
UNIT 3 ORTHOGRAPHIC PROJECTIONS-I

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3.1 INTRODUCTION

An engineering drawing is a graphic representation of engineering objects such as buildings, roads, machines, parts etc. on paper. Modern engineering produces enormous numbers of articles, each first designed and presented in the form of a technical drawing, and then manufactured on the basis of this drawing. Engineering drawing known as the language of engineers is one of the fundamental subjects of the engineering education.

Objectives

After studying this unit, you should be able to

- explain the concept of reference projection planes, i.e. HP, VP etc.,
- understand the line of intersection of reference plains, i.e. $X-Y$ line,
- understand the concept and position of four dihedral angles,

- understand the concept of projectors and their relation with reference planes,
- distinguish the difference between first angle projection method and third angle projection methods,
- locate the position and draw the projections of points,
- understand the concept of views,
- know the relative positions of different views,
- understand the concept of lines and their projections, and
- understand the concept of projection of planes.

3.2 ORTHOGRAPHIC PROJECTION

3.2.1 Projection

For making an engineering drawing of an object, projection is a common method. Imaginary straight lines are drawn from the various important points of the object on its contour which further reach the plane surface. The points of intersection so obtained on the plane surface are then joined in a sequence and the figure thus obtained is called the Projection. The imaginary lines starting from various points of the object are called the Projectors.

3.2.2 Orthographic Projection

When the projectors are perpendicular to the surface of projection on which they intersect, and all the projectors are parallel to each other, the projection so obtained is called the *orthographic projection*. If we compare a film projection with an orthographic projection, we see that in the former case all the rays of light that start from the film and reach the screen are neither perpendicular to the screen nor parallel to each other. Hence, a film projection can never be an orthographic projection.

Figure 3.1 shows two mutually perpendicular planes used in orthographic projection in which all four quadrants are marked.

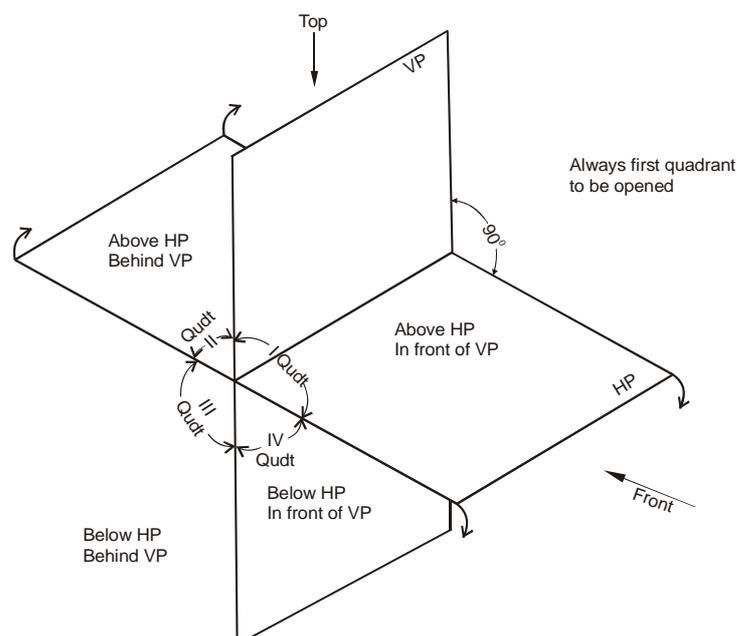


Figure 3.1

3.2.3 Projection Methods

A projection will be known as the first angle projection, if the object to be projected is considered to be situated in the first dihedral angle or the first quadrant. Its position will be defined as in front of the vertical plan and above the horizontal plane.

The projection will be termed as the second angle projection when the object is considered in the second quadrant. Its position will be behind the vertical plane and above the horizontal plane. The planes are considered to be transparent, thin and dimensionless. The horizontal plane is supposed to be hinged about the line of intersection with the vertical plane.

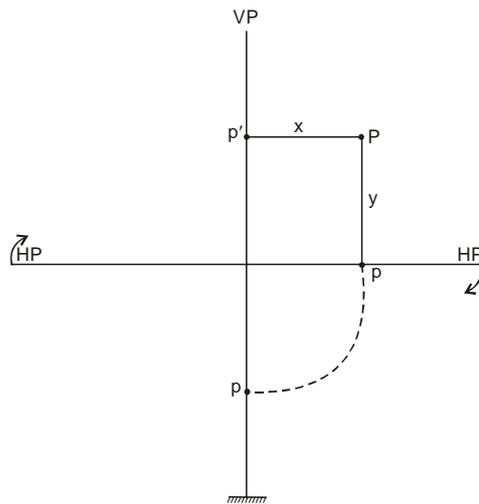


Figure 3.2

Imagine a point P situated in the first quadrant in Figure 3.2 (Figure 3.2 is the edge-wise view of the reference planes, i.e. HP and VP). Its co-ordinates are x - y . From P , a projector is drawn to the vertical plane. It will produce an image p' on the VP. The distance of p' from the line of intersection, i.e. from X - Y line will be y units from P , another projector is drawn perpendicular to horizontal plane. It will produce an image on the horizontal plane at x units away from the origin.

Now for obtaining these projections of P , on a single plane surface, it is obligatory to rotate the horizontal plane clockwise (as shown in Figure 3.2) till it coincides with the vertical plane. The portion of the horizontal plane which is in the first quadrant will go down and coincide with the vertical plane below the intersection line X - Y , and the portion of the horizontal plane which is in second quadrant will rotate clockwise and coincide with the vertical plane, above the reference line XY . When we look at the whole arrangement perpendicular to the planes, it will appear as shown in Figures 3.3(a) and (b).

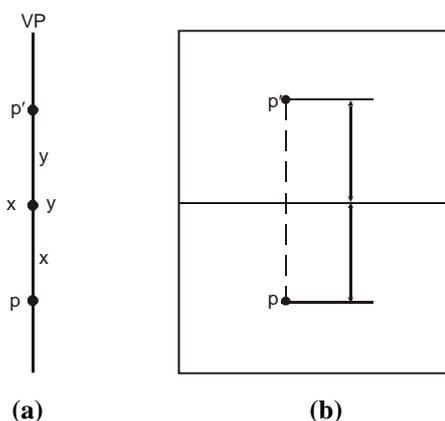


Figure 3.3

It may be noted that the projections of point p are lying on the same projector, which is perpendicular to the reference line XY . The projection of point p on the vertical plane, which is called elevation or front view is usually marked with lower case letter p' , is of y units above $X-Y$ line and the projection of P on the horizontal plane is termed as *Plan* or *Top view* and is usually marked with lower case letter p (without dash), is x units below the reference line $X-Y$.

When the object is considered in second quadrant, its front view will be above XY line and the top view obtained on the horizontal plane will cover the front view. The horizontal plane is rotated clockwise only. Imagine a drawing in which one view overlaps the other (here the top view overlaps the front view). In this case, no details can be read. Therefore, in practice, there cannot exist the second angle method of projection.

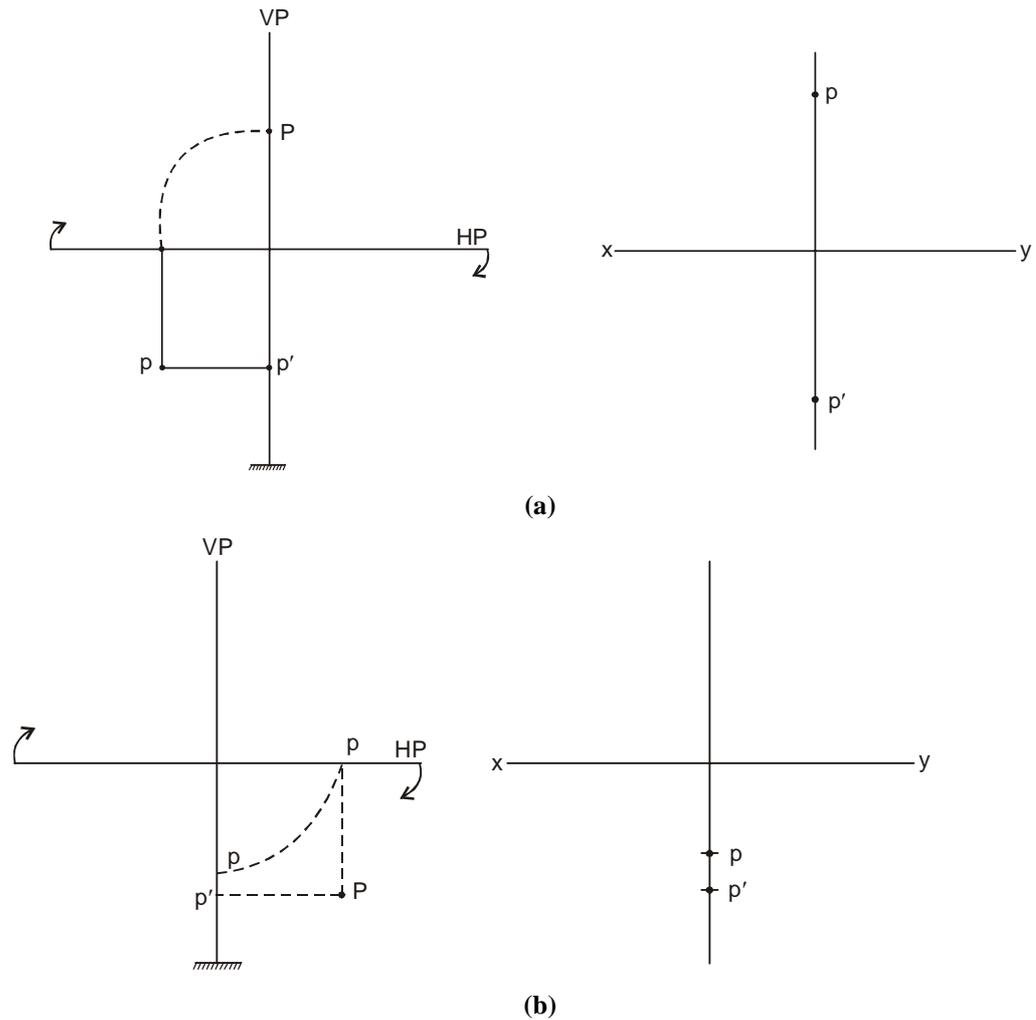


Figure 3.4

When the object is considered in the third quadrant (Figure 3.4(a)), its front view will be below $X-Y$ line and the top view will be above $X-Y$ line. This is called the third angle method of projection.

When the object is considered in the fourth quadrant, the front view will be below the $X-Y$ line and the top view will cover the front view. Therefore, in practice, the fourth angle projection method does not exist.

3.2.4 Designation of Views

As per the Bureau of Indian Standards (BIS-SP : 46-1988) which specifies general principles of presentation of technical drawings following the

orthographic projection methods, the designation of views is as follows (Figure 3.5).

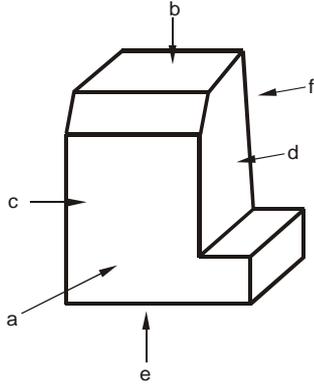


Figure 3.5

- View in direction a = View from the front called Front View or Elevation.
- View in direction b = View from above called Top View or Plan
- View in the direction c = View from the left side called Left Side View.
- View in the direction d = View from the right side called Right Side View
- View in the direction e = View from below called Bottom View
- View in the direction f = View from the rear or back called Back View.

3.2.5 Relative Position of Views

Two alternative orthographic projection methods of equal importance can be used. They are the *first angle projection methods* and the *third angle projection method*.

3.2.6 First Angle Projection Method

With reference to the front view (a), the other views are arranged as follows and are depicted in Figure 3.6.

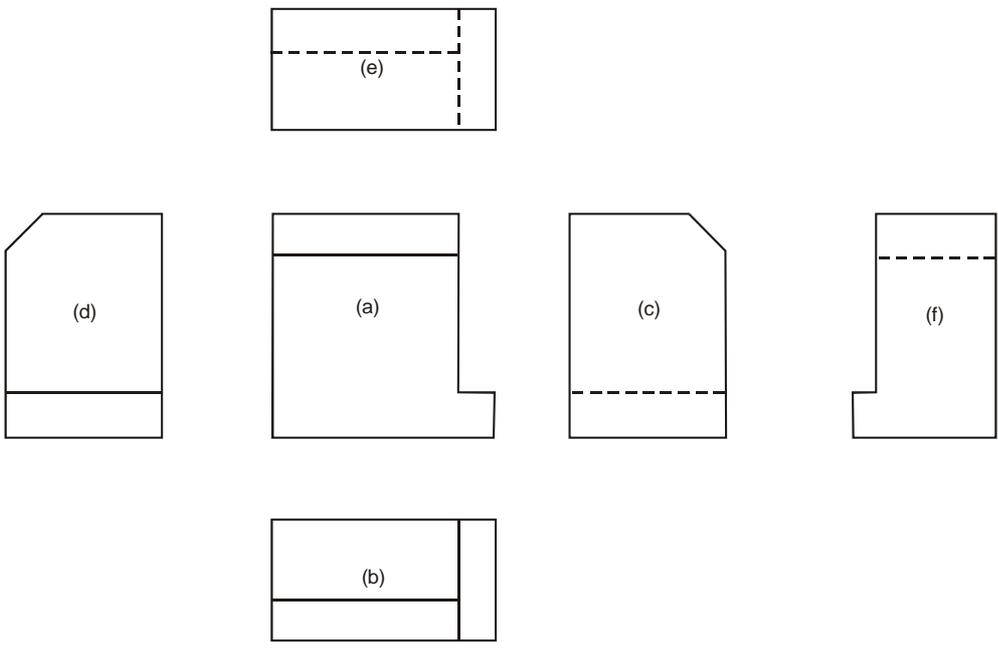


Figure 3.6

- (a) The view from top, i.e. plan, is placed below the front view.
- (b) The view from the below, i.e. bottom view, is placed above the front view.
- (c) The view from the left, i.e. left side view, is placed on the right side of the front view.
- (d) The view from the right side, i.e. right side view, is placed on the left side of the front view.
- (e) The view from the rear, i.e. back side view, may be placed on the right side or on the left side of the front view, as per convenience.

The symbol as adopted by BIS for first angle projection method is shown in Figure 3.7.

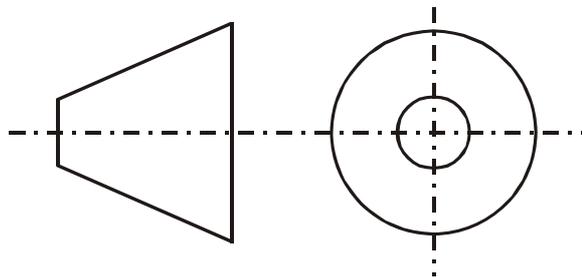


Figure 3.7

3.2.7 Third Angle Projection Method

With reference to the front view (a), the other views are arranged as follows (Figure 3.8).

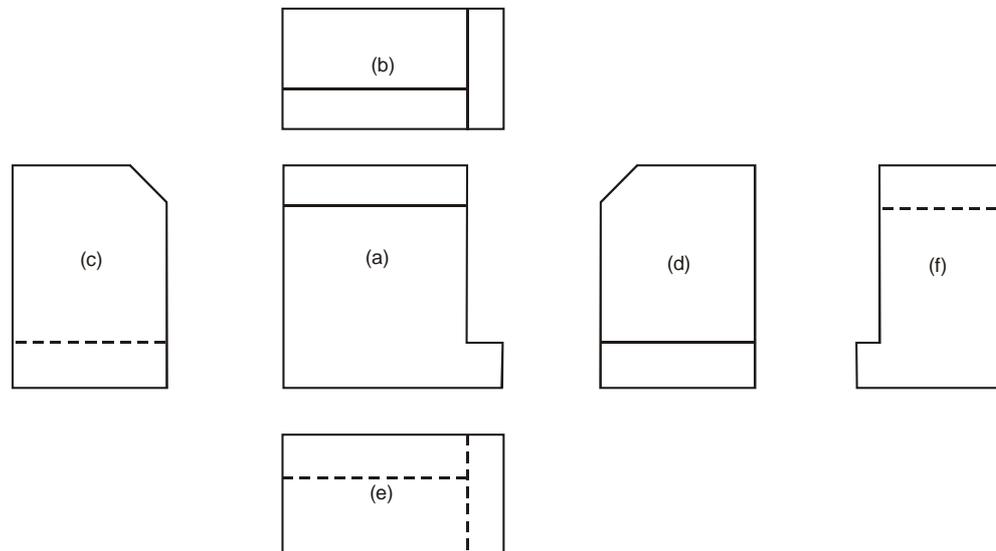


Figure 3.8

- (a) The view from the top, i.e. plan, is placed above the front view.
- (b) The view from the below, i.e. bottom view, is placed below the front view.

- (c) The view from the left side, i.e. left side view, is placed on the left side of the front view.
- (d) The view from the right side, i.e. right side view, is placed on the right side of the front view.
- (e) The view from the rear, i.e. back side view, is placed on the right side or left side of the front view as per convenience.

The symbol as adopted by BIS for third angle projection method is shown in Figure 3.9.

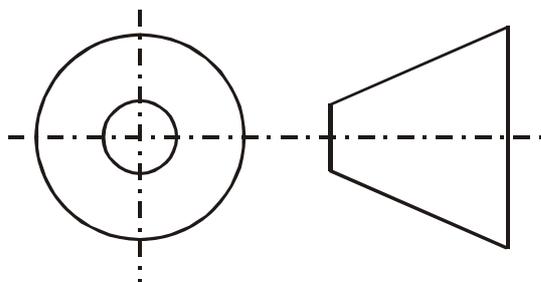


Figure 3.9

3.2.8 Indication of Method

Where one of the methods (first angle projection or third angle projection) is being used, the said method must be indicated on the drawing by means of its distinguishing symbols as shown in Figures 3.8, and 3.9. The symbol shall be placed in a space provided for the purposes in the title block of the drawing sheet.

3.2.9 General Principles of Orthographic Drawing

The following principles should be thoroughly understood before any attempt is made to prepare an orthographic drawing

- (a) The plan and elevation are always in a vertical line.
- (b) The elevation and side elevation are always in a line horizontally.
- (c) The length of the plan horizontally is always the same as the length of the elevation horizontally.
- (d) The height of the side elevation is always the same as the height of the elevation.
- (e) If a line is parallel to a plane of projection, then it will show its true length on the same plane.
- (f) If a line is inclined to a plane of projection, then projection on the same plane will be a line shorter than the length of line (true length). The projected length will be a function of the angle of the given line with the plane.
- (g) If a line is perpendicular to a plane, then its projection on the same plane will be a point.
- (h) If a thin surface (plane) is parallel to a plane, then its projection on the same plane will show its true shape and size.
- (i) If a surface (plane) is inclined to a reference plane, then the projection of the surface on that plane will be foreshortened (will not be true shape).

- (j) If a thin surface is perpendicular to a plane, then its projection on the same plane will be a straight line.

For an object consisting of flat, inclined and cylindrical surfaces, elevation, plan, and right side view are shown in the first angle projection method in Figure 3.10 and the projections in third angle projection method are shown in Figure 3.11.

(a)

(d) **Conventional Representation
of First Angle Projection**

(b)

(c)

Figure 3.10 : First Angle Method of Projection

(a)

(d) Conventional Representation of Third Angle Projection

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Projections-I

Figure 3.11

(b)

(c)

Figure 3.11 : Third Angle Projection

3.3 PROJECTION OF POINTS

As indicated in Sections 3.2.6 and 3.2.7, there are only two standard methods of projection. These are first angle method of projection and the third angle method of projection. In the first angle projection method, the object is considered to be situated in the first quadrant, i.e. above HP, and in front of VP. And in third angle projection method, the object is considered in the third quadrant, i.e. behind VP and below HP.

The object is never considered in the second or fourth quadrants. This statement is true for solid **objects** only. When we wish to draw the projection of **points** and also **lines**, these may be considered in any of the quadrant.

A point may be in I, II, III and IV quadrant, or it may also lie on any or both the reference planes at a time.

Similarly, a line say AB , may have its one end A or both ends A and B in any of the four quadrants or the entire line AB may be lying in one or both planes or the line may have one end in one quadrant and the other end in other quadrant.

3.3.1 Projections of Point Situated in the First Quadrant

The point P is situated in first quadrant, i.e. y units above HP and x units in front of VP when two projectors (imaginary lines one parallel to HP and other perpendicular to VP) are drawn from this point P , they will intersect VP and HP at p' and p respectively. These two points p' on VP and p on HP are the elevation and plan of point P .

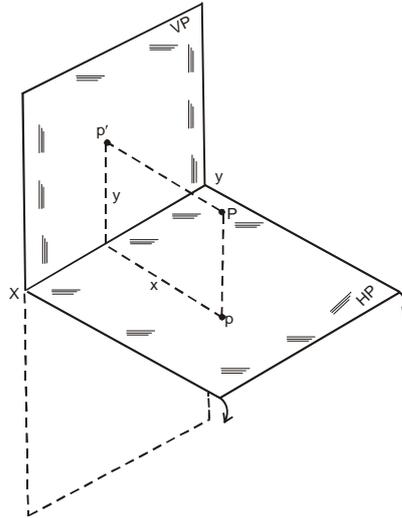


Figure 3.12(a)

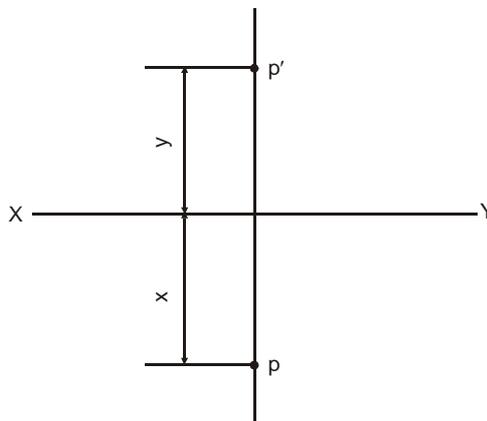


Figure 3.12(b)

The method of obtaining the projection of P is as follows (Ref Figure 3.12(b)).

- (a) Draw a horizontal line of any length and mark it $X-Y$. This is reference line.
- (b) Draw a thin and comparatively light line perpendicular to $X-Y$. This line is called projector.
- (c) Mark a point p' , on the projection at a distance y from $X-Y$ and above it.
- (d) Mark a point p on the projector at a distance x from $X-Y$ and below it.

p' and p are the projections of P in Ist quadrant.

3.3.2 Projections of a Point Q Situated in Second Quadrant

The point Q is situated in second quadrant, i.e. y units above HP and x units behind the vertical plane.

The elevation q' of the point Q will be at y units above the HP and on the projector perpendicular to $X-Y$.

The plan q of the point Q will be on the HP at a distance x units behind VP

The horizontal plane is rotated clockwise till it coincides with VP. The plan q will then come above $X-Y$ and on the same projector at a distance x units above $X-Y$ line.

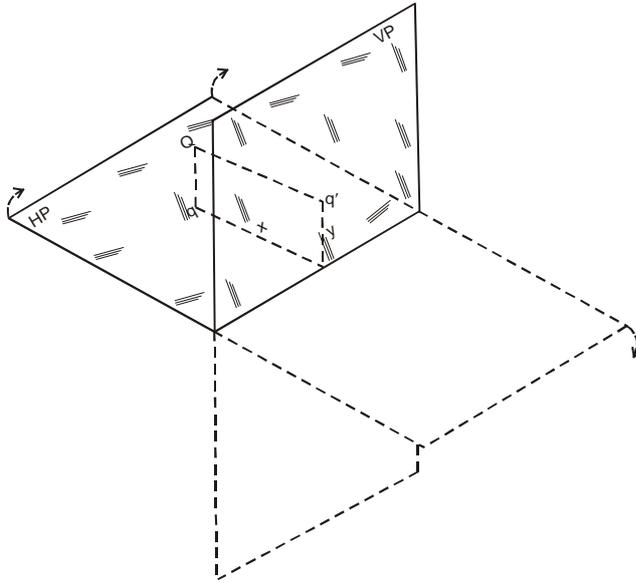


Figure 3.13(a)

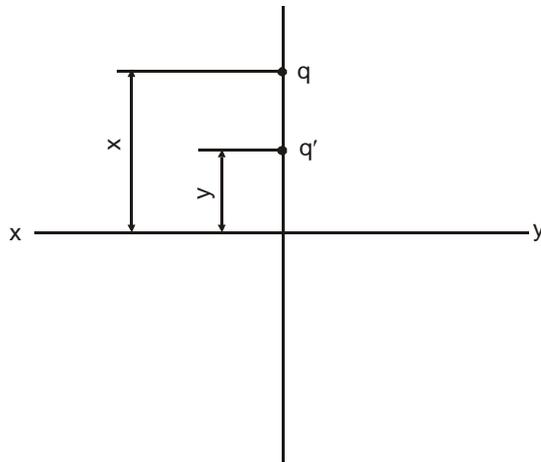
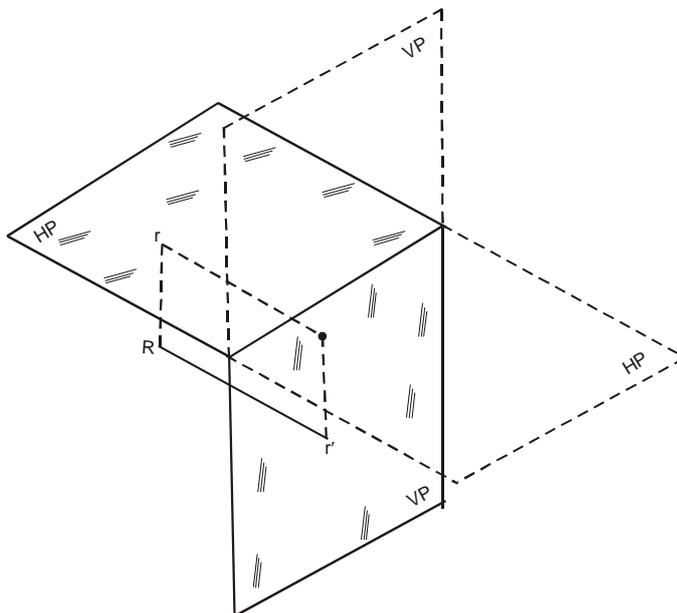


Figure 3.13(b)

3.3.3 Projections of a Point R Situated in the Third Quadrant

The point R is situated in third quadrant, i.e. x units behind the VP and y units below the HP.



(a)

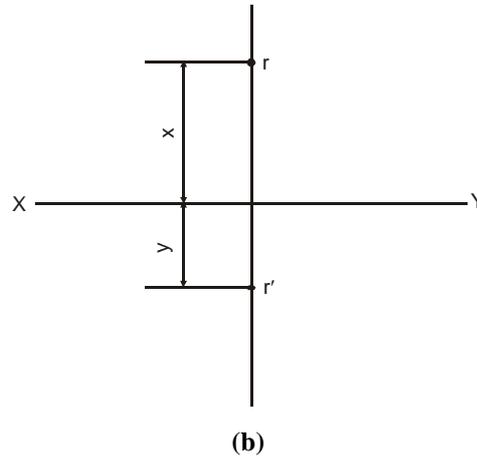
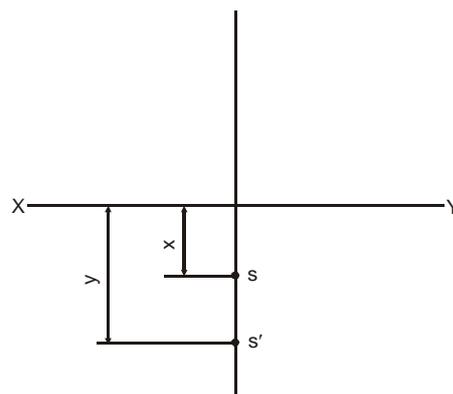
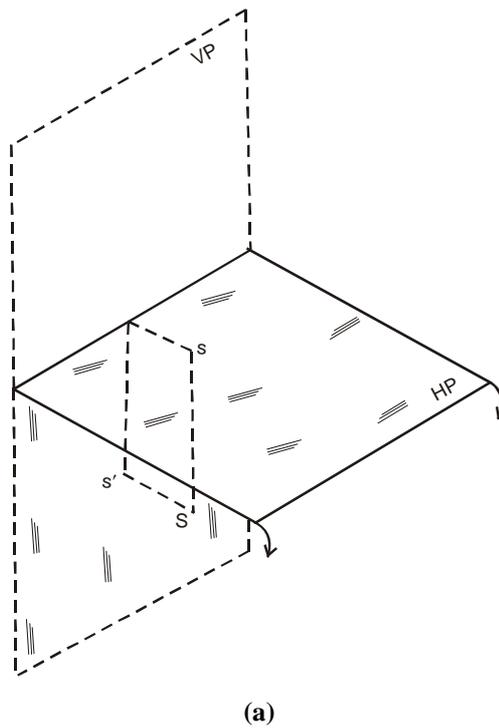


Figure 3.14

The elevation r' will be y units below $X-Y$ and plan r will be x units above $X-Y$. As the HP is rotated clockwise, the plan goes up.

3.3.4 Projections of a Point Situated in the Fourth Quadrant

The point S is situated in the fourth quadrant, i.e. x units in front of VP and y units below HP. The elevation s' will be y units below XY and when the HP is rotated clockwise, the plan goes down, i.e. s will be x units below $X-Y$.



(b)
Figure 3.15

3.3.5 Projections of a Point Situated in a Plane

If the point is situated in HP, then its elevation will be in $X-Y$ line. The position of plan will be related to the position of the point in HP, i.e. if the point is in HP and in front of VP, its plan will be below $X-Y$ line, and if the point is in HP and behind VP then its plan will be above $X-Y$ line.

If the point is situated in VP, its plan will be in $X-Y$ line. The position of elevation will be related to the position of the point in VP, i.e. if the point is in VP and above HP, its elevation will be above $X-Y$ line, and if the point is in VP but below HP, its elevation will be below $X-Y$ line.

If the point is situated simultaneously in HP and VP both, i.e. the point is on $X-Y$ line and under this condition, the plan and elevation of the point will coincide and will be in $X-Y$ line.

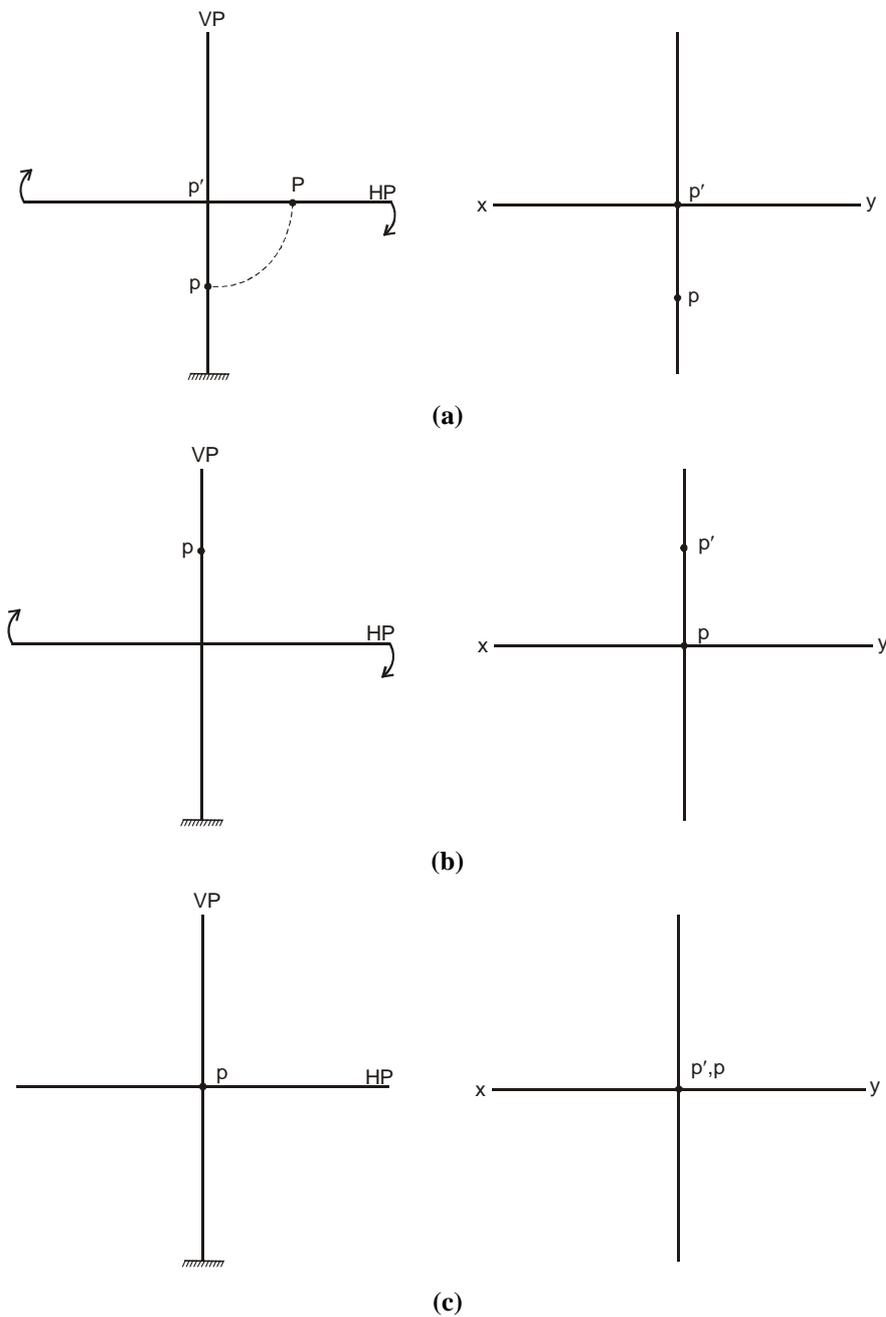


Figure 3.16



- (a) Draw the projections of the following points considering the reference line $X-Y$ to be the same. You may keep the distance between the projectors of the points equal to 20 mm.
- A , 30 mm above HP and 40 mm in front of VP
 - B , 35 mm below HP and 40 mm in front of VP
 - C , 20 mm below HP and 30 mm behind VP
 - D , 30 mm above HP and 40 mm behind VP
 - E , in HP and 30 mm in front of VP
 - F , in HP and 40 mm behind VP
 - G in VP and 30 mm above HP
 - H in VP and 40 mm below HP
- (b) Projections of points p, q, r, s, t and v are given in Figure 3.17.

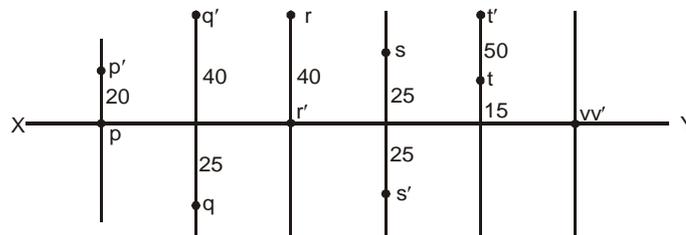


Figure 3.17

State the position of each point with respect to the planes of projection, showing their distances in cms.

- (c) A point C is 20 mm above HP and 25 mm in front of VP. Another point D is 30 mm behind VP and 40 mm below HP. Draw their projections keeping the distance between their projectors equal to 100 mm. Draw lines joining $c'd'$ and cd . You will know that line $c'd'$ is the elevation and line cd is the plan of a straight line CD .
- (d) The points P and Q are in the HP and distance between their projections is 80 mm. P is 40 mm in front of the VP while Q is behind the VP. If a line is drawn joining p and q , i.e. their plans, this line makes an angle of 45° with $X-Y$. Find graphically the distance of point Q from VP.
- (e) The top view of a point R is 40 mm above XY line and the front view is 15 mm below the top view. In which quadrant the point R is situated?

3.4 PROJECTION OF STRAIGHT LINES

A straight line is the shortest distance between two points which may be the ends of the line. The line is considered to be a thin line (as said no thickness).

3.4.1 Position of the Line with Respect to Planes of Projections, i.e. VP and HP

Position of the line with respect to planes of projections, i.e. VP and HP may be

- (a) parallel to one or both the planes,
- (b) contained by one or both the planes,
- (c) perpendicular to one of the planes,
- (d) inclined to one of the planes and parallel to the other plane, and
- (e) inclined to both the planes.

3.4.2 Traces of a Line

If a line is inclined to a plane, it will intersect the same plane either in natural course or after extension of the line towards the same plane. The point of intersection of the line and plane is called the trace of the line. If the line intersects with HP, the point of intersection on HP is called the Horizontal Trace or HT of the line. If the line intersects the VP, the point of intersection of the line and VP is called Vertical Trace or VT of the line.

If the line is given parallel to a plane, it will never intersect that plane and, therefore, no trace of the line on that plane.

If the line is given parallel to VP and inclined to HP, only HT will be obtained and no VT.

If the line is given parallel to HP and inclined to VP, only VT will be obtained and no HT.

If the line is given parallel to both the planes, neither HT nor VT will be obtained.

Method of Obtaining Traces of a Line

As shown in Figure 3.18, the line AB is given parallel to VP and making a θ° with HP say the line is in the first quadrant.

Under these conditions the elevation of the line $a'b'$ will be a line equal in length to AB and will be making an angle of θ° with $X-Y$.

The plan of the line will be a line parallel to XY (but it will be shorter than length AB).

Now imagine if the line AB is extended in its own direction \overline{AB} , it will intersect the horizontal plane at a point HT. Now at the same time when the line AB is being extended in the direction \overline{AB} , the elevation of the line $a'b'$ is also being extended towards $X-Y$, and will meet with $X-Y$ at point h' , when AB meets the HP, and HT. One more observation is important that when the line \overline{AB} is being extended the plan ab of the line is also being extended and ultimately will reach the point HT.

From this explanation, a good rule is being established which is like this.

For obtaining HT of an inclined line AB , inclined to HP, extend the elevation $a'b'$ towards $X-Y$ line till it intersects $X-Y$ (In some problems, depending upon the position of the given line, the elevation might intersect $X-Y$ line in natural course and it need not be extended. This may happen when one end of the line is above HP and the other end below HP. In this also the point of intersection of elevation with $X-Y$ will be h' .) The point of intersection of the extended elevation with $X-Y$ is h' .

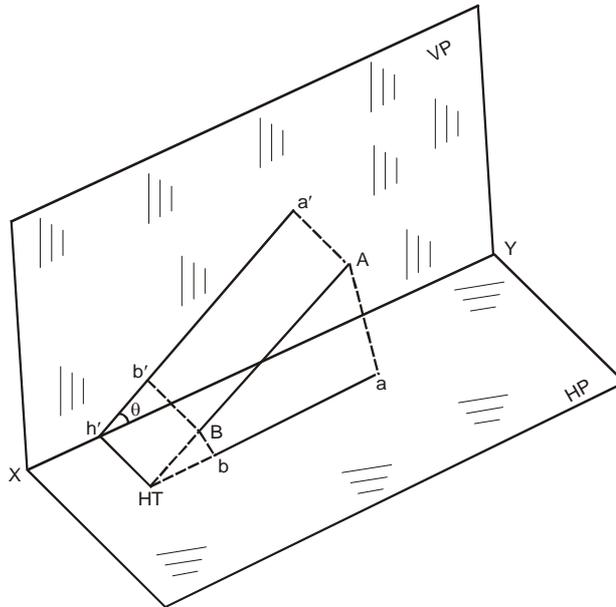


Figure 3.18(a)

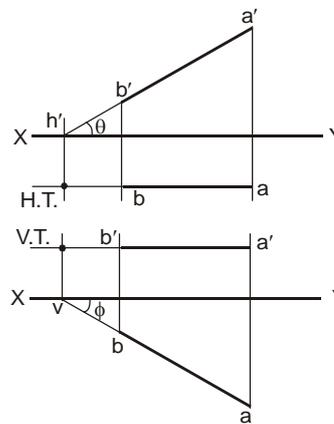


Figure 3.18(b)

Now from h' , draw a line perpendicular to $X-Y$. Then extend the plan ab in its own direction, till it intersects the perpendicular from h' .

The point of intersection of extended plan ab (in some cases the plan need not be extended but the perpendicular from h' will intersect the plan ab) and the perpendicular from h' will be HT.

For obtaining the VT the procedure is reversed

- (a) Extend the plan towards $X-Y$ line till it intersects $X-Y$. The point of intersection is v .
- (b) Draw a line perpendicular to $X-Y$ from h .
- (c) Extend the elevation in its own direction, so that it intersects the perpendicular drawn from h .
- (d) The point of intersection is VT.
(In some cases, the elevation need not be extended and the perpendicular from h will intersect the elevation. This point of intersection is VT)



The projections of lines AB , CD , EF are given in Figures 3.19, 3.20 and 3.21. Locate their Traces.

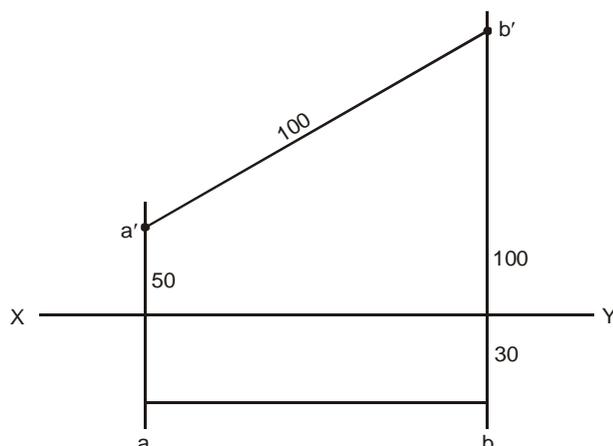


Figure 3.19

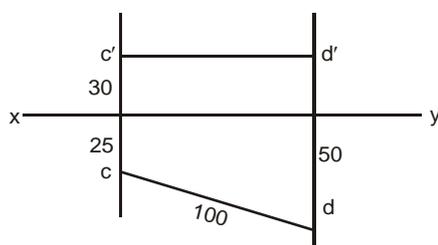


Figure 3.20

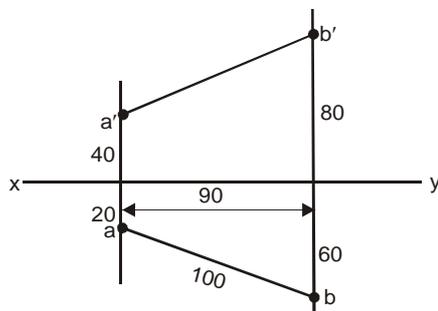


Figure 3.21

When the projections of the line are perpendicular to $X-Y$, i.e. when the sum of θ and ϕ is 90° , it is not possible to find the traces by the above methods. The another method for determining the HT and VT is as follows :

- Draw projections $a'b'$ and ab .
- Draw a horizontal line $a'-a_2$ (equal to $a'A$, the distance of A from VP).
- Draw a horizontal line $b'-b_2$ (equal to distance of B from VP).
- Join b_2-a_2 and extend it to intersect the extended elevation $b'-a'$, in point VT.
- Draw a horizontal line $a-a_1$ (equal to distance of A from HP).
- Draw a horizontal line $b-b_1$ (equal to distance of B from HP).
- Join a_1-b_1 to intersect the extended plan $a-b$ in point HT.

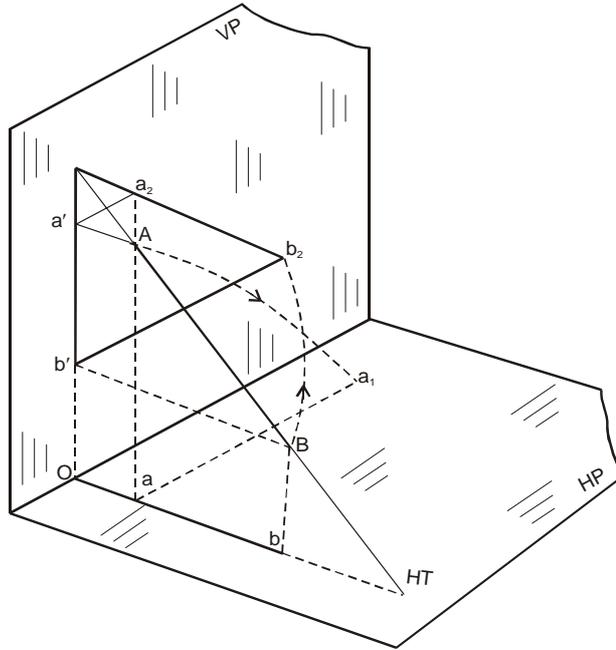


Figure 3.22

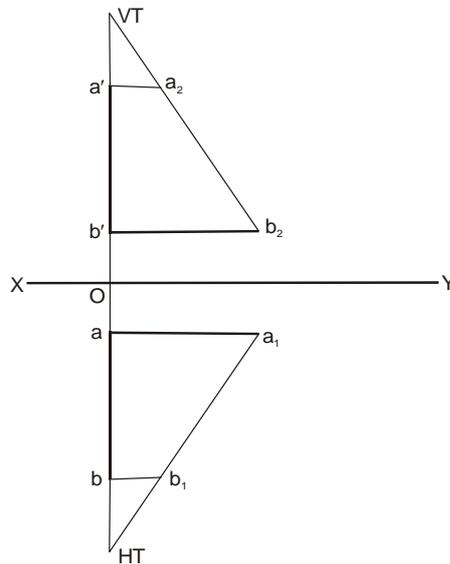


Figure 3.22

3.4.3 True Length of a Line

True length of a line is the given length of the line, i.e. distance between its ends.

If the line is given parallel to a reference plane, the projection on that plane will be of true length, i.e. equal to given length of the line.

If the line is inclined to a reference plane, its projection on that plane will be shorter than true length. This projected length or apparent length is a function of the angle made by the line with the plane. Go on increasing the angle, the projected length goes on reducing, and when the line makes 90° angle, i.e. perpendicular, the projected length has reduced to a point only.

Example 3.1

Hold a pencil in your hand and open a note-book in such a way that some pages are on the table and some pages are held perpendicular to the table top. Consider these as two planes HP and VP.

Solution

Put one end of the pencil on the HP and let it make some angle with HP. Make the pencil parallel to VP (imagine the pencil to be a line).

Under the condition if the line is projected on HP and VP, its projections will be as shown in Figure 3.23.

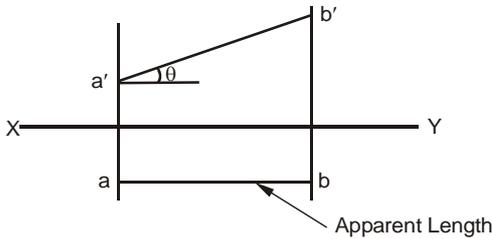


Figure 3.23

The elevation of the line $a'b'$ will be true length, the angle θ will also be true angle but plan ab will be shorter than true length. The length ab is a function of angle θ . If θ is increased, ab will be reduced further.

Now consider the pencil parallel to HP and making some angle ϕ with VP. Its projections will be as shown in Figure 3.24.

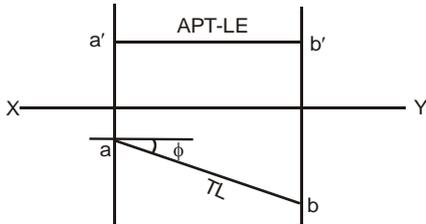


Figure 3.24

The length of plan is True length TL , the angle made by this plan with VP, i.e. with XY line is true angle ϕ , and the length of elevation is a function of angle ϕ . Elevation is parallel to XY (in second case, the line is given parallel to HP, while in this case the elevation is a line parallel to XY and the plan of the line is TL , the angle made ϕ is true angle).

From the example, we see that in first case when the line was parallel to VP, the elevation of the line is TL , the angle made, i.e. θ° , is also true angle and the plan is a line parallel to $X-Y$.

Hence, learn a rule that if the projections of the line are given and if one of the views (plan or elevation) is parallel to XY , the other view **MUST BE** true length and angle made by this TL will be true angle. Its reverse is also true, i.e. from among the views of a line, if one view is TL , the other view **MUST BE** parallel to $X-Y$.

Determining True Length and True Angle

Imagine a line AB in the first quadrant with its end A in the HP and making an angle θ° with H.P and parallel to VP. Its projections will be as shown in Figure 3.25.

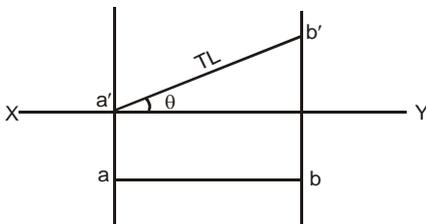


Figure 3.25

The length of plan is a function of angle θ . If θ is made constant, the length of plan will remain constant. Similarly, the height of point b' above $X-Y$ is also a function of θ . Therefore, if θ is made constant, the height of b' will be constant.

Keeping θ constant, if the relation of the line is now changed with VP, i.e. it is no more parallel to VP but makes some angle ϕ with VP. Under this condition (as θ is not changed), the length of plan will not change and also the height of b' will not change. What changes is the position of ab and b' .

Let the new position of plan ab be a_1-b_1 with a as center and radius equal to ab rotate ab so that it makes an angle β with $X-Y$ (Figure 3.26) and b moves to b_1 . Now the new position of plan is a_1b_1 , which is making angle β (apparent angle as ab is not true in length). The new elevation of b_1 has to be just above b_1 and at the same height as b' . Draw a projector (line perpendicular to XY) from b_1 and a line parallel to $X-Y$ from b' . (This line is called Locus.) The point of intersection of the perpendicular and the locus will be new elevation of b_1 . Call it b_1' . Join a' with b_1' and thus a new plan and elevation of this line is obtained. See that in this case the line is inclined to both the planes and, thus, plan and elevation both are apparent lengths, and angles made by these lines are also apparent.

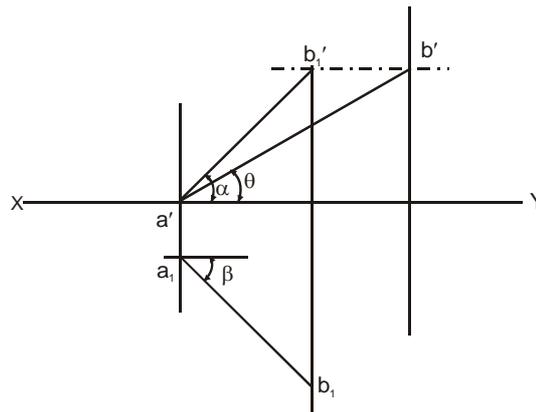


Figure 3.26

From this example, a rule is made, i.e. when the plan of constant length changes its position, one end of the plan say b moves along the arc of a circle (centre a and radius ab) and at the same time the elevation b' moves along a locus (i.e. a line through b' and parallel to $X-Y$).

This rule can be made applicable to the elevation also. Keeping one end of the elevation fixed the other end is moved along the arc of a circle (with a' as center and radius equal to $a'b'$), the plan b_1 will move along its locus (i.e. a line parallel to XY and passing through b_1).

Example 3.2

Figure 3.27 shows projections of a line AB .

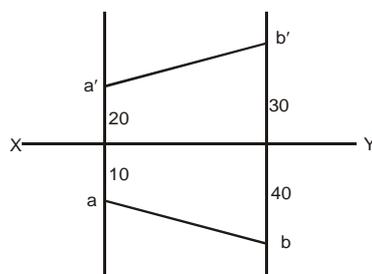


Figure 3.27

Find

- True length of the line.
- True angles made by the line with HP and VP.

Solution

As the given plan and elevation, both the lines are inclined to $X-Y$. Therefore none of them is true in length and also angles made by them are also apparent.

To determine true length in elevation do this :

- Draw a line xy and the plan and elevation as given.
- With a as center and radius equal to ab , rotate the plan till it becomes parallel to XY (Figure 3.28).

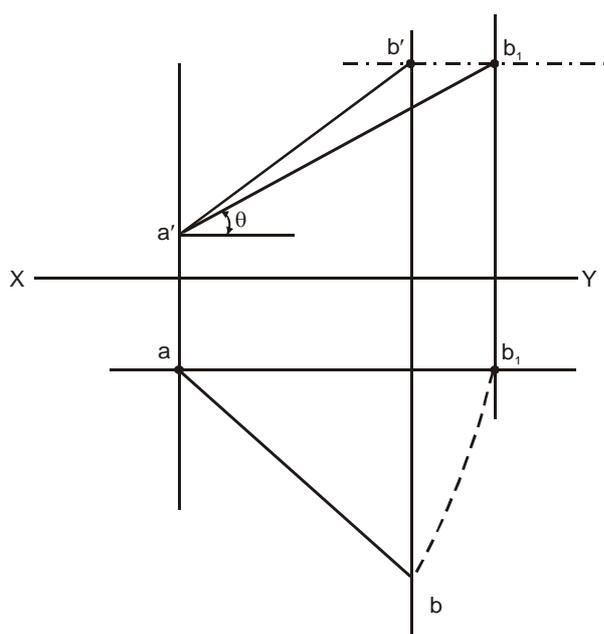


Figure 3.28

The new position of ab is ab_1 . As ab_1 is parallel to $X-Y$, its another view, i.e. $a'b_1'$ must be true in length. The angle made by the true length $a'b_1'$ with $X-Y$ is true angle θ (angle with HP). To determine true length in plan (it must be same as determined in elevation) the procedure is as follows :

- After drawing the given projections, make the elevation parallel to XY by rotating it about a' as center and $a'b'$ as radius. The new position of $a'b'$ is now $a'b_1'$.

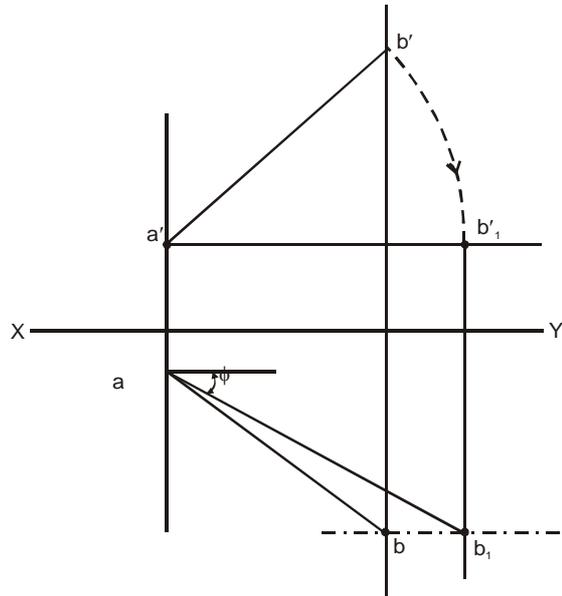


Figure 3.29

- (b) Draw a locus (line parallel to $X-Y$ through b) of b .
- (c) Draw a perpendicular to $X-Y$ from b'_1 . The point of intersection of the locus and perpendicular is b_1 . Join a, b_1 by a line. This line ab_1 is the plan true in length. The angle made by this true length ab_1 with $X-Y$ is true angle ϕ (i.e. angle of line with VP).

Instead of drawing two separate Figures 3.28 and 3.29, a combined figure can also be drawn.

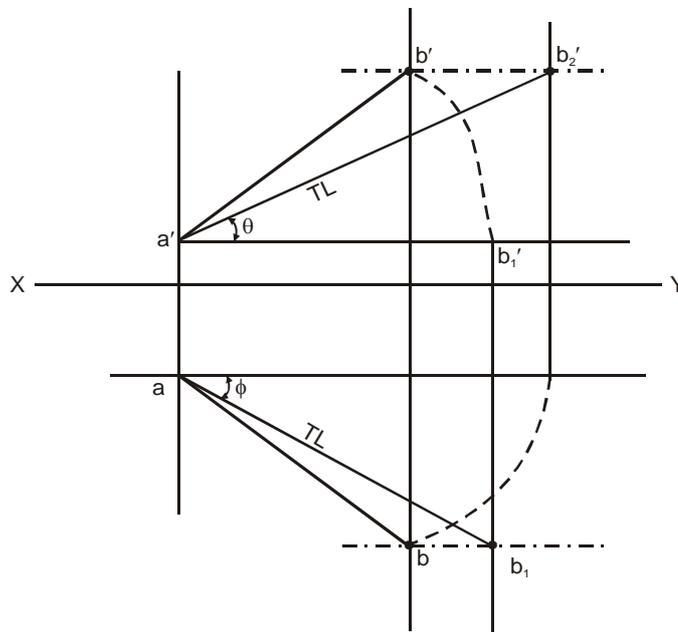


Figure 3.30

Example 3.3

Before attempting the examples of projections of lines, following important tips must be remembered.

- (a) To locate HT of the line, extend elevation up to $X-Y$, draw perpendicular to XY and extend plan. The point of intersection is HT.
- (b) To locate VT of the line-extend plan up to $X-Y$ draw perpendicular to XY , and extend elevation the intersection point is VT.
- (c) If one of the views in projection is parallel to XY , the other corresponding view is true in length.
- (d) Angles made by True lengths with $X-Y$ line are true angles. Angles made by apparent lengths are apparent angles.
- (e) Apparent lengths are always shorter than the true lengths. Apparent angles are always larger than the true angles.
- (f) For convenience, we will mark true angles made by the line with HP by letter θ and apparent angle with VP as α . True angles made with VP will be denoted as ϕ and apparent angles by β .
- (g) If the angles θ and ϕ are given, add their values and if this addition is 90° , the line lies in the profile plane (a plane perpendicular to HP and VP both) and the projections of this line (both, plan and elevation) will lie on a single projector. Plan and elevation both will be perpendicular to $X-Y$.

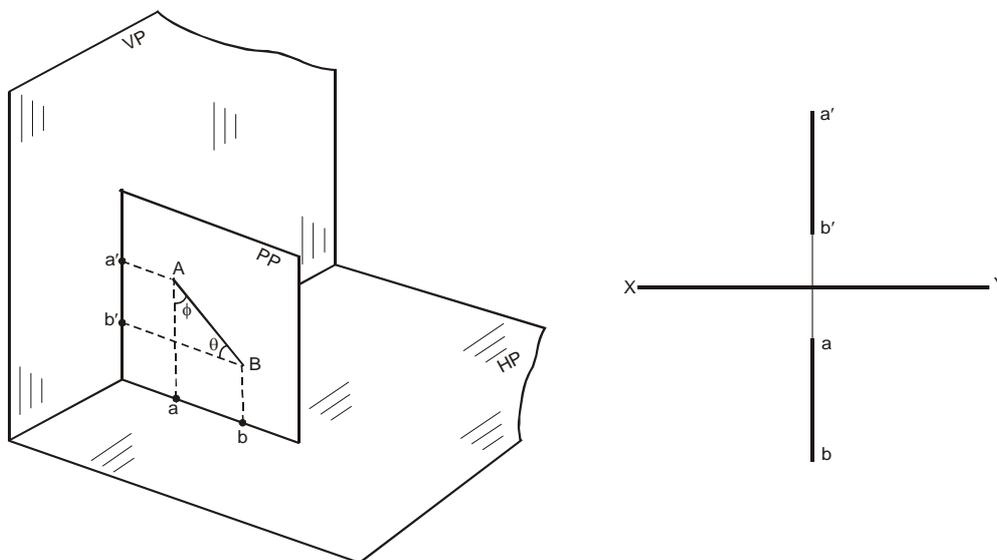


Figure 3.31

- (h) Note all the points which are given in the problem to be solved.

Example 3.4

A line ab is 75 mm long and is inclined to HP at 45° and to VP at 30° . Its end b is in HP and 40 mm in front of VP. Draw its projections and determine its traces.

Solution

Given TL $AB = 75$ mm
 $= 45^\circ$

$= 30^\circ$

B is in HP and 40 mm in front of VP

Required – Projections, HT and VT

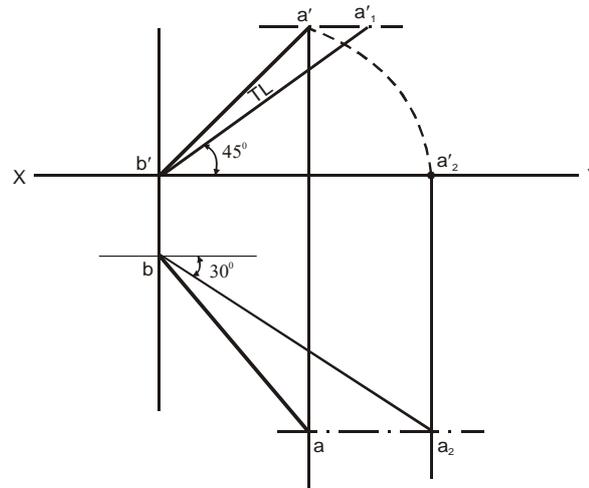


Figure 3.32

Steps to Solve the Problem

- (a) Draw X-Y line and a projector on it as the location of *B* is fully given, mark *b'* on the XY and *b* at 40 mm below XY on the projector.
- (b) From *b'*, draw a line making 45° with X-Y, and 75 mm long. Name this point as *a'1*.
- (c) From *b*, draw a line 75 mm long and making 30° angle with X-Y. Name this point as *a2*.
- (d) Draw loci of *a'1* and *a2* (lines parallel to XY from *a'1* and *a2*).
- (e) Project line *ba2* on XY, then *b'a'2* is the length of elevation. With *b'* as center and radius equal to *b'a'2*, draw an arc of the circle cutting the locus of *a'1* at *a'*. Join *a'* and *b'*, this is the required elevation.
- (f) Draw a projector through *a'* intersecting the locus of *a2* at point *a*. Join *a* and *b*. This is the required plan.
- (g) For obtaining VT, extend the plan *ab* till it intersects XY at point *V*. Draw a projector through *V* and extend the elevation *a'b'* to intersect the projector through *V* at point VT.
- (h) For obtaining HT, extend the elevation till it intersects X-Y. (It is already intersecting at *b'*, hence no extension needed.) From *b'* draw a projector and it is intersecting the plan at *b* (no need to extend plan). Hence *b* is the HT.

Example 3.5

The top view of a 75 mm long line *cd* measures 50 mm. *C* is 50 mm in front of the VP and 15 mm below the HP *d* is 15mm in front of the VP and is

above the HP. Draw the projections of the line and find its inclinations with HP and VP. Mark its traces also.

Solution

Given

- TL cd = 75mm
- Length of plan cd = 50 mm
- Location of c – 50 mm in front of VP and 15 mm below HP
- Location of d – 15 mm in front of VP and above HP

Required

- (a) Projections
- (b) θ
- (c) ϕ
- (d) HT
- (e) VT

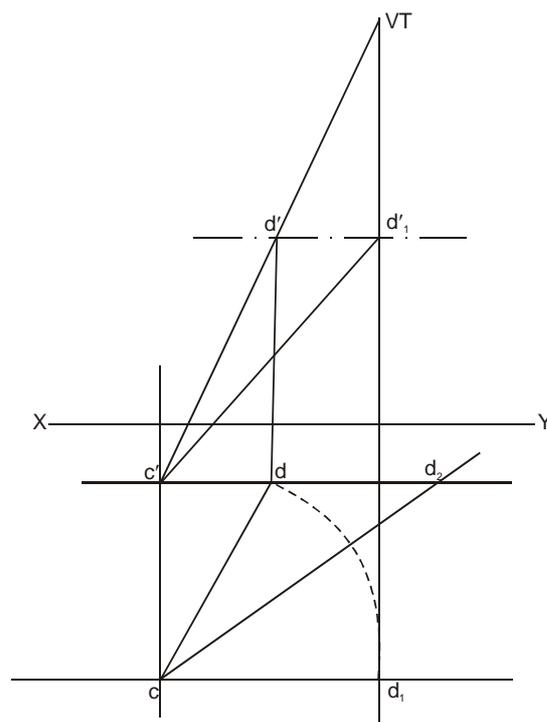


Figure 3.33

- (a) As location of c is fully given, draw X - Y line. Draw a projector and mark on this c' , 15 mm below XY and c , 50 mm below XY . (Point c is in fourth quadrant.)
- (b) Now draw the locus of d (d is 15 mm in front of VP hence d is 15 mm below XY).
- (c) With c as center and 50 mm radius, intersect the locus of d in point d . Join cd . This is the plan of the line.
- (d) Rotate this plan about c , till it becomes parallel to X - Y . The new position of plan is cd' .

- (e) Draw a projector through d_2 .
- (f) With c' as center and radius 75 mm (TL) intersect the projector at point d_2' .
- (g) Draw the locus of d_1' and a projector through d , intersecting the locus at d' . Join $c'd'$. This is the elevation.
- (h) Measure the angle made by $c'd'$, with $X-Y$. This angle is θ .
- (i) With c as center and radius equal to 75 (TL), intersect the locus of d at point d_2 . Join cd_2 . The angle made by cd_2 with $X-Y$ is angle ϕ .
- (j) The line $c'd'$ is intersecting the XY line at h' . Draw a line perpendicular to $X-Y$ from h' , intersecting the plan at HT.
- (k) Extend cd to intersect XY at V . Draw perpendicular to XY from V . Extend $c'd'$ to intersect the perpendicular at VT.

Example 3.6

A line AB , 65 mm long, has its end a in the HP and 15 mm in front of the VP. The end b is in the third quadrant. The line is inclined at 30° to HP and at 60° to VP. Draw its projections.

Solution

Given

- TL of line AB = 65
- θ = 30°
- ϕ = 60°
- a – in HP and 15 mm in front of VP
- b – in third quadrant

Required – Projections

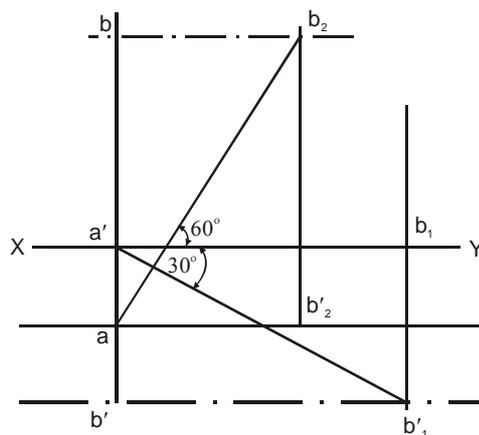


Figure 3.34

One important observation in the given data is that sum of $\theta + \phi$ is equal to $30 + 60 = 90^\circ$. Hence, the projections of this line AB will lie on same projectors, i.e. both will be perpendicular to $X-Y$ line.

- (a) Draw $X-Y$ line. Draw a projector on this and mark a' on XY and a 15 mm below XY .

- (b) As end b is in third quadrant, b will be above $X-Y$ and b' will be below XY .
- (c) Through a' , draw a line making 30° angle with XY (downwards as b' has to come below XY) and make this line 65 mm long (TL). Name this end b'_1 . Draw locus through b'_1 .
- (d) Through a , draw a line ab_2 , 65 mm long and making 60° angle with XY . Draw locus to b_2 .
- (e) Draw projectors through b'_1 intersecting XY in b_1 , and through b_2 , intersecting the locus through a in b'_2 .
- (f) The length $a'b_1$ is the length of plan. With center a , and length $a'b_1$ draw an arc of circle intersecting the locus of b_2 in b . Join ab , this is plan.
- (g) The length a, b'_2 is the length of elevation. With center a' and radius ab'_2 draw an arc, intersecting the locus of b'_1 at b' . Join $a'b'$. This is the elevation.

See that a_1b_1 and ab lie on the same projector.

Example 3.7

The front view of a line ab measures 65 mm and makes an angle of 45° with XY . a is in HP and the VT of the line is 15 mm below the HP. The line is inclined at 30° to VP. Draw the projection of ab and find its true length and inclination with HP. Also locate its HT.

Solution

Given

length of elevation $a'b'$	=	65 mm and $a'b'$ is making 45° with XY .
A	–	in the HP
VT	–	15 mm below HP
ϕ	=	30°

Required

- (i) Projections
- (ii) HT
- (iii) TL
- (iv) θ

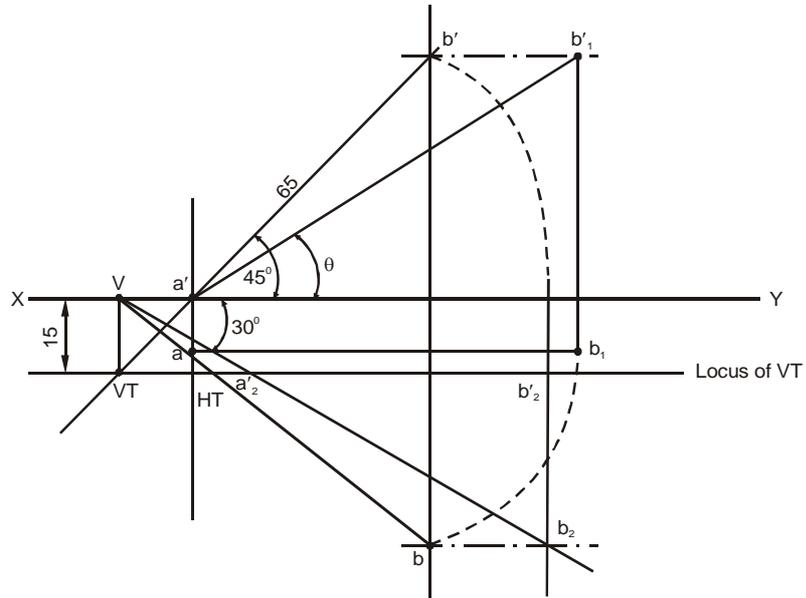


Figure 3.35

- (a) Mark a' on XY . Draw a line $a'b' = 65$ mm and making 45° with XY . This is elevation.
- (b) Draw a line parallel to and 15 mm below XY .
- (c) Extend elevation to intersect this parallel line in VT .
- (d) Draw projector through VT to intersect XY in V .
- (e) Through a' draw a projector.
- (f) The elevation $a'b'$ is to be made parallel to XY but it is to be rotated about VT and not about a' . (If it is rotated about a' , the VT will be dislocated.) With center as VT and radius equal to $VT b'$, draw an arc to intersect locus of VT at b'_2 . Again, with center at VT and radius equal to $VT a'$, draw an arc to intersect locus of VT at a'_2 .
- (g) Through V , draw a line making 30° angle with XY and intersecting the projector through b'_2 at b_2 .
- (h) Draw the locus of b_2 , intersecting the projector through b' in b .
- (i) Join b with V , intersecting the projector through a in a . Join ab . This is the plan.
- (j) Rotate plan ab about a till it is parallel to xy . The new position of plan is ab_1 . Draw a projector through b_1 to intersect the locus of b' in b'_1 . Join $a'b'_1$. Angle made by $a'b'_1$ with XY is angle θ .
- (k) Point a is HT also.
- (l) Length $a'b'_1$ is TL .

Example 3.8

The projectors drawn from the HT and VT of a straight line ab are 80 mm apart while those drawn from its ends are 50 mm apart. The HT is 35 mm in front of the VP , the VT is 55 mm above the HP and the end a is 10 mm

above the HP. Draw the projections of ab and determine its true lengths and inclinations with reference planes.

Solution

Given

Projections through HT and VT are 80 mm apart.

Projector through aa' and bb' are 50 mm apart.

HT is 35 mm in front of VP

VT is 55 mm above HP

A is 10 mm above HP

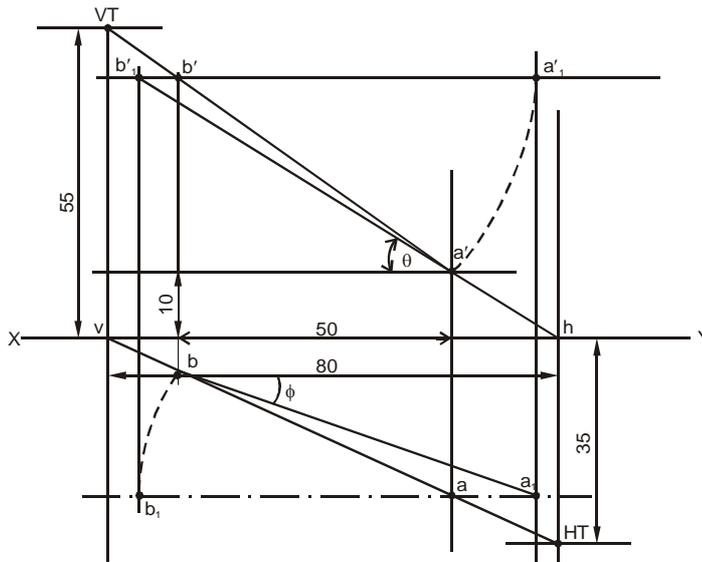


Figure 3.36

- Draw XY and also draw line projectors 80 mm apart. On one projector mark a point VT , 55 mm above XY and on the other projector mark HT 35 mm below XY . The points of intersection of these projectors with XY are v and h respectively.
- Join VT and h . On this line the elevation $a'b'$ will lie. As a is 10 mm above HP a' will be 10 mm above XY . Draw a line parallel to and 10 mm above XY intersecting line $h VT$ in a' . Draw a projector through a_1 , intersecting XY . From this point mark 50 mm distance on XY and draw a projector intersecting the line $h VT$ in b' join $a'b'$. This is elevation.
- Join V with HT . On this line the plan ab will lie. The projectors through $a'b'$ will intersect this line in a and b . Join ab . This is plan.
- For determining θ , make plan ab_1 , parallel to XY . The elevation $a'b'_1$, will give the angle θ .
- For angle ϕ , make elevation $a'b'_2$ parallel to XY . The plan ab_2 gives angle ϕ .



- (a) A line 80 mm long makes an angle of 35° with the HP and 45° with the VP. Its mid-point M is 30 mm below the HP and 25 mm behind the VP. Draw the projections of the line.
- (b) A room is $4.8\text{ m} \times 4.2\text{ m} \times 3.6\text{ m}$ high. Determine graphically the distance between a top corner and the bottom corner diagonally opposite to it.
- (c) A line AB is in the first quadrant. Its ends A and B are 20 mm and 60 mm in front of the VP respectively. The distance between the end projectors is 75 mm. The line is inclined at 30° to HP and its HT is 10 mm above XY . Draw the projections of AB and determine its True length and the VT.
- (d) A pipe line from a point A , running due north east has a downward gradient of 1 in 5. Another point B is 12 m away from and due east of A and on the same level. Find the length and slope of a pipe-line from B which runs due 15° east of north and meets the pipe line from A .

3.5 PROJECTION OF PLANES

A plane or surface has two dimensions only. The thickness is considered equal to the thickness of a paper.

The shape of the plane may be a regular shape which can be geometrically defined, viz. square, triangle, circle, rectangle pentagon, hexagon etc. The shape may also be irregular, but in this unit we will consider only regular shapes.

3.5.1 Position of the Plane with Reference to the Planes of Projection, i.e. HP and VP

The plane to be projected can be

- (a) Perpendicular to both HP and VP.
- (b) Perpendicular to one plane and parallel to the other.
- (c) Perpendicular to one plane and inclined to the other.
- (d) Inclined to both the reference planes.

The Given Plane is Perpendicular to Both Reference Planes

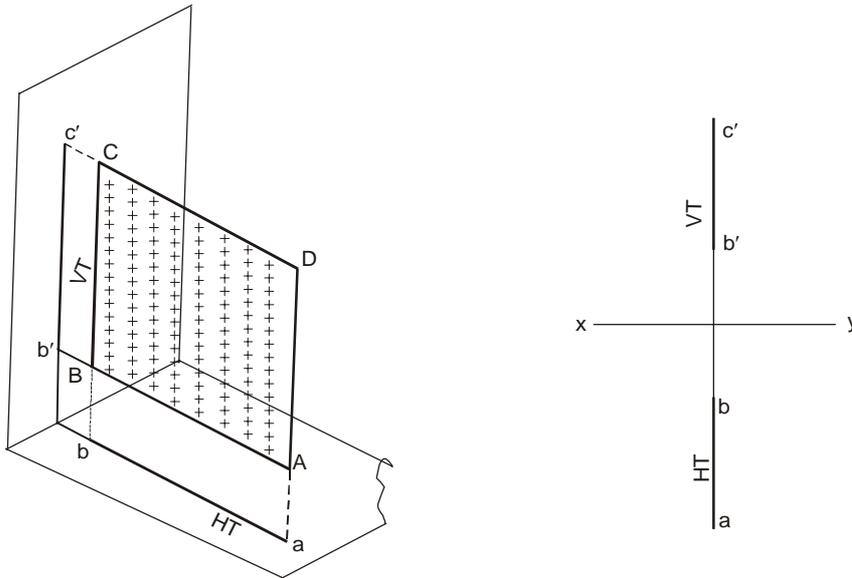


Figure 3.37

The square $ABCD$ is perpendicular to both the planes. The front view $b'c'$ coincides with VT and is perpendicular xy . Top view ab coincides with HT and is perpendicular to xy . Thus, when a plane is perpendicular to a reference plane, its projection on the same plane is a straight line.

The Given Plane is Perpendicular to HP and Parallel to VP

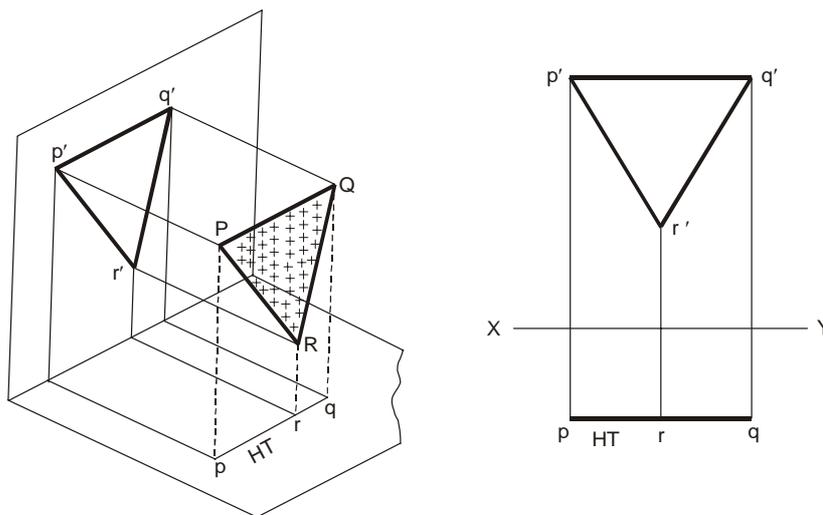


Figure 3.38

A triangle PQR is perpendicular to HP and is parallel to VP . Its HT is parallel to XY and it has no VT $p'q'r'$ is the front view (it is true shape) and pqr is the plan.

Thus, when a plane is given parallel to a reference plane, the projection on the same plane will be true shape and size.

The Given Plane is Perpendicular to HP and Inclined to VP

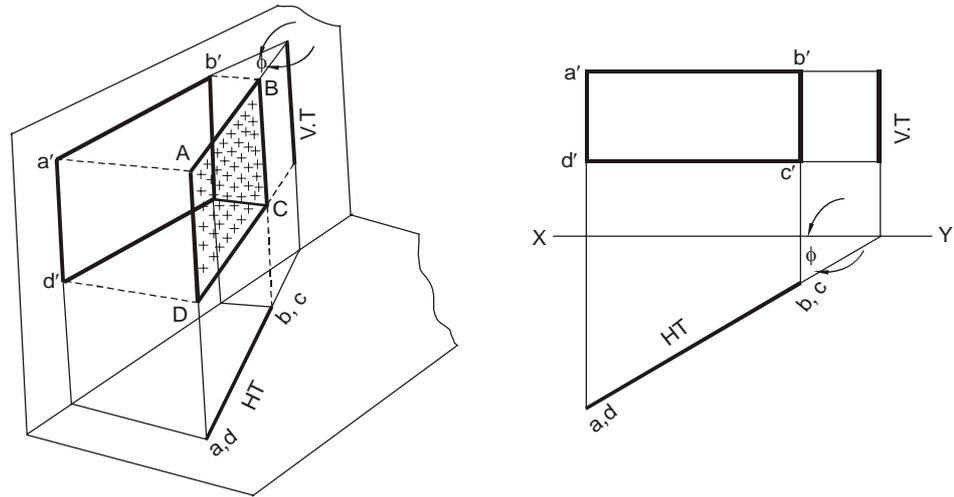


Figure 3.39

A rectangle $ABCD$ is given perpendicular to HP and inclined at ϕ° to VP. The longer sides of the rectangle are given parallel to HP, $a'b'c'd'$ is the front view while ad, bc is the top view. Its VT is perpendicular to xy . Its HT is inclined at ϕ° to xy .

While drawing projections of plane which is perpendicular to HP and inclined to VP, in the initial state, it is assumed to be parallel to VP. Its front view will show its true shape and the top view is a line parallel to XY. The plane is made inclined by rotating the top view through that angle.

Plane Inclined to Both HP and VP

For obtaining the projections of the plane under this condition, remember these steps :

The solution of such problems will be obtained by drawing three steps, e.g.

- (a) Initial,
- (b) Intermediate, and
- (c) final.

In case when the plane is given perpendicular to one reference plane and inclined to the other, the solution of such problems will be obtained in two steps such as (a) Initial, and (b) Final.

Rules for Drawing the Initial Step

Consider the given plane parallel to the same reference plane to which it is given inclined (If the plane is given inclined to VP, consider it parallel to VP, if the plane is given inclined to HP, consider it parallel to HP, and if the plane is given inclined to both, consider it parallel to anyone of the planes.) By doing so, the projection on the same plane will be true in shape.

For such planes which are having sides (triangle, square, rectangle etc.) it requires one more guideline for drawing the true shape.

- (a) If a side, or diagonal or diameter of the plane is given parallel to a reference plane, or contained by a reference plane, then for drawing the initial step, consider the same side perpendicular to the other plane.
- (b) If in the given problem instead of a side, a corner of the plane is given contained by a reference plane, then while drawing this

corner, draw it in such a way that the two sides making the corner are equally inclined to $X-Y$ line.

From the above rules it is very clear that the final solution of such problems in which the plane is given inclined to both reference planes, there will be three steps such as :

- (a) initial,
- (b) intermediate, and
- (c) final.

The method will be clear from the following examples.

Example 3.9

Draw the projections of a regular pentagon of 50 mm sides, having one of its sides parallel to HP and the surface of the pentagon is inclined at 30° to HP and perpendicular to VP.

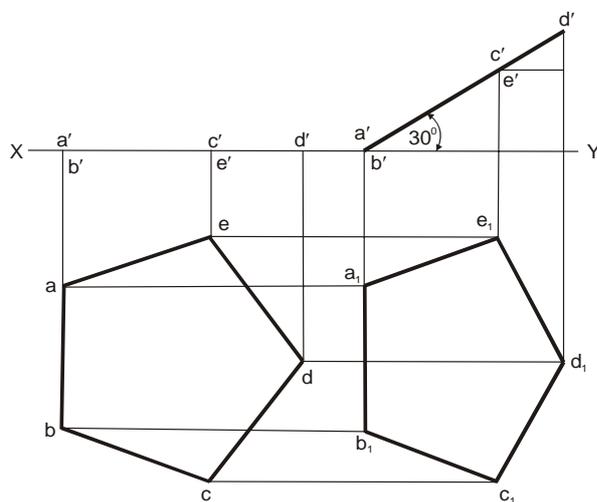


Figure 3.40

Solution

Given

The surface is given parallel to HP and perpendicular to VP. The problem is of IInd category and hence the solution will be obtained in two steps viz. (i) Initial, and (ii) Final.

One side of the pentagon is given parallel to HP. So for drawing initial step, Rule No. (i) and (ii) will be applicable.

As per Rule No. (i) the surface is considered parallel to HP. By doing so, the true shape is available in plan.

How to Draw?

Apply Rule No. (ii). One side is given parallel to HP, so as per rule consider it perpendicular to the other plane, i.e. VP.

Draw a line ab , 50 mm long below XY and complete a regular pentagon on this side. Its elevation will be a line $a'b' - c'e' - d'$, in $X-Y$. This is the initial position.

For drawing the final projection, the line $a'b' - c'e' - d'$ is drawn again keeping the point $a'b'$ in xy and line making 30° angle with $X-Y$. This is the second and final elevation. Draw projectors from all the corners, i.e. $a'b'c'd'e'$ and horizontal lines from the initial plan $abcde$. The points of intersection will be the final points (projector from a' and horizontal line from a will intersect and give a_1 , and so on).

Join $a_1 b_1 c_1 d_1$ and e_1 . This is the final plan.

Example 3.10

Draw the projections of a regular pentagon of 50 mm sides having its surface inclined at 30° to HP, one side of the pentagon is parallel to HP, and the same side making an angle of 60° to the VP.

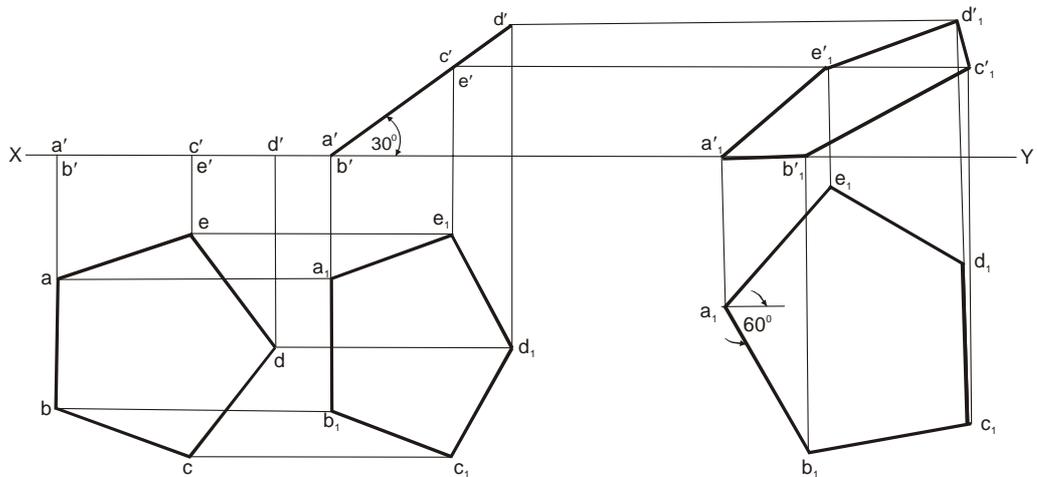


Figure 3.41

Solution

Given

- (a) The surface of the pentagon is inclined at 30° to HP.
- (b) One side of the pentagon is parallel to HP.
- (c) The side which is given parallel to HP, makes an angle of 60° to VP (Hence the surface will be inclined at 60° to VP).

As the surface is inclined to HP and VP both, the solution of this problem will be obtained in three steps :

- (a) Initial,
- (b) Intermediate, and
- (c) Final.

For drawing initial step, apply Rule No. (i) and (ii).

The initial step of Example 3.9 and this example is same.

For drawing intermediate step, draw the elevation and plan as drawn in Example 3.9.

For drawing the final step, rotate the plan obtained in intermediate step such that side $a_1 b_1$ is inclined at 60° to XY . This is the third and final plan.

Draw projectors from this plan and horizontal lines from second elevation. The points of intersection a'_1, b'_1, c'_1, d'_1 and e'_1 are obtained. Join these points in sequence. This is the final elevation.

Example 3.11

A regular hexagon of 50 mm side is resting on a corner in the HP and the surface of the hexagon is inclined at 45° to HP and perpendicular to VP.

Solution

The surface of the plane is inclined to HP and perpendicular to VP

The problem is of IInd category and hence its solution will be obtained in two steps :

- (a) Initial, and
- (b) Final.

For drawing initial step, Rule No. (i) and (iii) will be applicable

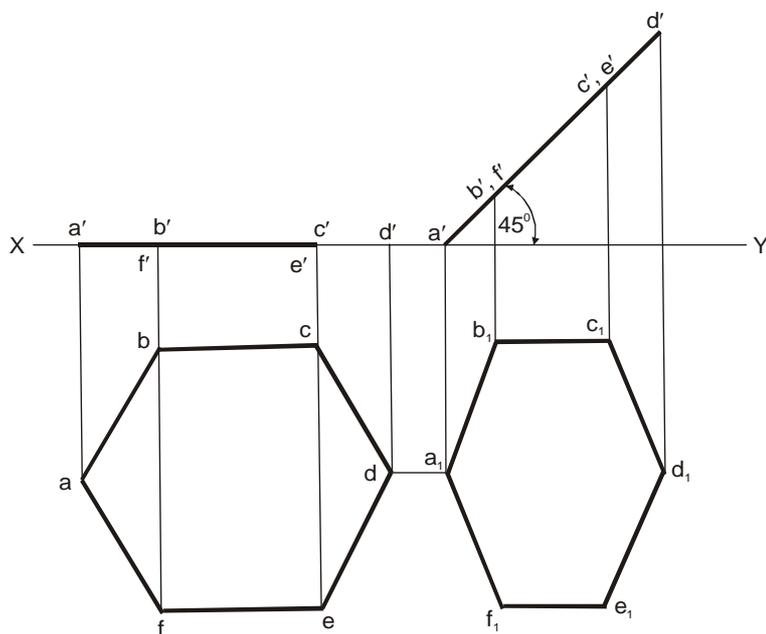


Figure 3.42

As per Rule No. (i) the surface will be considered parallel to HP. The true shape of the hexagon will be seen in the plan.

How to Draw this Hexagon?

Apply Rule No. (iii) :

Corner a of the hexagon (which is considered in HP) should be so drawn that the two sides making that corner, ab and af make equal angles with XY . (In this case it will be marked that after applying Rule (iii), the diagonal ad and two sides bc and ef have become parallel to xy).

Complete the regular hexagon and this is the initial plan.

Project the elevation in XY . Line $a' - b' f' - e' c' - d'$ is the initial elevation.

Draw second and final elevation same as the initial elevation but line $a' - d'$ now making 45° angle with XY .

Draw projections from this elevation and horizontal lines from initial plan.

The points of intersection from corresponding points will give $a' b' c' d' e' f'$ and f' . Join them in sequence and this is the final plan.

Example 3.12

A regular hexagon of 50 mm side has a corner in the HP. Its surface is inclined at 45° to the HP and the top view of the diagonal through the corner which is in the HP, makes an angle of 60° with the VP. Draw its projections.

Solution

Given

- (a) One corner of the hexagon is in HP.
- (b) The surface is given inclined to HP at 45° .
- (c) The plan of the diagonal through the corner is inclined at 60° to VP. Hence the surface is inclined to VP (angle is not known).

The problem is of III category hence the solution will be obtained in three steps :

- (a) Initial,
- (b) Intermediate, and
- (c) Final.

For drawing initial step Rule No. (i) and (iii) are applicable.

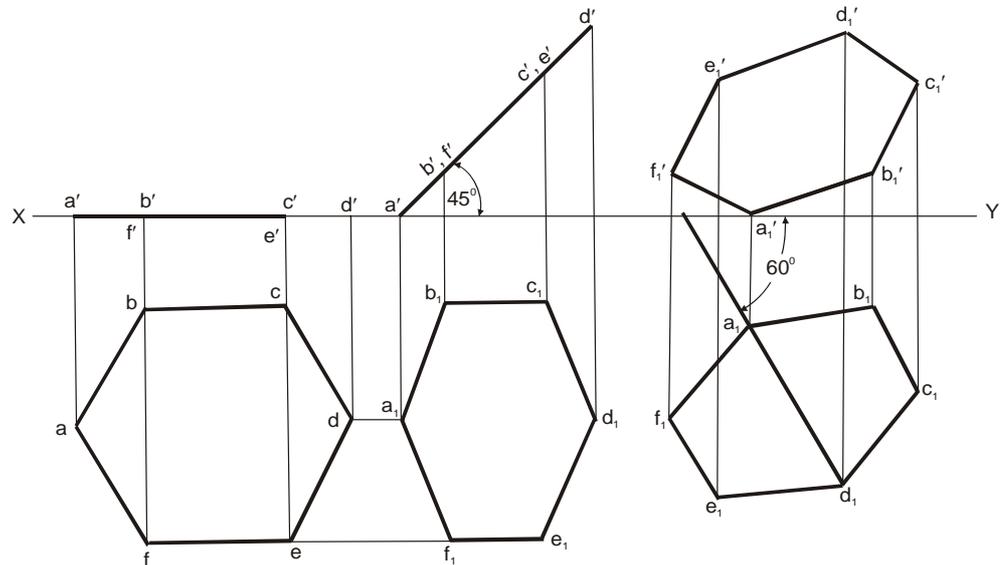


Figure 3.43

As per Rule No. (i) the surface is considered parallel to HP and hence, true shape of the hexagon will be seen in the plan.

How to Draw this Hexagon?

Apply Rule No. (iii).

One corner of the hexagon should be such drawn that the two sides making that corner are equally inclined to X-Y.

Complete the regular hexagon and name the corners as a, b, c, d, e and f . Project these points on XY . Line $a', b'f', e'c', d'$ is the initial elevation for drawing second (intermediate) elevation. Draw the line $a' - d'$ same as in initial elevation but inclined at 45° to XY . Draw projectors from second elevation and horizontal lines from the initial plan. The points of intersection from corresponding points will give a_1, b_1, c_1, d_1 , and f_1 . Join them in sequence. This is the second plan.

For obtaining third and final plan, redraw the second plan in such a way that line a, d is making an angle of 60° with xy . This is the final plan.

Draw projectors from final plan and horizontal lines from second elevation. The intersection points of these parallel lines and projectors from corresponding point will be $a'_1, b'_1, c'_1, d'_1, e'_1, f'_1$. Join them in a sequence and this is the final elevation.

Example 3.13

Draw an equilateral triangle of 60 mm side and inscribe a circle in it. Draw the projections of the figure, when its plane is vertical and inclined at 30° to VP and one of the sides of the triangle is inclined at 45° to the HP.

Solution

Given

- (a) The surface of the triangle is perpendicular to HP and inclined at 30° to VP. The problem is of second category. Hence, the solution will be obtained in two steps,
 - (i) Initial, and
 - (ii) Final.
- (b) One side of the triangle makes an angle of 45° with $X-Y$ (HP).

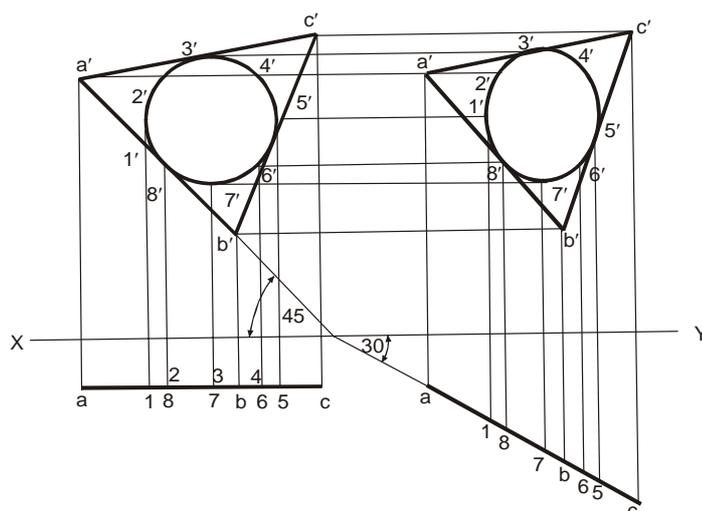


Figure 3.44

For drawing initial step the surface is considered parallel to V.P. So the true shape is seen in elevation.

How to Draw?

One side of the triangle is given inclined at 45° to XY .

Draw an equilateral triangle of 60 mm sides, above XY , with one side making 45° with XY . Inscribe a circle in the triangle. Divide this circle in some equal number of parts (say 8) and mark point $1' - 2' \dots 8'$, on the circumference. This is the initial elevation.

Draw projectors from this elevation. Draw a horizontal line at any distance below XY . The intersection points will give $a - c$ and $1 - 8$ on this line. This is initial plan.

Final Projections

Redraw the plan $a - c$, such that it makes 30° angle with XY .

Draw projectors from this plan and horizontal lines from initial elevation. The intersection points from corresponding numbers are to be marked. Join $a' b' c'$ with straight lines and $1' - 2' \dots 8'$ with a smooth curve. This is the final elevation.

SAQ 4



- (a) A square of 40 mm side has its one corner A in HP, its diagonal AC inclined at 30° to HP and diagonal BD inclined at 45° to the VP and parallel to the HP. Draw its projections.
- (b) A thin plate having shape of an isosceles triangle has base 50 mm long and attitude 70 mm. It is so placed that in front view it is seen as an equilateral triangle of 50 mm sides and one side inclined at 45° to xy . Draw its top view.
- (c) A 60° set-square of 100 mm longest side is so placed that the longest side is in the HP making an angle of 30° with VP and the surface of set-square making an angle of 45° to HP. Draw the projections of the set-square.
- (d) A thin rectangular plate of sides 50 mm \times 25 mm has its shorter side in the VP and inclined at 30° to the HP. Its front view is a square of 25 mm side. Draw its projections.

3.6 SUMMARY

While preparing the drawings of engineering importance, several methods of projections are used, viz. isometric projections, isometric drawings, oblique projections, etc. Among these, orthographic projections are very popular and very widely used. In learning process of orthographic projections, you have noticed the following :

- (a) In the orthographic projection system, the planes of projections are
- (i) Transparent
 - (ii) Very thin
 - (iii) Dimensionless
 - (iv) And most important that they are at 90° to each other.
- (b) The planes intersect and the line of intersection is called $X-Y$ line or the reference *line*.
- (c) When you see these planes edge wise, they are seen as horizontal and vertical lines crossing at a point, and making I, II, III and IV quadrants.
- (d) Irrespective of the position of the object to be projected, the vertical plane remains fixed and the HP is supposed to rotate clockwise.
- (e) If the object is in first quadrant, the plan will be below $X-Y$ and elevation above the $X-Y$.
- (f) If the object is in third quadrant, the plan will be above $X-Y$ and the elevation will be below $X-Y$.
- (g) If the object is considered in second or fourth quadrants, the plan overlaps the elevation and, therefore, no details of such a drawing can be observed.
- (h) There are only two methods of projections – *First angle*, when the object is considered in first quadrant and *third angle* when the object is considered in third quadrant. There are no method like second angle projection or fourth angle projection.
- (i) While considering the projections of points and projections of lines, they may be considered anywhere in the four quadrants. Even the lines may be intersecting the reference planes.
- (j) The intersection of the line and reference planes are called traces of the line. The line may be intersecting the planes in natural course or after extending the line in its own direction. Intersection with HP is called HT and intersection with VP is call VP.
- (k) True angles with the planes are made by true length. Apparent lengths make apparent angles.
- (l) If sum of the angles made by the line with HP and VP is 90° , the projections of such line will lie on one projector only.
- (m) The planes to be projected are considered very thin.
- (n) The shape of the plane may be regular, which can be defined geometrically or irregular.
- (o) If the plane is given perpendicular to a reference plane, its projection on that plane will be a line. If the plane is given parallel to a reference plane, the true shape of the plane will be obtained on that reference plane.
- (p) While solving the problems, note down all the parameters given and also what has been demanded.

3.7 ANSWERS TO SAQs

SAQ 1

(a)

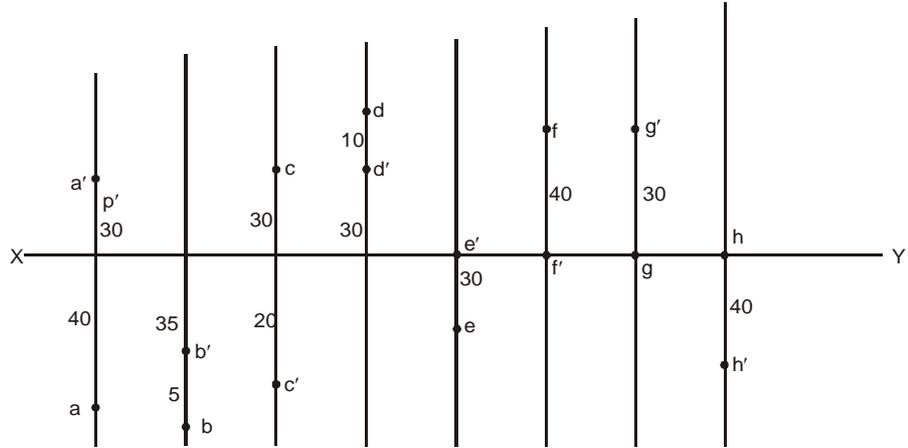


Figure 3.45

- (b) Point *P* is in VP and 2 cms above HP
- Point *Q* is 40 mm above HP and 2.5 in front VP
- Point *R* is in HP and 4 cms behind VP
- Point *S* is 2.5 cms below HP and 2.5 cms behind VP
- Point *T* is 5 cms above HP and 1.5 cms behind VP
- Point *V* is in X-Y.

(c)

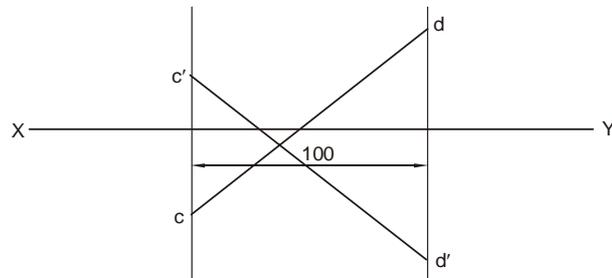


Figure 3.46

(d)

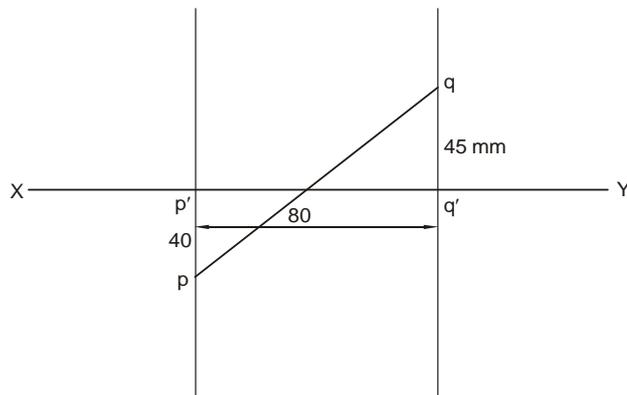


Figure 3.47

- (e) Point *R* is in the second quadrant.

(d)

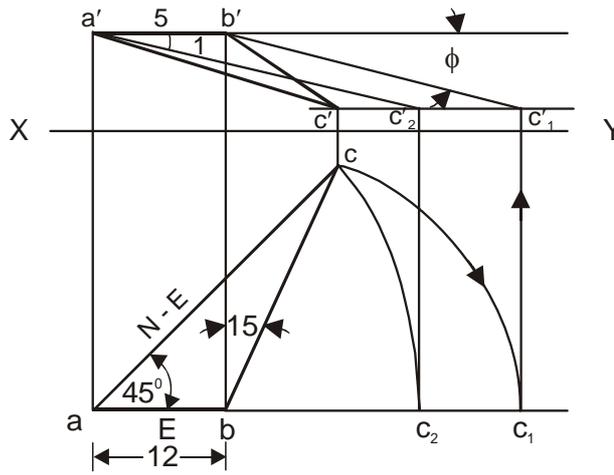


Figure 3.51

SAQ 4

(a)

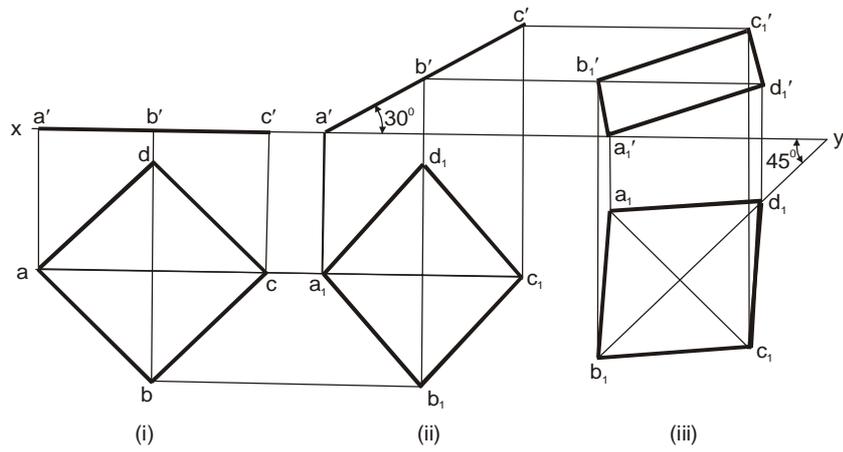


Figure 3.52

(b)

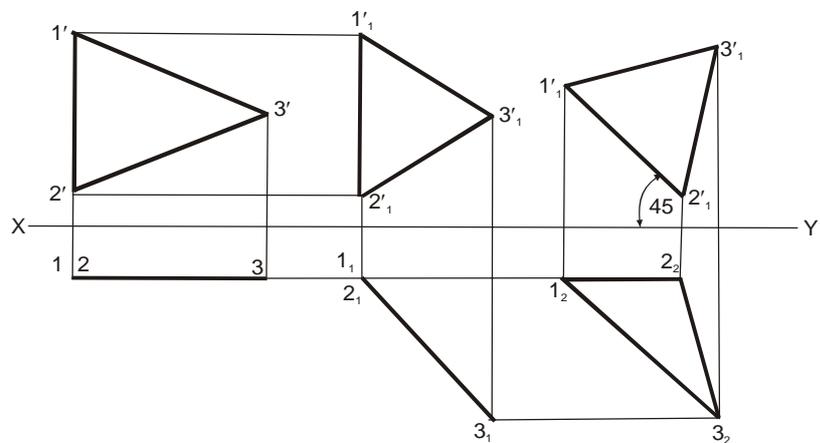


Figure 3.53

(c)

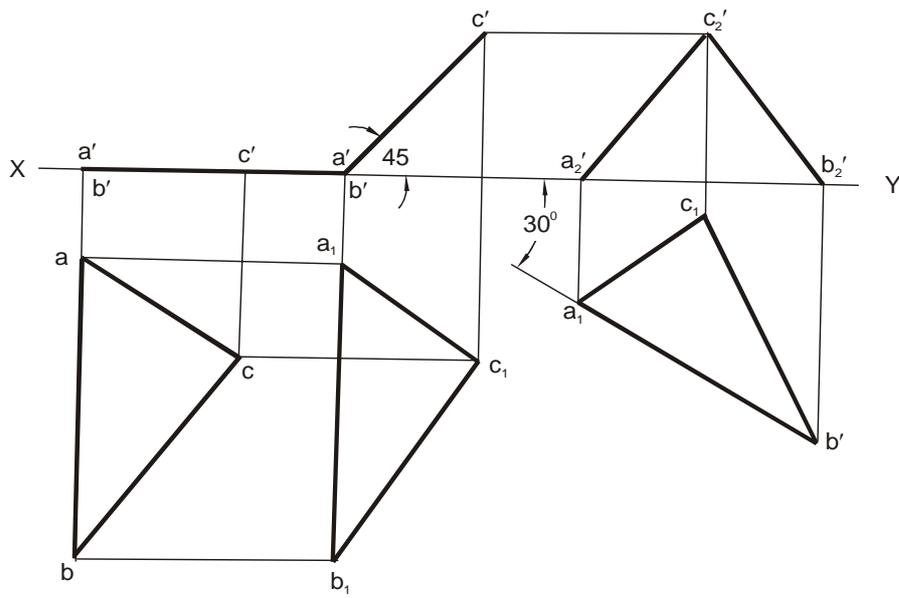


Figure 3.54

(d)

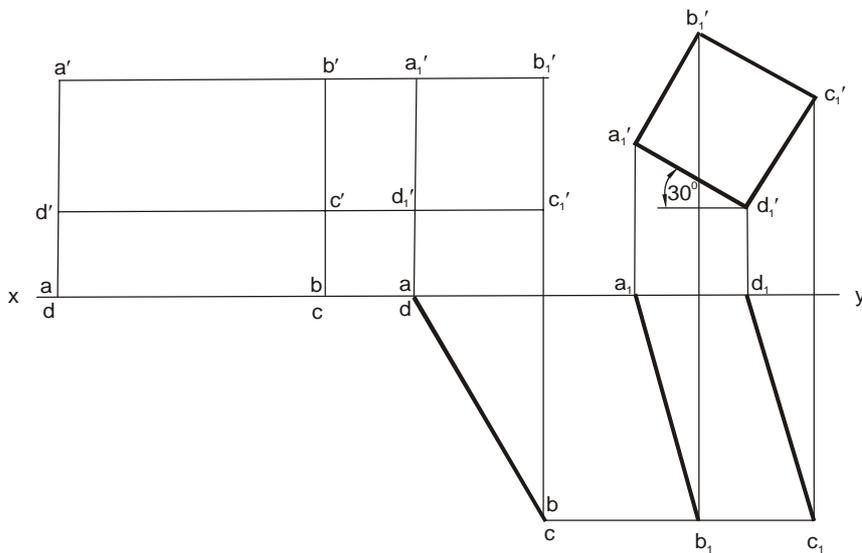


Figure 3.55