UNIT 3 SHEAR FORCES AND BENDING MOMENTS

Structure

3.1 INTRODUCTION

In practice, all beams are subjected to some type of external loadings. As the beam is loaded, it deflects from its original position, which develops the moment and shear on the beam. Shear force (SF) and bending moment (BM) are very important in designing any type of beam because the beam is designed for maximum bending moment and also the shear reinforcement is provided for maximum shear force. Strength of the beam is design criterion for bending moment and shear force, whereas the stiffness of the beam is the design criterion for deflection of the beam.

In this unit, you will be introduced to various types of beams, supports and loadings. Thereafter, methods of drawing Shear Force Diagram (SFD) and Bending Moment Diagram (BMD) for cantilever, simply supported beam and overhanging beam for different types of loadings have been discussed.

Objectives

After studying this unit, you should be able to

- identify the types of beam and types of support when it appears in building, structural system etc.,
- describe the shear force and bending moment and the relation between shear force, bending moment and rate of loading,
- draw the shear force and bending moment diagrams for any types of beam subjected to any types of loading system, and
- show the position of maximum bending moment and position of zero bending moment (point of contraflexure).

3.2 BEAMS

A beam is a structural member of sufficient length compared to lateral dimensions and supported so as to be in equilibrium and generally, subjected to a system of external forces to produce bending of the member in an axial plane. Ties, struts, shafts and beams are all one-dimensional or line-elements where the length is much greater than the depth or width. These are also known as rod-like or skeletal elements; and have different names depending upon the main action they are designed to resist. Thus, ties and struts resist uniaxial tension or compression, shafts resist torque and beams resist bending moments (and shear forces).

The clear horizontal distance between the supports is called the clear span of the beam. The horizontal distance between the centers of the end bearings is called the effective span of the beam. If the intensity of the bearing reaction is not uniform, the effective span is the horizontal distance between the lines of action of the end reactions.

Figure 3.1 : Clear and Effective Spans in a Beam

Whenever a horizontal beam is loaded, it bends due to the action of loads. The amount with which a beam bends, depends upon the amount and type of loads, length of the beam, elasticity properties of the beam material and the type of the beam.

Beams may be concrete, steel or even composite beam, having any type of sections such as angles, channels, I-section, rectangle, square, hat section etc.

3.2.1 Types of Support

The supports can be classified into following three categories :

(i) A simple or free support or roller support,

(ii) Hinged or pinned support, and

(iii) A built-in or fixed or encastre support.

A Simple or Free Support or Roller Support

It is a support in which beam rests freely. A roller support is the simplest and gives only one reaction, either in x direction or y direction (R_x or R_y) normal to the support and offers no resistance against rotation or lateral movements. The reaction will be taken as acting at a point, though actually it will be distributed over some length.

Figure 3.2 : Simple or Free Support or Roller Support

Hinged or Pinned Support

In this case, the beam is hinged or pinned to the support. A pinned or hinged support gives only two reactions, one against vertical movement and another against horizontal movement (say R_x or R_y) but offers no resistance to the angular rotation of the beam at the hinge. The reaction will pass through the centre of the hinge or pin.

H_A – Horizontal Reaction at the Support 'A'
V_A – Vertical Reaction at the Support 'A'

Figure 3.3 : Hinged or Pinned Support

A Built-in or Fixed or Encastre Support

It is a support which restrains complete movement of the beam both in position as well as direction. The support gives all the three relevant reactions (say R_x, R_y and M_z), i.e. the reactions in x and y directions and fixing moment M_z. In other words, the fixed support offers resistance against translation in both the directions and also against the rotation.

M_A – Fixing Moment at the Support 'A'

Figure 3.4 : A Built-in or Fixed or Encastre Support

3.2.2 Types of Beam

Depending upon the type and number of supports, the beams are divided into two categories :

(i) Statically determinate beam, and

(ii) Statically indeterminate beam.

Statically Determinate Beam

A beam is said to be statically determinate, when it can be analyzed using three static equilibrium equations, i.e. $\sum H_x = 0$, $\sum V_x = 0$ and $\sum M_x = 0$, where

$\sum H_x$ = algebraic sum of horizontal forces at section x,

$\sum V_x$ = algebraic sum of vertical forces at section x, and

$\sum M_x$ = algebraic sum of moment of all the forces about the section x.

Examples are as follows :

(i) Cantilevers,

(ii) Simply supported beams, and

(iii) Overhanging beams.

It can also be stated, if the number of unknown reactions or stress components can be found by means of three equilibrium equations, then the beam is called statically determinate beam.

Statically Indeterminate Beam

When the number of unknown reactions or stress components exceed the number of static equilibrium equations available, i.e. $\sum H_x = 0$, $\sum V_x = 0$ and $\sum M_x = 0$ then, the beam is said to be statically indeterminate. This means, the three equilibrium equations are not adequate to analyse the beam.

Examples are as follows :

(i) Fixed beams,

(ii) Propped cantilevers, and

(iii) Continuous beams.

Now, we will discuss briefly each type of beam in subsequent paragraphs.

Cantilever Beams

A beam fixed at one end and free at the other end is known as cantilever.

Figure 3.5 : Cantilever Beams

Simply Supported Beam

A beam supported or resting freely on the walls or columns at its both ends is known as simply supported beam. The support may be hinged or roller support.

Figure 3.6 : Simply Supported Beam

Overhanging Beam

A beam having its end portion (or portions) extended in the form of a cantilever beyond the support is known as overhanging beam. A beam may be overhanging on one side or on both sides. The support may be fixed, hinged or roller support.

Figure 3.7 : Overhanging Beam

l_1, l_2, l_3 are the lengths of overhanging portions.

Fixed Beam

A beam rigidly fixed at its both ends or built-in walls is known as rigidly fixed beam or a built-in beam.

Figure 3.8 : Fixed Beam

Propped Cantilever

If a cantilever beam is supported by a simple support at the free end or in between, is called propped cantilever. It may or may not be having overhanging portion.

Propped Cantilever **Propped Cantilever with Overhang**

Figure 3.9

l_1 = length of overhanging portions.

Continuous Beam

A beam which is provided with more than two supports is called a continuous beam. It may be noted that a continuous beam may or may not be an overhanging beam.

Figure 3.10 : Continuous Beam

l_1 = length of overhanging portion, and

l_2 = span AB; l_3 = span BC; l_4 = span CD.

3.2.3 Types of Loading

A beam may be subjected to the following types of loads.

(i) Concentrated or point load,

(ii) Uniformly distributed load, and

(iii) Uniformly varying load.

Concentrated or Point Load

A load acting at a point on a beam is known as concentrated or a point load. In practice, it is not possible to apply a load at a point, i.e. at a mathematical point as it will be distributed over a small area. But this very small area, as compared to the length of the beam, is negligible.

Figure 3.11 : Concentrated or Point Load

A beam may be subjected to one or more point loads.

Uniformly Distributed Load

A load which is spread over a beam in such a manner that each unit length is loaded to the same extent, is known as uniformly distributed load (briefly written as u.d.l.). Sometimes, the intensity of load may not be uniform and varying from point to point. For all calculation purposes, the uniformly distributed load is assumed to act at the centre of gravity of load.

Figure 3.12 : Uniformly Distributed Load

Uniformly Varying Load

A load which is spread over a beam in such a manner that its extent varies uniformly on each unit length is known as uniformly varying load. Sometimes, the load is zero at one end and increases uniformly to the other end. Such a load is known as triangular load.

Figure 3.13 : Uniformly Varying Load

A beam may carry anyone of the above load systems or a combination of two or more systems at a time. Loads may be static or dynamic. Static loads are those which are applied gradually and do not change their magnitude, direction or point of application with time. Dynamic loads are those, which vary in time with speed.

3.3 SHEAR FORCE AND BENDING MOMENT

3.3.1 Definitions

Shear Force : General

The static equilibrium of a space can be ensured if the algebraic sum of all the forces acting on the particle in the x, y, z directions are separately zero [Figure 3.14(a)]. Mathematically, $\sum F_x = 0$, $\sum F_y = 0$, and $\sum F_z = 0$.

where, $\sum F_x$ = algebraic sum of all the forces acting on the particle in the x direction $= F_{x_1} - F_{x_2}$,

$\sum F_y$ = algebraic sum of all the forces acting on the particle in the y direction $= F_{y_1} - F_{y_2}$, and

$\sum F_z$ = algebraic sum of all the forces acting on the particle in the z direction $= F_{z_1} - F_{z_2}$.

Figure 3.14(a) : Equilibrium of a Particle under a System of Forces

These forces are general forces and their positive or negative sign will depend on, whether the forces are directed away from the origin or towards the origin. Let us consider a section A-A parallel to y-z plane in the body (Figure 3.14(b)).

Figure 3.14(b) : Equilibrium of a Body under System of Forces

Figure 3.14(c) : A Section A-A Parallel to *y*-*z* Plane Cut and Forces Exposed at A-A

The right hand portion is removed and the forces exposed on section A-A in the left hand portion of the body are shown in Figure 3.14(c). The three resultant forces in the x, y, z directions, denoted as F_x, F_y and F_z do not create the same type of stresses on the section. As the area of the section A-A is normal to x axis, F_x will create axial stresses or normal stress τ_x on the section. However, the remaining two forces F_y and F_z act tangential or along the plane and create shear stresses τ_{xy} and τ_{xz}. Using the common nomenclature of beams and bending, these forces F_y and F_z are known as **shear forces**, whereas F_x is simply an **axial force.**

F_x = resultant of all the forces acting on the left portion of the body in the x direction.

F_y = resultant of all the forces acting on the left portion of the body in the y direction.

F_z = resultant of all the forces acting on the left portion of the body in the z direction.

Figure 3.14(d) : Development of Shear Force at Section C

Now let us consider a beam (for example, cantilever beam) subjected to point load P. Consider a section at C. At this section there is a possibility of failure by shear as shown in Figure 3.14(d). If such a failure occurs at section C, the cantilever is liable to be sheared off into two parts. For the equilibrium of the cantilever, the fixed support at A will provide a vertical reaction vertically upwards, of magnitude $R_A = P$. It is clear that the force acting normal to the centre line of the member on each part equals $S = P$. The force acting on the right part on the section C is downward. The resultant force acting on the left part is upward. The resultant force acting on anyone on the parts normal to the axis of the member is called **shear force** at the section C.

Thus, the shear force (briefly written as SF) at the cross section of a beam may be defined as the unbalanced vertical force to the right or left of the section.

Bending Moment : General

For a rigid or deformable body, the rotational equilibrium can be ensured if the algebraic sum of moments of all the forces acting on the body in the x, y, z directions are separately zero (Figure 3.14(e)).

Mathematically, $\sum M_x = 0$, $\sum M_y = 0$, and $\sum M_z = 0$.

where M_x, M_y, M_z are the moments of forces taken about x, y, z axes.

Figure 3.14(e) : Equilibrium of a Body under a System of Moments

These moments are general moments and their positive or negative sign will depend on, whether the moments are counter-clockwise or clockwise.

$\sum M_x$ = Algebraic sum of moments of all the forces about x axis = $M_{x_1} - M_{x_2}$

$\sum M_y$ = Algebraic sum of moments of all the forces about y axis = $M_{y_1} - M_{y_2}$

$\sum M_z$ = Algebraic sum of moments of all the forces about z axis = $M_{z_1} - M_{z_2}$

Now, consider a section A-A parallel to y-z plane in the body (Figure 3.14(f)). The right hand portion is removed and the moments exposed on section in the left hand portion of the body are shown in Figure 3.14(f). The three moments, M_x, M_y, M_z are shown to be positive for the sake of convenience. A little thought will show that all of the moments do not create same type of stresses on the section. "M_x, acting on section x" creates a twisting action on the section. M_x renamed as M_{xx} is a torque and produces torsional shear stresses on the section. The remaining moments M_y and M_z acting on section x, renamed as M_{xy}, and M_{xz} produce a bending of the body around y or z axis and create linearly varying tensile and compressive stresses on the section. These moments are known as **bending moments** on a section A-A. Thus, we see that same moments can be twisting or bending moments, depending on the orientation of the section considered.

M_x = Algebraic sum of the moments of all the forces acting on the left portion of the body, taken about x axis.

M_y = Algebraic sum of the moments of all the forces acting on the left portion of the body, taken about y axis.

M_z = Algebraic sum of the moments of all the forces acting on the left portion of the body, taken about z axis.

Figure 3.14(f) : A Section A-A Parallel to *y-z* Plane Cut and Moments Exposed at A-A

Let us now study another effect of load applied on the cantilever. The cantilever is liable to bend due to the load on it. The cantilever has a tendency to rotate in clockwise direction about '*A*' (Figure 3.14(g)). Hence, the fixed support of A has to offer a resistance against this rotation.

Figure 3.14(g)

Moment of the applied load P about '*A*' is equal to Pl_1 (clockwise). For the equilibrium of the cantilever, the fixed support at A will provide a reacting or resisting anticlockwise moment of Pl_1.

Now consider, for instance, the section D. Suppose the part DB was free to rotate about D. Obviously the load on the part DB would cause the part DB to rotate in a clockwise order about D. Moment of any force is calculated as the product of the force and the perpendicular distance between the line of action of force and the section (or point) about which moment is required. Considering the part DB, taking moments about D, we find that there is a clockwise moment of $P\,(l_1 - l_2)$ about D. Hence, for the equilibrium of the part DB it is necessary that the part DA of the cantilever should provide a reacting or resisting anticlockwise of $P(l_1 - l_2)$ about D.

Taking moments about D, considering the part DA, we have the following moments about D.

(i) $RA \times DA = P \times l_2$ (clockwise)

(ii) Couple $= P \times l_2$ (anticlockwise)

$\therefore$ Net moment about $D = Pl_1 - Pl_2 = P(l_1 - l_2)$ (anticlockwise).

Figure 3.14(h)

Hence, for equilibrium of the part DA, the part DB should provide a clockwise moment of $P(l_1 - l_2)$. Hence, at the section D, the part DB provides a clockwise moment of $P(l_1 - l_2)$ and the part DA provides an anticlockwise moment of $P(l_1 - l_2)$. This moment is known as **bending moment** at D.

Thus, the bending moment (briefly written as BM) at the cross section of a beam may be defined as the algebraic sum of the moment of the forces, to the right or left of the section.

While calculating the shear force or bending moment at a section, the end reactions must also be considered along with other external loads.

3.3.2 Sign Conventions

Sign Convention for Shear Force (SF)

Since the shear force is the unbalanced vertical force, therefore it tends to slide one portion of the beam, upward or downward, with respect to the other as shown in Figure 3.15(a).

Figure 3.15(a) : Sign Convention for Shear Force (SF)

It is said to be positive, at a section, when the left hand portion tends to slide upwards with respect to the right hand portion. Or, in other words, all the upward forces to the left of the section will cause positive shear. It is said to be negative, at a section, when the left hand portion tends to slide downwards with respect to the right hand portion. Or in other words, all the downward forces to the section will cause negative shear.

On the other hand, all the upward forces to the right of the section will cause negative shear and those acting downwards will cause positive shear.

Sign Convention for Bending Moment (BM)

At sections, where the bending moment is such that it tends to bend the beam at that point to a curvature having concavity at the top as shown in Figure 3.15(b) is taken as positive, and where the bending moment is such that it tends to bend at that point to a curvature having convexity at the top as shown in Figure 3.15(b) is taken as negative.

Figure 3.15(b) : Sign Convention for Bending Moment (BM)

On the other hand, a loading which causes the beam to hog, will create negative bending moment, a loading which causes the beam to sag, will create positive bending moment.

In other words, the BM is said to be positive, at a section when it is acting in an anticlockwise direction to the left and clockwise direction to the right of the section. The BM is said to be negative, at a section, when it is acting in clockwise direction to the left and an anticlockwise direction to the right of the section.

3.3.3 Relation between Loading, Shear Force and Bending Moment

Consider a beam subjected to external loading of intensity w per unit length. Consider a small portion of the beam between sections 1-1 and 2-2, δx apart, at a distance x from the left end support as shown in Figure 3.16. The load acting on the small portion is equal to $w\ \delta x$.

Let F and M be the SF and BM at left end to the element and $F + \delta F$ and $M + \delta M$ be the SF and BM at right end of the element

The forces and moments acting on the element of the beam are as follows :

(i) upward force F at section 1-1,

(ii) downward force $F + \delta F$ at section 2-2,

(iii) downward load $w\ \delta x$,

(iv) anticlockwise moment M at section 1-1, and

(v) clockwise moment $M + \delta M$ at section 2-2.

Since the element of the beam is in equilibrium, therefore the system of forces and moments acting on the element, must obey the laws of equilibrium. Now equating the unbalanced vertical force at section 2-2, we get

$$F + \delta F = F - w\,\delta x$$

or $$\frac{\delta F}{\delta x} = -w \qquad \text{. . . (i)}$$

Thus, the rate of change of shear force (or in other words, the slope of shear force curve) is equal to the intensity of the loading.

Figure 3.16

Taking moments about the section 2-2,

$$M + \delta M = M + F\delta x - \frac{w(\delta x)^2}{2}$$

Ignoring higher powers of small quantities and simplifying the relation, we get

$$M + \delta M = M + F\delta x$$

or $$\frac{\delta M}{\delta x} = F \qquad \text{. . . (ii)}$$

Thus, the rate of change of bending moment (or in other words, the slope of the bending moment curve) is equal to the shear force at the section.

It is clear that SF curve can be obtained by integrating the loading curve and BM curve can be obtained by integrating the SF curve.

3.3.4 Maximum Value of Bending Moment

If the BM diagram is a continuous curve, where $\frac{dM}{dx} = 0$, the SF will be zero, and BM will be maximum positive or maximum negative.

For maximum value of bending moment,

$$\frac{\delta M}{\delta x} = 0$$

In other words, for maximum bending moment, the shear force is zero. But in actual practice, the bending moment may be maximum where shear force changes sign. However, the above relation is very important to obtain the maximum value of bending moment over the beam.

Equating the shear force at a distance of x from the support to zero, we get the point where the maximum bending moment will occur. Taking the moments at that section, maximum bending moment can be obtained.

3.4 SHEAR FORCE AND BENDING MOMENT DIAGRAMS

A Shear Force diagram for a structural member is a diagram which shows the values of shear forces at various sections of the member.

A bending moment diagram for a structural member is a diagram which shows the values of bending moment at various sections of the member.

The shear force and bending moment, at any section, can be obtained analytically. The values of shear force and bending moment are plotted as ordinates against the position of the cross section of the beam as abscissa. Such ordinates are drawn at important points of the beam and a straight line or a curve is drawn joining the tops of all such ordinates. These diagrams give the clear picture of the distribution of shear forces and bending moment along the length of the beam.

Procedure for Drawing the SF Diagram (SFD) and BM Diagram (BMD)

Step 1

Find the reaction of any one of the support, by taking moments about the hinged or pinned support and equating to zero ($\sum M = 0$).

Step 2

Find the reactions of other supports by means of static equilibrium equation ($\sum V = 0$, $\sum H = 0$).

Step 3

Draw a base line of length equal to the length of the beam to some scale.

Step 4

Starting from one end (left end), calculate the shear force at all salient points. If there is a vertical load (including reaction) at a section, calculate the shear force just left as well as right of the vertical load (or reaction). The shear force diagram will increase or decrease suddenly at the point of application of load depending upon the direction of load, i.e. by a vertical straight line at the section.

Step 5

Calculate the bending moment at salient points.

Step 6

Plot the positive values of shear force and bending moment above the base line and negative below it. The shear force diagram and bending moment diagram can be obtained by joining these ordinates.

The following points should be kept in mind for drawing the shear force and bending moment diagrams :

(i) If there is no loading on the portion of the beam the shear force diagram will be horizontal. The bending moment diagram will be inclined. The shear force diagram will remain unchanged between any two vertical loads provided there is no loading between the vertical loads.

(ii) If there is a uniformly distributed load between two sections, the shear force diagram will be inclined straight line and the bending moment diagram will be a curve.

(iii) The bending moment will be zero at the free end of the cantilever and at the two supports of simply supported beam.

(iv) The bending moment diagram will consist of either straight lines or smooth curves.

Cantilevers

Cantilevers with a Point Load at the Free End

Let us consider a cantilevers beam AB of length 'l' subjected to a point load W at the free end as shown in Figure 3.17.

Figure 3.17

Reaction at the support A, $R_A = W \uparrow$ (since, $\sum V = 0$).

Consider a section XX at a distance x from the free end. SF at this section is equal to the unbalanced vertical force either to the right or to the left of the section.

Consider right side of the section, $F_x = + W$ (plus sign indicates right downward).

The bending moment at this section, $M_x = -Wx$ (minus sign indicates right anticlockwise).

The SF just on the left side of the point load is $+ W$. Since there is no loading between A and B, the shear force will remain same over the length of the beam, i.e. the shear force diagram will be horizontal for the length AB.

We know that $M_x = -Wx$, then we get

BM at B, when x is zero, $M_B = 0$

BM at A, when x is 'l', $M_A = -Wl$

Since there is no other load between A and B, the bending moment diagram will be inclined, and the bending moment equation is straight line equation.

Cantilever with a Uniformly Distributed Load

Consider a cantilever beam AB of length l carrying uniformly distributed load of w per unit run, over the entire length of the beam as shown in Figure 3.18.

Figure 3.18

Consider a section XX at a distance x from the right end B,

SF at this section, $F_x = + w\, x$ (plus sign indicates right downward)

SF at B, $F_B = 0$ (at $x = 0$)

SF at A, $F_A = + w\, l$ (at $x = l$)

Reaction at the support A, $R_A = wl$.

Since the loading is uniformly distributed load, the SF diagram will be inclined and also, the shear force equation is straight line equation.

BM at section xx, $M_x = - w \times l \times \dfrac{l}{2}$ (minus sign indicates right anticlockwise).

SF at B, $M_B = 0$ (at $x = 0$)

SF at A, $M_A = -\dfrac{wl^2}{2}$ (at $x = l$)

The bending moment diagram will be in the form of parabolic curve.

Example 3.1

A cantilever beam of 8 m length is subjected to point loads of 10 kN, 15kN, 25 kN and 20 kN at distances of 2 m, 4 m, 6 m and 8 m respectively from the fixed end. Draw the shear force diagram and bending moment diagram.

Solution

Reaction at the support A = R_A = 10 + 15 + 25 + 20 = + 70 kN.

Figure 3.19

Shear Force

SF at A = F_A = + 70 kN

SF just left of C = + 70 kN

SF just right of C = + 70 – 10 = + 60 kN

SF just left of D = + 60 kN

SF just right of D = + 60 – 15 = + 45 kN

SF just left of E = + 45 kN

SF just right of E = + 45 – 25 = + 20 kN

SF just left of B = + 20 kN

SF just left of B = + 20 kN (considering right side of the section)

Bending Moment

BM at B, M_B = 0 (since the moment at the free end is equal to zero)

BM at E, M_E = – 20 × 2 (considering the right side)

= – 40 kN m

BM at D, M_D = – (20 × 4) – (25 × 2)

= – 130 kN m

BM at C, M_C = – (20 × 6) – (25 × 4) – (15 × 2)

= – 250 kN m

BM at A, M_A = – (20 × 8) – (25 × 6) – (15 × 4) – (10 × 2)

= – 390 kN m.

Example 3.2

Draw the shear force diagram (SFD) and bending moment diagram (BMD) for the beam shown in Figure 3.20.

Solution

Reaction at the support A, $R_A = +4 + (1.5 \times 6) = +13$ kN.

Figure 3.20

Shear Force

SF at A, $F_A = +13$ kN (considering right side)

SF at C, $F_C = +13 - 1.5 \times 6$

$= +4$ kN

SF just left of D $= +13 - 1.5 \times 6$

$= +4$ kN

SF just right of D $= +4 - 4 = 0$

Bending Moment

BM at B, $M_B = 0$ (since, the moment at the free end is equal to zero)

BM at D, $M_D = 0$ (considering right side of D, there is no loading on BD)

BM at C, $M_C = -4 \times 2 = -8$ kN m (– ve sign due to right anticlockwise)

BM at A, $M_A = -4 \times 8 - 1.5 \times 6 \times \frac{6}{2} = -59$ kN m.

Example 3.3

A cantilever beam carries a uniformly distributed load of 2 *t*/m over the entire length of 6 m and point loads of 5 *t*, 3 *t*, 7 *t* and 2 *t* at a distance of

2 m, 4 m, 5 m, and 6 m respectively from the fixed end. Draw SFD and BMD for the beam.

Solution

Reaction at the support A, $R_A = +5 + 3 + 7 + 2 + (2 \times 6) = 29\ t$

Figure 3.21

Shear Force : Starting from end A,

SF at A, $R_A = 29$ t

SF just left of C = + 29 – 2 × 2 = + 25 t

SF just right of C = + 25 – 5 = + 20 t

SF just left of D = + 20 – 2 × 2 = + 16 t

SF just right of D = + 16 – 3 = + 13 t

SF just left of E = + 13 – 2 × 1 = + 11 t

SF just right of E = + 11 – 7 = + 4 t

SF just left of B = + 4 – 2 × 1 = + 2 t

SF just right of B = + 2 t (considering right side)

Bending Moment : Starting from B,

BM at B, $M_B = 0$

BM at E, $M_E = -2 \times 1 - 2 \times 1 \times \frac{1}{2} = -3$ t m

BM at D, $M_D = -2 \times 2 - 7 \times 1 - 2 \times 2 \times \frac{2}{2} = -15$ t m

BM at C, $M_C = -2 \times 4 - 7 \times 3 - 3 \times 2 - 2 \times 4 \times \frac{4}{2} = -51$ t m

BM at A, $M_A = -2 \times 6 - 7 \times 5 - 3 \times 4 - 5 \times 2 - 2 \times 6 \times \frac{6}{2} = -105$ t m.

SAQ 1

(a) A cantilever beam of length 8 m carrying a u.d.l. of 3 kN/m over a length of 6 m from free end and 1.5 kN/m on a span of 2 m at a distance of 2 m from the fixed end and a point load of 6 kN at a distance of 1 m from the fixed end. Draw the SFD and BMD for the cantilever beam.

(b) A cantilever of 3 m length is loaded with a uniformly varying load of intensity 2000 N/m at free end to 600 N/m at fixed end. Draw SFD and BMD for the cantilever.

(c) A cantilever beam 5 m long caries point loads of 2 kN, 3 kN, and 3 kN at 1 m, 3 m and 5 m respectively from the fixed end. Construct SFD and BMD for the beam.

(d) Draw SFD and BMD for the cantilever beam of 3 m long which carries a uniformly distributed load of 2 kN/m over a length of 2 m from the free end.

(e) A cantilever beam 5 m long carries point loads of 3 kN, 3 kN and 3 kN at distances of 1.5 m, 3 m and 4.5 m respectively from the fixed end. In addition to this the beam carries an uniformly distributed load of 1 kN/m over the entire length of the beam. Draw SF and BM diagrams for the beam.

Simply Supported Beams

In case of simply supported beam, the point of contraflexure is very important. Where the BM changes sign at some point, the bending moment will be zero and that point is called the **point of contraflexure.**

Simply Supported Beam with a Point Load at its Mid-span

Let us consider a simply supported beam AB, span *l*, carries a point load *W* at its mid-point ‘C’ as shown in Figure 3.22.

Figure 3.22

Let us first find the reaction at the support B by taking moments about A, and equating to zero.

$$R_B \times l - W \times \frac{l}{2} = 0$$

$$\therefore \qquad R_B = +\frac{W}{2}$$

$$R_A = W - R_B = W - \frac{W}{2} = +\frac{W}{2}$$

(On the other hand, since the load is at the mid-point of the beam, the reactions at the supports $R_A = R_B = W/2$.)

Consider a section XX between B and C at a distance 'x' from the end 'B'. The shear force equation is

$$F_x = -\frac{W}{2} \qquad \text{('–' sign indicates right upward)}$$

This equation is valid for all values of x from 0 to $\frac{l}{2}$. Since there is no loading between B and C, the shear force diagram is horizontal, i.e. SF remains unchanged.

Consider a section XX between A and C at a distance 'x' from the end 'B'.

The shear force equation is

$$F_x = -\frac{W}{2} + W = +\frac{W}{2}$$

This equation is valid for all values of x from $\frac{l}{2}$ to l. The shear force diagram is horizontal.

Alternatively, the shear force equation can also be written by considering a section between A and C at a distance 'x' from the end A.

The bending moment at A and B is zero. It increases by a straight line law, and is maximum at the centre of the beam where shear force changes sign.

The bending moment equation is

$$M_x = +\frac{W}{2} \cdot x \qquad \text{('+' sign indicates right clockwise)}$$

BM at C, $\qquad M_C = \frac{Wl}{4} \qquad \left(\text{when } x = \frac{l}{2}\right)$

Simply Supported Beam with an Eccentric Point Load

Consider a simply supported beam AB of length l subjected to eccentric point load W at C at a distance 'a' from the end A and B from the end B as shown in Figure 3.23.

To find the reaction at the support B, taking moment about 'A'

$$R_B \times l - W \times a = 0$$

$$\therefore \qquad R_B = + \frac{Wa}{l}$$

$$R_A = W - \frac{Wa}{l} = + \frac{W(l-a)}{l} = + \frac{Wb}{l} \quad \text{(where } l - a = b\text{)}$$

Consider a section XX between B and C at a distance x from the end B, i.e. $x < b$.

Figure 3.23

The shear force equation is as follows :

$$F_x = - \frac{Wa}{l} \qquad \text{('–'sign indicates right upward)}$$

This equation is valid for the portion BC. The shear force diagram is horizontal. Consider a section XX between A and C at distance x from the end B, i.e. $x > a$.

The shear force equation is as follows :

$$F_x = - \frac{Wa}{l} + W = \frac{W(l-a)}{l} = + \frac{Wb}{l} \quad \text{(where } l - a = b\text{)}$$

This is valid for the portion AC.

Alternatively, the shear force equation for the portion AC may also be obtained by considering the section between A and C at a distance 'x' from the end A.

The bending moment equation for the portion BC is as follows :

$$M_x = + \frac{Wa}{l} \times x \qquad \text{('+'sign indicates right clockwise)}$$

The bending moment at A and B is zero. It increases by a straight line law, and is maximum at C where the SF changes sign.

BM at C, $M_C = + \frac{Wab}{l}$ (at $x = b$)

Simply Supported Beam with a Uniformly Distributed Load

Consider a simply supported beam AB of span l carrying a uniformly distributed load of w per unit length over the entire length as shown in Figure 3.24.

Figure 3.24

Let us find the reaction at the support B, taking moments about A and equating to zero,

$$R_B \times l - w \times l \times \frac{l}{2} = 0$$

$$\therefore \quad R_B = + \frac{wl}{2}$$

$$R_A = wl - R_B = wl - \frac{wl}{2} = + \frac{wl}{2}$$

Consider a section XX at a distance x from the end B.

The shear force equation,

$$F_x = - \frac{wl}{2} + wx$$

SF at B, $F_B = - \frac{wl}{2}$ (at $x = 0$)

SF at C, $F_C = - \frac{wl}{2} + w\left(\frac{l}{2}\right) = 0$ (at $x = \frac{l}{2}$)

SF at A, $F_A = - \frac{wl}{2} + wl = + \frac{wl}{2}$ (at $x = l$)

The shear force at B is equal to $-\frac{wl}{2}$ and increases uniformly by a straight line law to zero at C and continuous to increase uniformly to $+\frac{wl}{2}$ at A.

The bending moment at section *xx*,

$$M_x = \frac{wl}{2}x - \left(w \times x \times \frac{x}{2}\right)$$

$$= \frac{wl}{2}x - \frac{wx^2}{2}$$

BM at A and B is zero

$$M_A = M_B = 0$$

$$M_C = \frac{wl}{2}\left(\frac{l}{2}\right) - \frac{wl}{2}\left(\frac{l}{2}\right)^2$$

$$= \frac{wl^2}{4} - \frac{wl^2}{8} = +\frac{wl^2}{8}$$

Since the shear force is zero at mid point of the beam, the maximum bending moment occurs at ‘C’

$$\therefore \qquad M_{\max} = M_C = +\frac{wl^2}{8}$$

The bending moment diagram in the form of parabolic curve as the bending moment equation is parabolic equation.

Example 3.4

A simply supported beam of 7 m length carries point loads 2 kN, 4 kN and 6 kN at distances of 1 m, 2 m and 4 m from the fixed end respectively. Draw SFD and BMD and also calculate the maximum bending moment that will occur.

Solution

Figure 3.25

Taking moments about A to find R_B,

$R_B \times 7 - (6 \times 4) - (4 \times 2) - (2 \times 1) = 0$

Thus, $R_B = 4.857$ kN, and

$R_A = 12 - 4.857 = 7.143$ kN

Shear Force (Starting from left end A)

SF at A, $F_A = + 7.143$ kN

SF just left of C = + 7.143 kN

SF just right of C = + 7.143 – 2 = + 5.143 kN

SF just left of D = + 5.143 kN

SF just right of D = + 5.143 – 4 = + 1.143 kN

SF just left of E = + 1.143 kN

SF just right of E = + 1.143 – 6 = – 4.857 kN

SF just left of E = – 4.857 kN = Reaction at B

Bending Moment (Starting from right end B)

BM at A, $M_A = 0$

BM at E, $M_E = + 4.857 \times 3 = + 14.571$ kN m

BM at D, $M_D = + (4.857 \times 5) - (6 \times 2) = + 12.285$ kN m

BM at C, $M_C = + (4.857 \times 6) - (6 \times 3) - (4 \times 1) = + 7.142$ kN m

Or $M_C = 7.143 \times 1 = + 7.143$ kN m (considering left side)

BM at A, $M_A = 0$

Maximum Bending Moment

Maximum Bending Moment will occur at a point where the shear force changes sign. Here, SF changes from positive to negative at *E*. The Bending Moment at E will be Maximum Bending Moment.

Thus, $M_{max} = + 14.571$ kN m

Example 3.5

A 12 m span simply supported beam is carrying a uniformly distributed load of 2 kN/m over a length of 6 m from the left end and point loads 6 kN, 3 kN and 4 kN at distances of 7 m, 8 m and 9 m respectively. Draw SF diagram and BM diagram for the beam and find the maximum bending moment.

Figure 3.26

Solution

Taking moments about A, and equating to zero.

$$R_B \times 12 - (4 \times 9) - (3 \times 8) - (6 \times 7) - 2 \times 6 \times \frac{6}{2} = 0$$

Thus, $R_B = 11.5$ kN, and

$R_A = (2 \times 6) + 6 + 3 + 4 - R_B = 25 - 11.5 = 13.5$ kN

Shear Force (Starting from the left end)

SF at A, $F_A = +13.5$ kN

SF at C, $F_C = +13.5 - 2 \times 6$

$= +1.5$ kN

SF just left of D = + 1.5 kN

SF just right of D = + 1.5 – 6

= – 4.5 kN

SF just left of E = – 4.5 kN

SF just right of E = – 4.5 – 3

= – 7.5 kN

SF just left of F = – 7.5 kN

SF just right of F = – 7.5 – 4

= – 11.5 kN

SF just left of B = – 11.5 kN = Reaction at B

Bending Moment

BM at A, $M_A = 0$

BM at C, $M_C = (13.5 \times 6) - \left(2 \times 6 \times \frac{6}{2}\right) = 45$ kN m (considering left side)

BM at D, $M_D = (11.5 \times 5) - (4 \times 2) - (3 \times 1) = 46.5$ kN m (considering right side)

BM at E, $M_E = (11.5 \times 4) - (4 \times 1) = 42$ kN m

BM at F, $M_F = (11.5 \times 3) = 34.5$ kN m

The maximum bending moment occurs at D where the shear force changes sign.

$M_{max} = 46.5$ kN m.

Example 3.6

Construct the SFD and BMD for 10 m span simply supported beam subjected to a system of loads as shown in Figure 3.27.

Figure 3.27

Solution

Taking moment about A and equating to zero.

$$R_B \times 10 - 2 \times 4 \times \left(6 + \frac{4}{2}\right) - 3 \times 8 - 4 \times 6 - 5 \times 5 - 1.2 \times 5 \times \frac{5}{2} - 1 \times 2 = 0$$

$$R_B = \frac{154}{10} = 15.4 \text{ kN}$$

$$R_A = (1.2 \times 5) + 1 + 5 + 4 + 3 + (2 \times 4) - R_B$$

$$R_A = 27 - 15.4 = 11.6 \text{ kN.}$$

Shear Force (starting from left end)

SF at A, $F_A = +\ 11.6$ kN

SF just left of C, $F_C = +\ 11.6 - (1.2 \times 2) = +\ 9.2$ kN

SF just right of C, $F_C = +\ 9.2 - 1 = +\ 8.2$ kN

SF just left of D, $F_D = +\ 8.2 - (1.2 \times 3) = +\ 4.6$ kN

SF just right of D, $F_D = +\ 4.6 - 5 = -\ 0.4$ kN

SF just left of E, $F_E = -\ 0.4$ kN

SF just right of E, $F_E = -\ 0.4 - 4 = -\ 4.4$ kN

SF just left of F, $F_F = -\ 4.4 - (2 \times 2) = -\ 8.4$ kN

SF just right of F, $F_F = -8.4 - 3 = -11.4$ kN

SF just left of B, $F_B = -11.4 - (2 \times 2) = -15.4$ kN

Bending Moment

BM at A and B, $M_A = M_B = 0$

BM at F, $M_F = +(15.4 \times 2) - \left(2 \times 2 \times \frac{2}{2}\right) = +26.8$ kN m

$= +39.6$ kNm

BM at E, $M_E = +(15.4 \times 4) - (3 \times 2) - \left(2 \times 4 \times \frac{4}{2}\right) = +39.6$ kN m

BM at D, $M_D = +(11.6 \times 5) - (1 \times 3) - \left(1.2 \times 5 \times \frac{5}{2}\right)$ (considering left side)

$= +40$ kN m

BM at C, $M_C = +(11.6 \times 2) - \left(1.2 \times 2 \times \frac{2}{2}\right)$ (considering left side)

$= +20.8$ kN m

Since the SF changes sign at D, the maximum bending moment occurs at D.

$$M_{\max} = M_D = +40 \text{ kN m}$$

SAQ 2

(a) A beam 6 m long is simply supported at the ends carries a uniformly distributed load of 2 kN/m over the middle 2 m length and point loads of 1 kN and 4 kN at distances of 1 m and 5 m from the left end respectively. Draw SFD and BMD and determine the magnitude and position of the maximum BM.

(b) A beam simply supported at its ends has a span of 6 m. It is loaded with a gradually varying load of 750 N/m from the left hand support to 1500 N/m to the right hand support. Construct the SF and BM diagrams and find the magnitude and position of the maximum BM over the beam.

(c) A simply supported beam of 6 m span is loaded with a uniformly distributed load of 1.5 kN/m over the entire span and concentrated load of 4 kN and 5 kN at distances of 2 m and 4 m from the left hand support respectively. Find the magnitude and position of the maximum BM.

(d) Draw the shear force and bending moment diagram for the simply supported beam shown in figure below. Indicate the numerical values at all salient points.

Figure For SAQ 2(d)

Overhanging Beams

An overhanging beam is a beam which overhangs from the support either one side or both sides. The overhanging portion of the beam will be treated as cantilever.

Point of Contraflexure

For the purpose of shear force and bending moment, the overhanging beam will be analyzed as a combination of simply supported beam and a cantilever beam. We have discussed that the B.M. is negative for the cantilever and positive for the simply supported beam. It implies that in an overhanging beam, there is a point at which the BM changes from positive to negative and vice-versa. Such a point where the BM changes sign is known as contraflexure. There may be one or more points of contraflexure.

Example 3.7

Draw the shear force and bending moment diagrams for the overhanging beam shown in Figure 3.28.

Figure 3.28

Solution

Taking moments about A and equating to zero,

$$R_B \times 6 - (6 \times 8) - 0.8 \times 3 \times \left(3 + \frac{3}{2}\right) - (5 \times 2) = 0$$

R_B = 11.47 kN and R_A = 5 + (0.8 × 3) + 6 – 11.47 = 25 – 11.5 = 1.93 kN

Shear Force (Starting from the left end A)

SF at A, F_A = + 1.93 kN

SF just left of D, F_D = + 1.93 kN

SF just right of D, F_D = + 1.93 – 5 = – 3.07 kN

SF at E, F_E = – 3.07 kN

SF just left of B, F_B = – 3.07 – (0.8 × 3) = – 5.47 kN

SF just right of B, F_B = – 5.47 + 11.47 = + 6 kN

SF just left of C, F_C = + 6 kN = Load at E

Bending Moment

BM at C, M_C = 0

BM at B, M_B = – 6 × 2 = – 12 kN m

$$\text{BM at E, } M_E = +(11.47 \times 3) - (6 \times 5) - \left(0.8 \times 3 \times \frac{3}{2}\right)$$

$$= +0.81 \text{ kNm}$$

BM at D, M_D = + 1.93 × 2 = + 3.86 kN m

BM at A, M_A = 0

Maximum Bending Moment

Maximum positive bending moment occurs at D and maximum negative bending moment occurs at B.

M_{max} (positive) = + 3.86 kN m

M_{max} (negative) = – 12 kN m

Point of Contraflexure

Since the bending moment changes sign between E and B, consider a section XX between E and B at a distance x from C.

BM at section XX,

$$M_x = -6x + 11.47(x-2) - 0.8(x-2)\frac{(x-2)}{2}$$

$$= -6x + 11.47x - 22.94 - 0.4(x-2)^2$$

$$= -0.4x^2 + 7.07x - 24.54$$

By equating this equation to zero, we get the point of contraflexure,

$$-0.4x^2 + 7.07x - 24.54 = 0$$

$$x^2 - 17.67x + 61.35 = 0$$

Solving by trial and error,

$x = 4$ Value of $(x^2 - 17.675x + 61.35) = 6.65$

$x = 4.9$ Value of $(x^2 - 17.675x + 61.35) = -1.2475$

$x = 4.8$ Value of $(x^2 - 17.675x + 61.35) = -0.45$

$x = 4.7$ Value of $(x^2 - 17.675x + 61.35) = 0.3675$

$x = 4.75$ Value of $(x^2 - 17.675x + 61.35) = -0.04375$

$x = 4.745$ Value of $(x^2 - 17.675x + 61.35) = -0.00285$

Point of contraflexure is at a distance of 4.745 m from the end C.

Example 3.8

Draw the shear force and bending moment diagram for 10 m span overhanging beam having overhanging portion of 4 m subjected to a system of loads as shown in Figure 3.29. Calculate the maximum bending moment and also locate the point of contraflexure.

Solution

Taking moments about A and equating it to zero,

$$R_B \times 6 - (8 \times 10) - 3 \times 4 \times \left(6 + \frac{4}{2}\right) - (20 \times 4) - (10 \times 2) = 0$$

$$R_B = 46 \text{ kN}$$

$$R_A = 10 + 20 + (3 \times 4) + 8 - R_B = 50 - 46 = 4 \text{ kN}$$

Figure 3.29

Shear Force (Starting from the end A)

SF at A, $F_A = +4$ kN

SF just left of D, $F_D = +4$ kN

SF just right of D, $F_D = +4 - 10 = -6$ kN

SF just left of E, $F_E = -6$ kN

SF just right of E, $F_E = -6 - 20 = -26$ kN

SF just left of B, $F_B = -26$ kN

SF just right of B, $F_B = -26 + 46 = +20$ kN

SF just left of C, $F_C = +20$ kN = Load at C

Bending Moment

BM at C, $M_C = 0$

BM at B, $M_B = -8 \times 4 - \left(3 \times 4 \times \frac{4}{2}\right) = -56$ kN m

BM at E, $M_E = +(4 \times 4) - (10 \times 2) = -4$ kN m

BM at D, $M_D = +(4 \times 2) = +8$ kN m

Maximum Bending Moment

Since SF changes sign at D and B, the maximum positive bending moment will occur at D and maximum negative bending moment will occur at B.

$M_{\max}$ (positive) = + 8 kN m

$M_{\max}$ (negative) = – 56 kN m

Point of Contraflexure

BM changes sign between D and E. Therefore, consider a section XX at a distance x from the end A.

BM at section XX,

$M_x = +4x - 10(x - 2)$

Equating this to zero, we get

$4x - 10(x - 2) = 0$

$4x - 10x + 20 = 0$

$-6x + 20 = 0$

$\therefore \quad x = 3.333$ m

The point of contraflexure is at a distance of 3.333 m from the left end A.

Example 3.9

An overhanging beam of 15 m span is carrying an uniformly distributed load of 1 kN/m over the length of 10 m at a distance 5 m from the left free end and point loads 7 kN and 4 kN, at free end and at a distance 2 m from the free end respectively. Sketch the SFD and BMD for the beam. Locate the point of contraflexure.

Solution

Taking moments about B and equating to zero,

$$R_B \times 10 - (7 \times 15) - 4 \times 13 - \left(1 \times 10 \times \frac{10}{2}\right) = 0$$

$$R_A = 20.7 \text{ kN}$$

$$R_B = 7 + 4 + (1 \times 10) - 20.7 = 0.3 \text{ kN}$$

Figure 3.30

Shear Force (Starting from the left end C)

SF just right of C, $F_C = -\,7$ kN

SF just left of D, $F_D = -\,7$ kN

SF just right of D, $F_D = -\,7 - 4 = -\,11$ kN

SF just left of A, $F_A = -\,11$ kN

SF just right of A, $F_A = -\,11 + 20.7 = 9.7$ kN

SF just left of B, $F_B = +\,9.7 - (1 \times 10) = -\,0.3$ kN = Reaction at the support B

Bending Moment

BM at B, $M_B = 0$

BM at A, $M_A = +\,(0.3 \times 10) - \left(1 \times 10 \times \frac{10}{2}\right) = -\,47$ kN m

BM at D, $M_D = -\,(7 \times 2) = -\,14$ kN m

BM at C, $M_C = 0$

Maximum Bending Moment

Consider a section XX at a distance x from the end B as shown in Figure 3.31.

Shear force at section XX,

$$F_x = -\,0.3 + 1(x)$$

For maximum bending moment, the shear force is zero.

$$-\,0.3 + x = 0$$

$$x = 0.3 \text{ m}$$

We have, $M_x = + 0.3(x) - 1 \times (x) \times \frac{x}{2} = 0.3x - \frac{x^2}{2}$

$$M_{\max} = 0.3 \times (0.3) - \frac{(0.3)^2}{2} = + 0.045 \text{ kN m}$$

Figure 3.31

It is seen that the maximum negative bending moment occurs at A and maximum positive bending moment occurs at a distance 0.3 m from the right end B.

$$M_{\max} \text{ (positive)} = + 0.045 \text{ kN m}$$

$$M_{\max} \text{ (negative)} = - 47 \text{ kN m}$$

Point of Contraflexure

It is observed that the BM changes sign between A and B,

BM at any section XX,

$$M_x = 0.3\, x - \frac{x^2}{2}$$

Equating this to zero,

$$0.3\, x - \frac{x^2}{2} = 0$$

$$0.6\, x - x^2 = 0$$

$$\therefore \quad x = 0.6 \text{ m}$$

So, the point of contraflexure is at a distance of 0.6 m from the right end B.

Example 3.10

Draw the shear force and bending moment diagrams for 15 m span overhanging beam, which overhangs on both sides. It is subjected to a u.d.l. of 5 kN/m on left side overhanging portion of length 5 m and a u.d.l. of 4 kN/m on right side overhanging portion of length 4 m. Indicate the numerical values at all salient points.

Solution

To Find Reaction at B

Considering right side, the moments about A,

$$M_A = + R_B \times 6 - 4 \times 4 \times \left(6 + \frac{4}{2} \right)$$

$$M_A = 6\ R_B - 128$$

Considering left side, take moments about A,

$$M_A = -\left(5 \times 5 \times \frac{5}{2} \right) = -62.5$$

Equating these two values, we get

$$6\ R_B - 128 = -62.5$$

$$6\ R_B = 65.5$$

$$R_B = 10.92 \text{ kN}$$

$$R_A = (5 \times 5) + (4 \times 4) - R_B$$

$$R_A = 41 - 10.92 = 30.08 \text{ kN}$$

Shear Force (Starting from the left end C)

SF at C, $F_C = 0$

SF just left of A, $F_A = 0 - 5 \times 5 = -25$ kN

SF just right of A, $F_A = -25 + 30.08 = +5.08$ kN

SF just left of B, $F_B = +5.08$ kN

SF just right of B, $F_B = +5.08 + 10.92 = +16$ kN

SF at D, $F_D = +16 - (4 \times 4) = 0$

Bending Moment

BM at C and D, $M_C = M_D = 0$

BM at A, $M_A = -5 \times 5 \times \dfrac{5}{2} = -62.5$ kN m

BM at B, $M_B = -4 \times 4 \times \dfrac{4}{2} = -32$ kN m

Example 3.11

Sketch the shear force and bending moment diagrams for the beam overhanging on both sides as shown in Figure 3.32. Find the magnitude of maximum positive and maximum negative bending moment and also locate the point of contraflexure, if any.

Solution

Considering right side, take moments about A,

$$M_A = R_B \times 12 - 2 \times 12 \times 10$$

$$M_A = 12\, R_B - 240$$

Considering left side, take moments about A,

$$M_A = -\, 6 \times 4 = -\, 24 \text{ kN m}$$

Figure 3.32

Equating these two values, we get the reaction at the support B,

$$12\, R_B - 240 = -\, 24$$

$$R_B = 18 \text{ kN}$$

$$R_A = 6 + (2 \times 12) - R_B = 30 - 18 = 12 \text{ kN}$$

Shear Force (Starting from the left end C)

SF at C, $F_C = -\, 6$ kN

SF just left of A, $F_A = -\, 66$ kN

SF just right of A, $F_A = -\, 6 + 12 = +\, 6$ kN

SF at D, $F_D = +\, 6$ kN

SF just left of B, $F_B = +\, 6 - (2 \times 8) = -\, 10$ kN

SF just right of B, $F_B = -\, 10 + 18 = +\, 8$ kN

SF at E, $F_E = +\, 8 - (2 \times 4) = 0$.

Bending Moment

BM at C and E, $M_C = M_E = 0$

BM at B, $M_B = -\left(2 \times 4 \times \frac{4}{2}\right) = -\, 16$ kN m

BM at D, $M_D = +\, (18 \times 8) - \left(2 \times 12 \times \frac{12}{2}\right) = 0$

BM at A, $M_A = -\, (6 \times 4) = -\, 24$ kN m

Maximum Bending Moment

Consider a section XX at a distance ‘x’ from the end E.

SF at section XX,

$$F_x = -18 + 2x$$

For maximum bending moment, F_x should be zero.

$$-18 + 2x = 0$$

$$\therefore \qquad x = 9 \text{ m}$$

BM at any section XX between D and B,

$$M_x = 18(x - 4) - 2x \times \frac{x}{2}$$

$$= 18(x - 4) - x^2$$

$$M_{max} = 18(9 - 4) - 9^2 = 9 \text{ kN m}$$

Maximum negative bending moment = –24 kN m

Maximum positive bending moment = + 9 kN m

Points of Contraflexure

Let M and D be the points of contraflexure where the BM changes sign.

To find the position of M, equate the moment equation between D and B to zero.

$$18(x - 4) - x^2 = 0$$

$$-x^2 + 18x - 72 = 0$$

or, $$x^2 - 18x + 72 = 0$$

we get, $x = 6$ m and $x = 9$ m.

There are two points of contraflexure, one at a distance of 6 m and the other at a distance of 9 m from the end E.

SAQ 3

(a) Draw shear force and bending moment diagrams for the beam shown in figure below. Indicate the numerical values at all salient points.

Figure for SAQ 3(a)

(b) A simply supported beam with overhanging ends carries transverse loads as shown in figure below.

Figure for SAQ 3(b)

(c) If $W = 10w$, what is the overhanging length on each side, such that the bending moment at the middle of the beam is zero? Sketch the shear force and bending moment diagrams.

Characteristics of the Diagrams

Shear force diagrams (SFDs) and bending moment diagrams (BMDs) are very useful which give the clear picture of the distribution of shear force and bending moment along the length of the beam. Some important points regarding these diagrams are as follows :

(i) Where the rate of loading is zero, the SF curve will have constant ordinates and BM curve will vary linearly. In other words, if there is no increase or decreases in SF curve between any two points, i.e. SF line is horizontal and consists of rectangle, it indicates that there is no loading between the two points.

(ii) If there is a sudden increase or decrease, i.e. a vertical line of SF diagram, it indicates that there is either a point load or reaction of the support at that point or in other words, if there is any point load or reaction, it will cause sudden increase or decrease in the SF diagram with respect to upward or downward direction of point load or reaction, and it will not cause any distinct variation in the BM diagram.

(iii) Where the intensity of load is constant, i.e. uniformly distributed load, the SF curve will vary linearly and the BM curve will be a parabola. In other words, if the SF line is an inclined straight line between any two points, and the BM curve is a parabolic curve between any two points, it indicates that there is uniformly distributed load between the two points.

(iv) Where the loading curve is varying linearly, i.e. uniformly varying load, the SF curve will be a parabola and the BM curve will be a cubic parabola. In other words, if the SF line is a parabolic curve between any two points and BM curve is a cubic parabola between any two points, it indicates that there is a uniformly varying load between the points.

(v) SF and BM diagrams can be drawn by successive integration for complex case of loading.

(vi) The area of loading curve in elementary length will be equal to change in shear force and area of shear force curve in elementary length will be equal to change in bending moment.

(vii) The BM will be equal to zero at the free end of the cantilever and at the simply supported ends.

(viii) At the intermediate supports, the bending moments are always negative.

(ix) The maximum bending moment occurs at the point where the shear force changes sign and the point of contraflexure is the point where the BM changes sign.

Sometimes, instead of load diagram, a SF diagram will be given. In such cases, a loading diagram is drawn first. After drawing the load diagram for the beam, the BM diagram may be drawn as usual.

Example 3.12

The shear force diagram for the overhanging beam is shown in Figure 3.33. Draw the loading diagram and bending moment diagram. Find the magnitude of maximum bending moment and locate the point of contraflexure.

Figure 3.33

Solution

Let us analyse the shear force diagram given in Figure 3.34.

At A

The shear force diagram suddenly decreases from 0 to – 400 N. It indicates that there is a downward point load of 400 N at A.

Between A and B

The shear force diagram is an inclined straight line and decreases from – 400 N to – 560 N. It indicates that there is a uniformly distributed load of (560 – 400=160) $\Rightarrow$ 160 × 1 = 160 kN/m between A and B.

At B

There is a sudden increase from – 560 N to + 440 N at B. It indicates that there is a support reaction of 1000 N at B.

Between B and C

Since the shear force diagram varies linearly from + 440 N to – 520 N between B and C. It indicates that a u.d.l. of (440 + 520 = 960) $\frac{960}{6}$ = 160 kN/m is acting between B and C.

At C

At C, the shear force diagram increases suddenly from – 520 kN to 480 N. It indicates that there is a support reaction of 1000 N(520 + 480) at C.

Between C and D

The shear force diagram is an inclined straight line which indicates that there is a uniformally distributed load of 480/3 = 160 kN/m from C to D.

Bending Moment

BM at A and D, $M_A = M_D = 0$.

BM at B, $M_B = -(400 \times 1) - \left(160 \times 1 \times \frac{1}{2}\right) = -480$ N m

BM at C, $M_C = -\left(160 \times 3 \times \frac{3}{2}\right) = -720$ N m

Maximum Bending Moment

SF at any section XX between B and C,

$$F_x = +1000 - 400 - 160x$$

For maximum bending moment, F_x should be equal to zero.

$$600 - 160x = 0$$

$$\therefore \quad x = 3.75 \text{ m}$$

$$\therefore \quad M_{\max} = (1000 \times 2.75) - (400 \times 3.75) - \left(160 \times 3.75 \times \frac{3.75}{2}\right)$$

$$= +125 \text{ N m}$$

Maximum positive bending moment occurs at a distance of 3.75 m from the end A, where SF changes sign.

Maximum negative bending moment occurs at a support C where SF changes sign.

$$\therefore \quad M_{\max} \text{ (negative)} = -720 \text{ N m}$$

Points of Contraflexure

Let M_1 and M_2 be the points of contraflexure, where BM is zero. But at any section XX between B and C at a distance x from the end A.

$$M_x = 1000\,(x-1) - 400x - \left(160 \times x \times \frac{x}{2}\right)$$

or $$1000x - 1000 - 400x - 80x^2 = 0$$

$$80x^2 - 600x + 1000 = 0$$

$$8x^2 - 60x + 100 = 0$$

$$2x^2 - 15x + 25 = 0$$

$$(2x - 5)\,(x - 5) = 0$$

Thus, $x = 2.5$ m and 5 m.

3.5 SUMMARY

- A beam is a structural member, subjected to a system of external forces (including inclined load) to produce the bending of the member in an axial plane.
- The shear force at the cross-section of the beam is defined as the unbalanced vertical force either to the right or to the left of the section.
- The bending moment at the cross section of the beam is defined as the algebraic sum of moments of all the forces acting on the beam either to the right or to the left of the section.
- All the upward forces to the left of the section and all the downward forces to the right of the section cause positive shear force. All the downward forces to the left of the section and all the upward forces to the right of the section cause negative shear force.
- The BM is said to be positive, when it is acting in an anticlockwise direction to the left of the section and clockwise direction to the right of the section. The BM is said to be negative, when it is acting in clockwise direction to the left of the section and an anticlockwise direction to the right of the section.
- While drawing SF and BM diagrams, all the positive values are plotted above the base line and the negative values below it.
- The maximum BM occurs, where the SF is zero or changes sign.
- If the SF diagram line is horizontal between two points, the BM diagram is inclined. It indicates there is no load between two points. If the SF diagram is inclined between two points, the BM diagram is a parabola of second degree. It indicates that there is a uniformly distributed load between the two points. If the SF diagram is in the form of parabola of second degree between two points, the BM diagram is in the form of parabola of third degree (cubic parabola). It indicates that there is a uniformly varying load between the two points.
- The point of contraflexure is a point where the BM is zero or changes sign.
- If there is a point load or reaction, the BM does not change at that point. But the SF suddenly changes (either decreases or increases) in magnitude equal to that of point load or support reaction.

3.6 ANSWERS TO SAQs

SAQ 1

(a) **Shear Force Diagram Ordinates**

SF at A, F_A = + 27 kN

SF just left of C, F_C = + 27 kN

SF just right of C, F_C = + 21 kN

SF at D, F_D = + 21 kN

SF at E, F_E = + 12 kN

SF at B, F_B = 0

Bending Moment Diagram Ordinates

BM at B, M_B = 0

BM at E, M_E = – 24 kN m

BM at D, M_D = – 57 kN m

BM at C, M_C = – 78 kN m

BM at A, M_A = – 105 kN m

(b) Maximum Shear Force = 10500 N

Maximum Bending Moment = 13500 N m

(c) Maximum Shear Force = + 8 N

Maximum Bending Moment = – 26 kN m

(d) Maximum Shear Force = + 4 N

Maximum Bending Moment = – 8 kN m

(e) Maximum Shear Force = + 14 N

Maximum Bending Moment = – 39.5 kN m

SAQ 2

(a) Maximum Bending Moment = + 7.6 kN m at 3.25 m from the left hand support

(b) Maximum Bending Moment = + 5077.5 kN m at 3.165 m from the left hand support.

(c) Maximum Bending Moment = + 15.70 kN m at 3.22 m from left the hand support

(d) Maximum Bending Moment = + 12750 N m at D

SAQ 3

(a) Maximum Positive Bending Moment = + 37500 N m at C

Maximum Negative Bending Moment = –5000 N m at B

(b) l = 1.25 m