliary arcs passing through the points of division of the rolling circle. From O_1 through O_{12} as centers and with a radius equal to $\frac{D}{2}$ draw arcs to intersect the corresponding auxiliary arcs at the points A_1 through A_{12} of the required hypocycloid. A smooth curve is then drawn through these points to obtain the required hypocycloid.

2.15 TANGENT AND NORMAL TO CURVES

The outlines of many parts of machines, apparatus, instruments and devices consist of straight lines and arcs gradually merging into one another. Such gradual continuation of one line (whether straight or curved) is termed tangency. It is practically impossible for a draftsmen to carryout tangency construction of required accuracy without knowing the appropriate rules. Here methods of drawing tangent and normal to common engineering curves like circle, ellipse, parabola and hyperbola is explained (Figure 2.36).



(a) Circle **(b) Ellipse** **(c) Parabola** **(d) Hyperbola**

Figure 2.36

2.15.1 To Draw Line Tangent to a Circle through a Given Point *A* Located Outside the Circle

Connect the given point *A* (Figure 2.37) to the center *O* of the circle with a straight line, *AO*. Divide this line into two equal parts and from its mid-point O_1 with the radius $O_1 A$, describe an auxiliary circle which will intersect the given circle at two points *D* and *E*. Join these points with point *A* by lines *AD* and *AE* which will be tangent to the given circle.

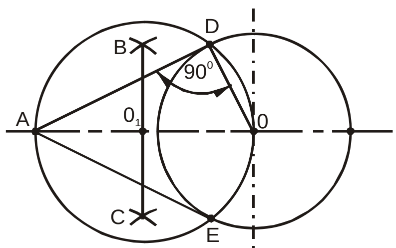
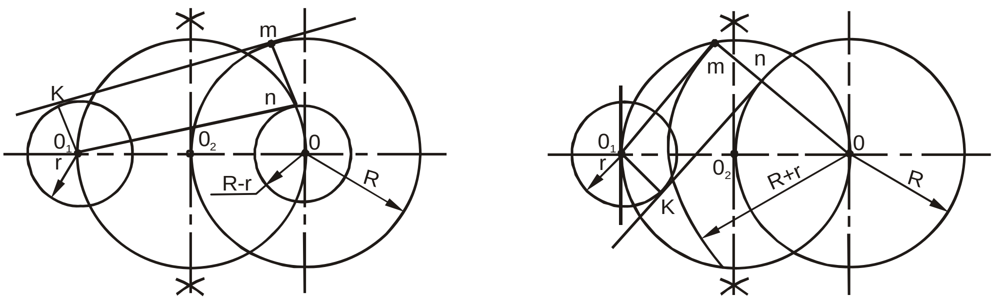


Figure 2.37

2.15.2 To Draw a Line Tangent to two Circles of Different Radius (*r* and *R*)

Two cases are possible here : uncrossed tangents (Figure 2.38(a)) and crossed tangents (Figure 2.38(b)). Uncrossed tangents are constructed in the following way. From O as the center and with a radius equal to the difference of the radii of the given circle ($R - r$), draw an auxiliary circle. From the center O_1, draw a line $O_1 n$, tangent to this circle at n. From the center O, draw a radius, On, and extend it to intersect the circle R at m. Draw the radius O_1K of the second circle parallel to the line Om. Join the points K and m with a straight line which will be tangent to the given circles. Crossed tangents are constructed in a similar way, except that the auxiliary circle is described with a radius $R + r$. On drawing a line from O_1 tangent to the auxiliary circle m, this point is joined with O by radius Om, which intersects the circle OR at n. Draw a line parallel to o_1m through n which gives line nK which is the required cross-tangent.



(a) **(b)**

Figure 2.38

2.15.3 To Draw a Tangent to an Ellipse at Any Given Point

Refer Figure 2.39, given any point such as P on the perimeter of the ellipse $ABCD$. Using C as a center and a radius equal to OA (one half the major diameter), strike arcs across the major axis at F_1 and F_2. From these points, which are foci of the ellipse, draw F_1P and F_2G. The bisector of the angle GPF_1 is the required tangent to the ellipse.

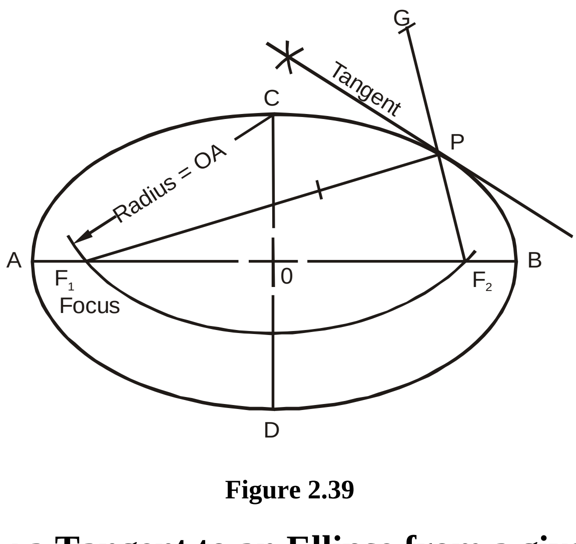


Figure 2.39

2.15.4 To Draw a Tangent to an Ellipse from a given Point *P* Outside of the Ellipse

Refer Figure 2.40, with the end of the minor axis as a center and a radius R equal to one-half of the length of the major axis, strike an arc to find the foci F_1 and F_2. With point P as a center and the length AB as a radius, strike arcs cutting the arc with center of P at point G and H. Draw line GF_1 and HF_1 to establish the location of the tangent point T_1 and T_2. Draw the required tangent.

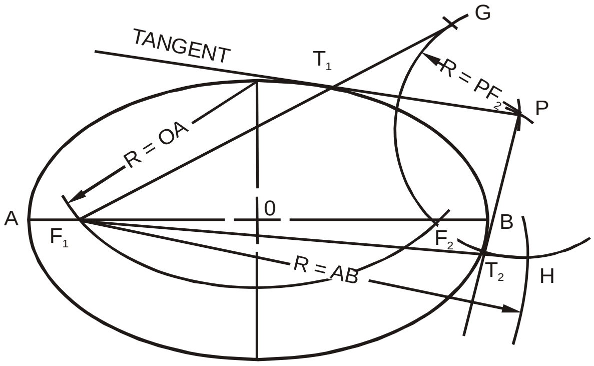


Figure 2.40

2.15.5 To Construct a Tangent to a Parabola

Refer Figure 2.27, to construct a tangent to a parabola, say at point P_6, draw the line P_6D parallel to the axis, then bisect the angle DP_6F. The bisector of the angle is the required tangent.

2.15.6 To Construct a Tangent to a Hyperbola

Refer Figure 2.32, the tangent to the hyperbola at any point such as P, is the bisector of the angle between the focal radii F_1P and F_2P.

SAQ 1

(a) Construct a regular hexagon having a 50 mm distance across flats. Select the most practical procedure.

(b) Construct a regular hexagon having a 75 mm distance across corners. Select the most practical method.

(c) Construct a regular pentagon having 15 mm sides.

(d) Using a line 75 mm long as the base line, construct a triangle having sides 50 mm, 75 mm and 100 mm long respectively. Trisect the angle between the 75 mm and 100 mm sides of the triangle.

SAQ 2

(a) Draw a 75 mm dia circle. Inside this circle and tangent to it draw a 40 mm dia circle. See that the centers of both circle are on the same vertical center line. Draw two 25 mm dia circles tangent to the 75 mm and 40 mm circle.

(b) Construct an ellipse having a major diameter using

  (i) Concentric circle method

  (ii) Four center method

(c) Construct a parabola with vertical axis, having the focus 20 mm from the directrix. Select a point on the curve and draw a line tangent to the parabola.

(d) Construct a hyperbola having a transverse axis of 25 mm and foci 40 mm apart.

SAQ 3

(a) Construct the cycloid generated by a 40 mm diameter circle.

(b) Construct the epicycloids generated by 40 mm diameter circle rolling on a 125 mm diameter circle.

(c) Construct the hypocycloid generated by a 40 mm diameter circle rolling in a 125 mm diameter circle.

(d) A bicycle having wheel diameter 650 mm passes over a segmental arched culvert of radius 3 m at a speed of 16 km per hr. Draw the locus of the tip of the value for inflating the tyre which is situated at a distance of 7 cm from the periphery of the tyre. Assume that the tyre remains circular under the load.

2.16 SUMMARY

The simplified geometrical constructions presented in this unit are those with which an engineer should be familiar, for they frequently occur in engineering drawing. The method are applications of the principles found in text books on plane geometry. The construction have been modified to take advantage of time saving methods made possible by the use of drawing instruments.

2.17 ANSWERS TO SAQs

Refer the relevant preceding text in the unit or other useful books on the topic listed in the 'Further Reading' given at the end to get the answers of SAQs.