UNIT 3 AC CIRCUITS

Structure

3.1 INTRODUCTION

In Unit 2, we have studied Electromagnetism. The practical useful mode of storage of electrical energy at the present time is through DC storage batteries (rechargeable DC cells) and are widely used in transport vehicles, electronic instruments and equipment, portable tools, etc.

However, the bulk of electrical energy utilisation, whether in domestic installations, or in industry or in public service organisations is through alternating current (AC) systems involving sinusoidal voltages and currents. Strictly speaking, the adjective alternating indicates any signal whose direction alternates with time but in practice, it is invariably used to refer to sinusoidal signals.

In this Unit, you will first learn about the characteristics and representation of sinusoidal voltages and currents. You will then get to know the types of response of standard circuit elements to sinusoidal excitation. This will be a prelude to the study of methods of analysis of circuits formed by various combinations of simple circuit elements and operating under the influence of AC sources. The *phasor concept* which simplifies the expression of steady-state response-excitation relations in these circuits would be adopted as the framework for the development of the various techniques of analysis. You will also be introduced to the various ramifications of power in an AC circuit.

Objectives

After studying this unit, you should be able to

- explain the reasons for the widespread use of sinusoidal signals in electrical engineering,

- deduce the different parameters of a sinusoidal waveform like peak value, phase, effective value, form factor etc.,
- work with complex numbers as needed for the manipulation of phasors in AC circuit analysis,
- express the steady state response both in time domain and in phasor domain of R, L, C elements to sinusoidal inputs,
- calculate the impedance $\overline{Z}$ and admittance $\overline{Y}$ of simple elements and combinations thereof in 2-terminal networks,
- explain the factors on which power in an AC circuit depends and distinguish between active power, apparent power and reactive power,
- describe characteristics of a 3-phase system,
- describe the features of a 3-phase system,
- distinguish between delta and star-connections of sources and loads, and
- distinguish between balanced and unbalanced systems.

3.2 SINUSOIDAL SIGNALS

Sinusoidal signals have a vital role both in electrical power engineering and in communication engineering. In power supply systems, the voltages and currents are invariably of this waveform with a frequency of 50 Hz in most countries including India. In the field of communication engineering, we have to deal with sinusoidal signals having wide frequency range extending from a few Hz to a few GHz (1 GHz = 10^9 Hz).

Formally, a sinusoidal function of time is defined as one having the general form $A \sin (\omega t + \theta)$ and is characterised by three parameters viz., **amplitude A,** angular frequency ω and phase angle θ. The argument (angle) of the sine function viz., $(\omega t + \theta)$ is measured in radians and increases at the rate of ω radians per second. Since a sine function repeats itself at intervals of 2 π radians of its angle, we immediately see that the period T of the sinusoid is related to ω by

$$\omega T = 2\pi$$

We also know that the frequency f of the signal is the reciprocal of the period T. Therefore, we have

$$\omega \times \frac{1}{f} = 2\pi \text{ or } \omega = 2\pi f$$

The parameters ω is, thus, a measure of the frequency of the signal and is called **angular frequency.** The phase angle θ is the value of the argument of the sine function at the origin of time and governs the instantaneous value at $t = 0$ of the sinusoidal signal of a given amplitude. Strictly speaking, θ should be expressed in radians as ωt is expressed in radians. However, since many of us have a better feel for angles measured in degrees, we often indulge in somewhat irregular practice of mixing up units by writing expressions like 325 sin (314 t + 60°) instead of the more proper 325 sin $\left(314\, t + \frac{\pi}{3}\right)$. However, the actual evaluation of the sine function is done only after expressing the two terms in its argument in the same units (radians or degrees) before their addition.

Figure 3.1 shows the waveform of a general sinusoidal voltage of frequency *f*. As shown therein, the x-axis can be graduated either in terms of time *t* in the conventional manner or alternatively in terms of the angle ω*t*. The latter has the effect of normalizing the period of the waveform to 2 π radians irrespective of the actual value of frequency *f*.

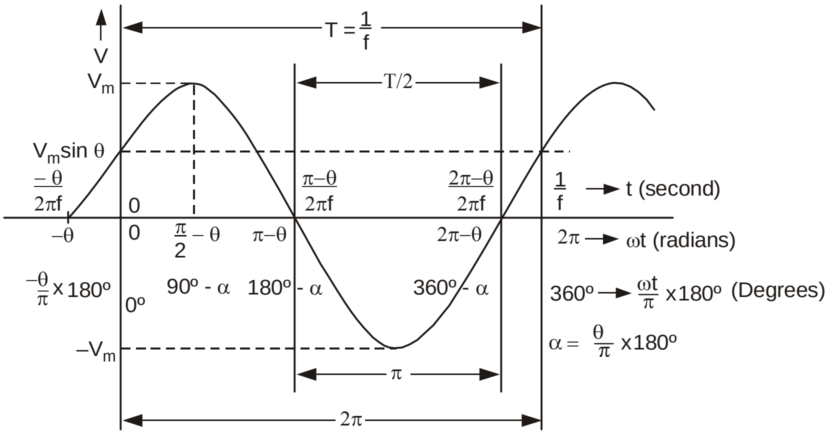


Figure 3.1 : Waveform of a General Sinusoidal Voltage $v(t) = V_m \sin(\omega t + \theta)$

Example 3.1

Express the following signals in the standard form, $A \sin(\omega t + \theta)$.

(a) $v_1 = 20 \cos \omega t$

(b) $v_2 = 100 \sin \omega t + 75 \cos \omega t$

(c) $v_3 = 10 \cos \omega t \left(\omega t - \frac{\pi}{2} \right)$

Solution

(a) $v_1 = 20 \cos \omega t = 20 \sin \left(\omega t + \frac{\pi}{2} \right)$

(b) $$100 \sin \omega t + 75 \cos \omega t = \sqrt{100^2 + 75^2} \left(\frac{100 \sin \omega t}{\sqrt{100^2 + 75^2}} + \frac{75 \cos \omega t}{\sqrt{100^2 + 75^2}} \right)$$

$$= 125 (\sin \omega t \cos \theta + \cos \omega t \sin \theta)$$

where, $$\cos \theta = \frac{100}{\sqrt{100^2 + 75^2}} = \frac{100}{125}$$

$$\sin \theta = \frac{75}{\sqrt{100^2 + 75^2}} = \frac{75}{125}$$

and $$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$= \frac{75}{100} = 0.75$$

or, $\theta = \tan^{-1}(0.75) = 37^o = 0.64$ radians

Hence, $v_2 = 125 \sin(\omega t + \theta)$,

or, $v_2 = 125 \sin(\omega t + 37^o)$ or $125 \sin(\omega t + 0.64)$

(c) $v_3 = 10 \cos\left(\omega t - \frac{\pi}{2}\right) = 10 \cos\left(\frac{\pi}{2} - \omega t\right)$

$= 10 \sin \omega t$

3.2.1 Importance of Sinusoidal Signals

A sinusoidal function of time represents the simplest periodic process that occurs in the physical world like the vibration of a tuning fork, the small amplitude oscillation of a pendulum, and the current in a simple *LC* (inductor-capacitor) circuit supplied with some initial energy. A superposition of such functions is the characteristic or natural behaviour of any non-dissipative linear system of arbitrary complexity, a network comprised pure inductors and capacitors being an example of such a system.

A sinusoidal function has certain unique characteristics which no other periodic function can claim. If two sinusoids of the same frequency are added to or subtracted from each other, the result is another sinusoid of the same frequency. If a sinusoid is integrated or differentiated, once again a sinusoid of the same frequency arises. Therefore, when a sinusoidal voltage is applied to resistor, an inductor or a capacitor, the resulting current waveform is sinusoidal of same frequency and vice-versa. That the waveform and frequency are retained under the above four linear operations is a very significant and unique property. What is true of a single element, R, L or C, is also true of a complex interconnection of such elements. Thus, it turns out that if any arbitrary linear electric network (i.e. a network composed of linear elements like resistors, capacitors and inductors) is excited by a sinusoidal source A sin ($\omega t + \theta$), the resulting steady state current/voltage response in any element of the network is another sinusoid of the same frequency. The response can differ from the excitation only in its amplitude and phase but in no other characteristic, i.e. it should have the form $B \sin(\omega t + \alpha)$. This property leads to particularly simple techniques of AC circuit analysis using concepts like impedance and sinusoidal transfer function, which we shall get to know later on.

The voltage developed by a rotating electrical generator in a power station is necessarily periodic. Leaving aside the case where this voltage is converted to DC form through a commutator, the particular periodic form employed in a commercial electrical generator is the simplest form of alternating quantity, namely one which varies sinusoidally with respect to time. The preference for this form is obvious from what is stated in the previous paragraph. An electrical power system may be enormously complex, containing scores of generators, hundreds of kilometers of transmission lines and different kinds of motors and other user equipment. But as long as all the generators develop sinusoidal voltages of the same frequency, as indeed they do, and the system is linear, then the voltage available at every power outlet is a sinusoid of the same frequency. If the generator voltages are of any other waveform, the voltages at different locations could be a maddeling medley of complex waveforms with no easily recognizable relation between one and other.

The foregoing observations clearly highlight the importance of developing efficient techniques for analysis of circuits under sinusoidal excitation. Further, these can be extended to find effective solutions for the behaviour of circuits and systems working under non-sinusoidal periodic and aperiodic excitations.

3.2.2 Effective Value and Form Factor

The *effective value* of a periodic signal which counts power calculations are of concern and that for the particular case of a sinusoidal signal, the effective value is $\left(\frac{1}{\sqrt{2}}\right)$ times the peak value. It is conventional to indicate the strength of a sinusoidal voltage or current in terms of its effective value or Root Mean Square value (RMS value) and to simply use the capital letter V or I as the symbol for this quantity, discarding the subscripts in the symbols V_{eff} and I_{eff}. A 230 V AC voltage would mean a voltage having an effective value of 230 V. This point has to be clearly understood that, unless otherwise stated, any numerical value assigned to an AC signal implies that it is the rms value and not either its peak value or its absolute average value. A 5 Amp, 50 Hz AC current would have a time variation $\sqrt{2} \times 5 \sin (100\, \pi\, t + \theta)$ since the peak value is $I_m = \sqrt{2} \times 5$ and $\omega = 2\pi\,(50) = 100\pi$. We hereafter indicate a general sinusoidal voltage and current as $\sqrt{2}\, V \sin (\omega t + \theta)$ and $\sqrt{2}\, I \sin (\omega t + \theta)$ respectively, V and I being the corresponding effective values.

The only meaningful average value that can be associated with a sinusoid is the absolute average value (also equal to the average over the positive half cycle and hence called half-cycle average) and that the latter is equal to $(2/\pi)$ times the peak value.

We now define the form factor of a symmetric periodic waveform as follows :

$$\text{Form factor} = \frac{\text{RMS Value}}{\text{Absolute Average Value}} \qquad \text{. . . (3.1)}$$

From the name of the term, it is clear that form factor gives an indication of the shape of the wave. The sharper is the peak of the waveform, the larger is the form factor. For a flat waveform like the symmetrical square wave the form factor has a value equal to 1, since the RMS and absolute average values of this wave are equal. For a sinusoid, we have

$$\text{Form factor of a sinusoid} = \frac{\text{Peak Value} / \sqrt{2}}{(\text{Peak Value})\,(2/\pi)} = \frac{\pi}{2\sqrt{2}} = 1.11 \qquad \text{. . . (3.2)}$$

The form factor is a parameter to be considered in certain applications like the determination of the effective value of voltage induced in a coil due to changing magnetic flux of a given amplitude.

Example 3.2

Find form factor of a symmetric triangular voltage waveform shown in Figure 3.2.

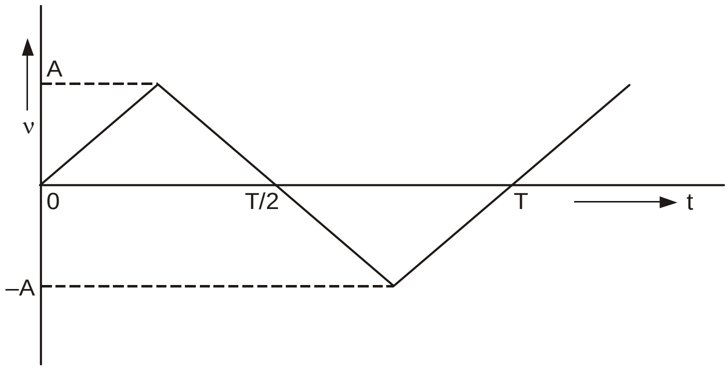


Figure 3.2 : Symmetric Triangular Waveform

Solution

To find $(V_{rms})^2$ we need to find the average of ν^2. As the area under the curve of ν^2 for the first quarter period is equal to the areas for the subsequent 3 quarter periods, we can find the required average of ν^2 by considering only the interval $0 < t < T/4$.

Thus, $$(V_{\text{rms}})^2 = \frac{4}{T}\int_0^{T/4}\left(\frac{4At}{T}\right)^2$$

$$dt = \frac{64A^2}{T^3}\,\frac{1}{3}\left(\frac{T}{4}\right)^3 = \frac{A^2}{3}$$

$$V_{\text{rms}} = \frac{A}{\sqrt{3}}$$

Likewise, $$V_{\text{absav}} = \int_0^{T/4}\left(\frac{4A}{T}\,t\right)$$

$$dt = \frac{A}{2}$$

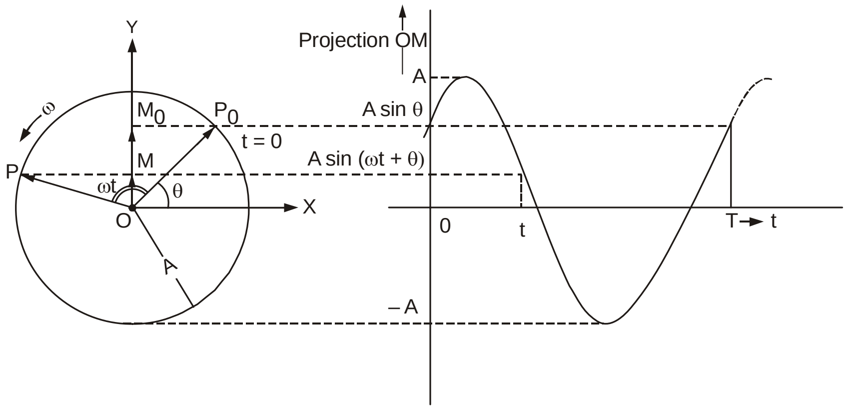
Hence, form factor of the waveform = $\dfrac{\frac{A}{\sqrt{3}}}{\frac{A}{2}} = \dfrac{2}{\sqrt{3}} = 1.154$.

3.2.3 Phasor Representation

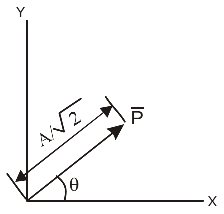
If a point moves around a circle with uniform angular velocity, its projection on a straight line varies sinusoidally. By virtue of this property, a sinusoid permits an extremely simple graphical representation viz., a directed line segment *P* of magnitude *A* and indicating the position of the revolving point.

Let the point *P* move around a circle of radius *A* in the anticlockwise direction with an angular velocity of ω rad/sec and pass through the angular position θ at $t = 0$, as shown in Figure 3.3(a). Its position at $t = 0$ and a general *t* are marked in the figure.

The displacement from origin ‘O’ the projection of *P* on the vertical axis (viz., OM) clearly varies as $A \sin(\omega t + \theta)$ as shown in Figure 3.3(b), taking upward displacement as positive.



(a) Revolving Point *P* and its Projection *M* on Y-axis; (b) Variation of *OM* with Time



(c) Phasor Representation

Figure 3.3 : Revolving Point, its Vertical Projection and Phasor Representation

On this basis, we evolve a simple representation of a sinusoid A sin ($\omega t + \theta$) in the form of a directed line segment at an angle θ with respect to X-axis (i.e., directed towards the position of the revolving point at $t = 0$) and having a length equal to $\frac{A}{\sqrt{2}}$ is dictated by convenience as we invariably deal with the RMS values of sinusoids rather than peak values. Such a representation is called the **phasor** of the particular sinusoid. For the sinusoidal signal A sin ($\omega t + \theta$), the phasor would, thus, be a directed line of length $\left(\frac{A}{\sqrt{2}}\right)$ at an angle θ relative to the horizontal as shown in Figures 3.3(c) and 3.4. The phasor of P is represented as $\overline{P} = \frac{A}{\sqrt{2}} \angle \theta$, where $\frac{A}{\sqrt{2}}$ is its RMS value and θ is the phase angle. If we imagine the phasor to rotate anticlockwise about the origin with an angular velocity ω, starting from the position shown at $t = 0$, then the projection of the phasor on the vertical axis at any time t has a length equal to $\left(\frac{1}{\sqrt{2}}\right)$ times the instantaneous value of the related sinusoid at that time.

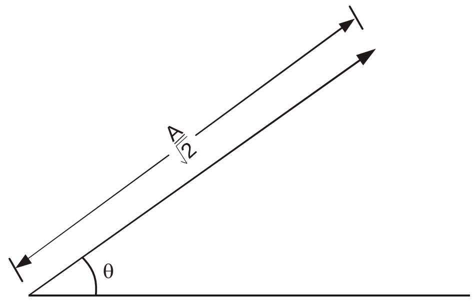
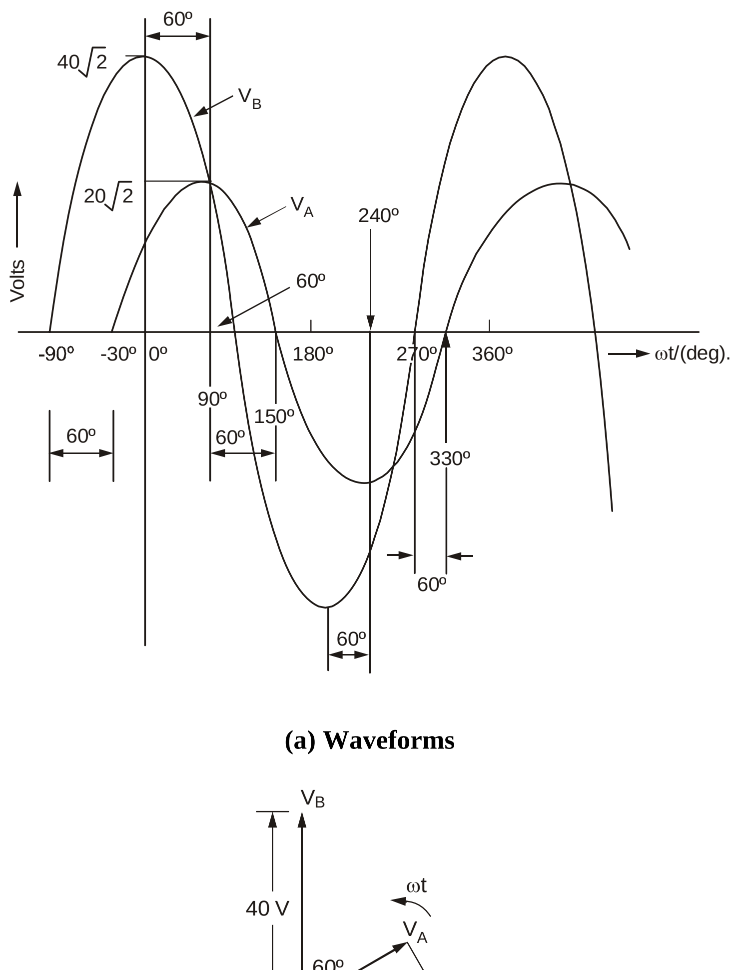


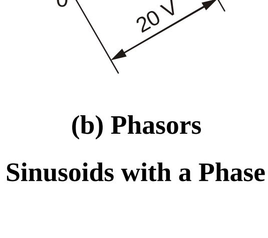
Figure 3.4 : Phasor or a Sinusoid A sin ($\omega t + \theta$)

Any sinusoid is uniquely determined by three quantities viz., RMS value, frequency and phase. In AC circuit analysis, we normally deal with situations in which all currents and voltages have the same frequency and the value of this frequency is known. In this situation, one sinusoid differs from another only in respect of RMS value and phase, both of which are prominently displayed by a phasor. There is therefore a one-to-one correspondence between a sinusoid of a given frequency and its phasor and we can deduce one from the other. Different voltages and currents occurring in an AC circuit (operating with a single frequency excitation) can, therefore, be represented by an assembly of directed line segments having a common origin, a representation which is far more concise and clearer than a display of all the pertinent waveforms on a common time base. For example, the phase difference between two voltages is clearly visualized as the angle between the respective phasors. Several mathematical operations on phasors either graphically or analytically can be easily performed on phasors as we shall see in the subsequent sections.

As an example, consider two voltages $\nu_A = 20\sqrt{2}\ \sin(\omega t + 30^o)$ and $\upsilon_B = 40\sqrt{2}\ \sin(\omega t + 90^o)$ with $V_A = 20$ and $V_B = 40$, whose waveforms are shown in Figure 3.5(a).



(a) Waveforms



(b) Phasors

Figure 3.5 : Two Sinusoids with a Phase Difference of 60º

The phase difference between v_A and v_B at an angular interval of 60^o. v_B is said to lead v_A by 60^o as successive similar events (e.g. upward zero crossings, positive peaks, negative peaks, downward zero crossings) occur with v_B at an angular interval of 60^o earlier than with v_A (Figure 3.5(a)). By the same token, v_A is said to lag v_B by 60^o.

This relationship between v_A and v_B can also be represented very simply with the help of phasors V_A and V_B as shown in Figure 3.5(b), which are supposed to rotate in anticlockwise direction with an angular speed ω. Thus, V_B, which is moving ahead of V_A by exactly 60^o is said to lead V_A by 60^o. Similarly, V_A is said to lagging V_B by 60^o. Thus, $\overline{V}_A = 20 \angle 30^o$ and $\overline{V}_B = 40 \angle 90^o$ and phase difference $= 90^o - 30^o = 60^o$.

SAQ 1

Sketch the phasors of the following sinusoidal signals :

(a) $-100 \sin (\omega t + 30^o)$

(b) $4\sqrt{2} \cos (\omega t + 30^o)$

3.3 IMPEDANCE CONCEPT

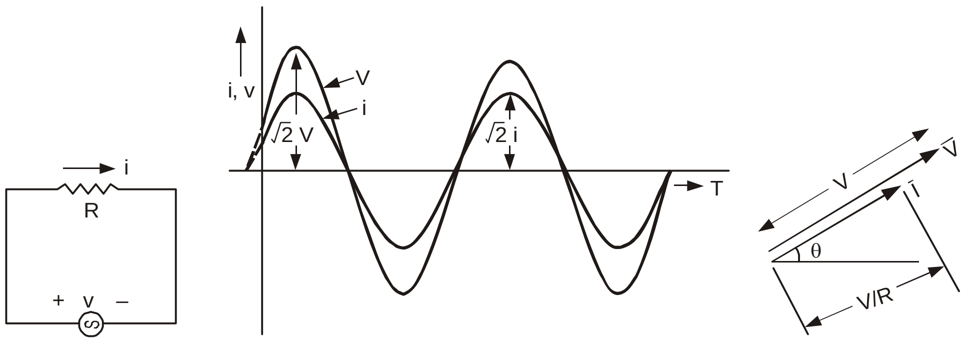
In the previous section, we saw how the phasor concept provides an effective means of representing voltages and currents in AC circuits. In this section, we extend the application of this concept to characterisation of terminal relations of single elements and two-terminal networks. To this end, let us first examine the response of RLC elements to sinusoidal excitation.

3.3.1 Response of Single Element to Sinusoidal Excitation

Resistance

Consider a resistor of R ohms connected to a sinusoidal voltage source of $v = \sqrt{2}\, V \sin (\omega t + \theta)$ volts as shown in Figure 3.6(a). From the fundamental terminal relationship $v = Ri$ of a resistor it follows that

$i = (v/R) = \sqrt{2}\, (V/R) \sin (\omega t + \theta)$ or $i = \sqrt{2}\, I \sin (\omega t + \theta)$, where $I = \dfrac{V}{R}$.



(a) Circuit **(b) Waveform** **(c) Phasor Representation of $\overline{V}$ and $\overline{I}$**

Figure 3.6 : Response of a Resistor to Sinusoidal Excitation

We note the following :

(a) The current and voltage are in phase (i.e., they have zero phase difference). Both vary in step. The positive peaks, negative peaks, zero crossing etc. occur in both at the same time as shown in Figure 3.6(b).

(b) RMS value of $I = \dfrac{V}{R}$ and, thus, the RMS values of voltage and current are related by

$$V = RI \qquad \text{. . . (3.3)}$$

a formula identical to Ohm's law in DC domain.

(c) The voltage and current phasors being $\overline{V} = V \angle \theta$ and $\overline{I} = \left(\dfrac{V}{R}\right) \angle \theta$, as shown in Figure 3.6(c), they are related by

$$\overline{V} = R\overline{I} \qquad \text{. . . (3.4)}$$

Note that the above relationship is independent of ω and θ.

Inductance

Now, let an inductor of L henrys have a voltage $\upsilon = \sqrt{2}\, V \sin (\omega t + \theta)$ applied across it as shown in Figure 3.7. From the fundamental terminal relationship of an inductor, we have

$$\upsilon = L \frac{di}{dt} \text{ or } i = \frac{1}{L} \int \upsilon \, dt$$

or

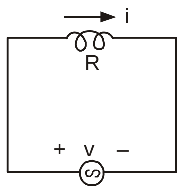
$$i = \frac{1}{L} \int \sqrt{2}\, V \sin (\omega t + \theta)\, dt$$

$$= - \frac{\sqrt{2} V}{\omega L} \cos (\omega t + \theta) = \frac{\sqrt{2} V}{\omega L} \sin \left(\omega t + \theta - \frac{\pi}{2} \right)$$

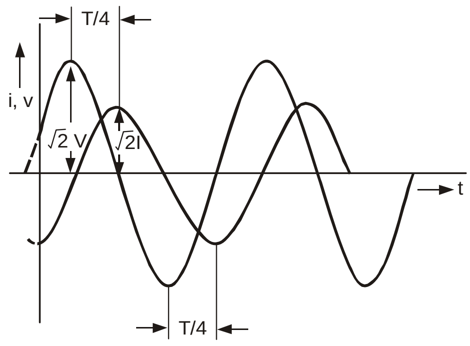
$$= \sqrt{2}\, I \sin \left(\omega t + \theta - \frac{\pi}{2} \right),$$

where

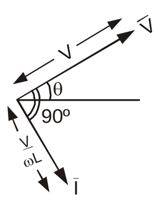
$$I = \frac{V}{\omega L}$$



(a) Circuit



(b) Waveform



(c) Phasor $\overline{V}$ and $\overline{I}$

Figure 3.7 : Response of an Inductor to Sinusoidal Excitation

We take the constant of integration in the above to be zero as for the AC circuits of concern to us, there cannot be a DC or constant current in any element.

The main characteristics of this response are :

(a) The current has phase difference of 90^o with respect to the voltage and lags behind it. Similar events (positive peaks, negative peaks, upward zero crossings, downward zero crossings etc.) occur in the voltage wave a quarter-period (equivalent to 90^o of the angle ωt) earlier than the current wave, as shown in Figure 3.7.

(b) RMS value of $I = \frac{V}{\omega L}$, and, thus, the RMS values of voltage and current are related by

$$V = (\omega L)\, I \qquad \ldots (3.5)$$

(c) The phasors of v and i being $\overline{V} = V \angle \theta$ and $\overline{I} = I \angle \left(\theta - \frac{\pi}{2}\right)$

They are related by

$$\overline{V} = (j\,\omega L)\, \overline{I} \qquad \ldots (3.6)$$

where, j is 90^o operator.

Capacitance

Figure 3.8 shows a capacitor of C farads applied with a voltage $v = \sqrt{2}\, V \sin(\omega t + \theta)$. The current through the capacitor would, therefore, be

$$i = C\,\frac{dv}{dt} = C\,\frac{d}{dt}\,(\sqrt{2}\, V \sin\, \omega t + \theta)$$

$$= \sqrt{2}\,\omega\, C\, V \cos(\omega t + \theta) = \sqrt{2}\,\omega\, C \sin\left(\omega t + \theta + \frac{\pi}{2}\right)$$

$$= \sqrt{2}\, I \sin\left(\omega t + \theta + \frac{\pi}{2}\right),$$

where $\quad I = \omega\, C\, V$

The main characteristics of this response are :

(a) The current has a phase difference of 90^o with respect to the voltage and leads the latter. Similar events occur in the current wave a quarter-period (equivalent to 90^o) earlier than the voltage wave as shown in Figure 3.8.

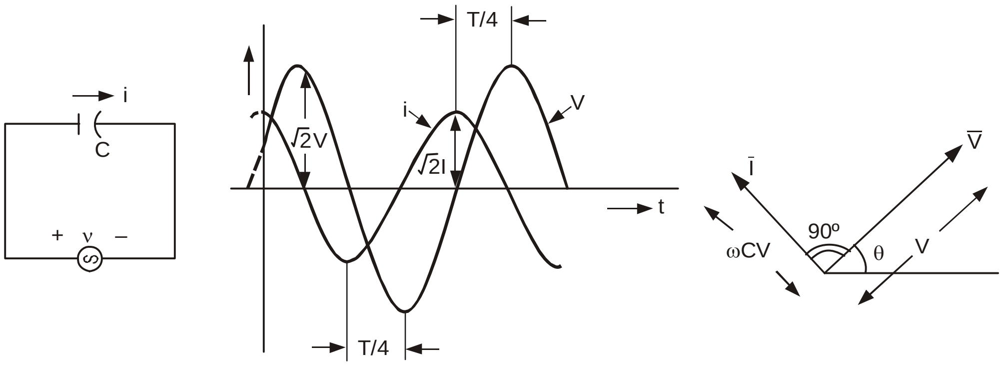
(b) Since RMS value of current $I = \omega\, C\, V$, the RMS values of voltage and current are related by

$$V = \left(\frac{1}{\omega C}\right) I \qquad \ldots (3.7)$$

(c) The phasors $\overline{V}$ and $\overline{I}$ being $V \angle \theta$ and $\omega C V \angle \left(\theta + \frac{\pi}{2}\right)$ respectively, they are related by

$$\overline{V} = \frac{1}{j\,\omega C}\, \overline{I} \qquad \ldots (3.8)$$

The properties derived earlier are characteristic of the three elements in AC circuits and hold irrespective of where they are connected in a circuit. A little reflection will show that item (c) in each case is a complete statement of the relevant properties and incorporates in itself the properties specifically stated under (a) and (b).



(a) **(b)** **(c)**

Figure 3.8 : Response of a Capacitor to Sinusoidal Excitation

To summarise, the three elements R, L and C respond differently to sinusoidal excitations (current or voltage). **The current and voltage in a resistor are in phase, while they are in quadrature (i.e., have a phase difference of 90°) in an inductor or a capacitor. In an inductor, the voltage leads the current by 90° while in a capacitor, the current leads the voltage by 90°.** The RMS values of the voltage across and the current through the element satisfy the proportionality relationship given in each case by Eqs. (3.3), (3.5) and (3.7) respectively. For a given value of current to be driven through it, an inductor requires more voltage as the frequency increases. The capacitor on the other hand, needs only a smaller voltage as the frequency increases.

Example 3.3

A capacitor draws a current of 5 mA from 200 V, 50 Hz AC supply. What current does it draw from 40 V, 400 Hz supply?

Solution

As mentioned earlier, all values relating to voltages and currents in AC circuit are to be taken as RMS values unless specifically stipulated otherwise, we have

$$I = \omega\, C\, V$$

for a given C,

$$\frac{I_2}{I_1} = \frac{\omega_2}{\omega_1} \cdot \frac{V_2}{V_1} = \frac{400}{50} \times \frac{40}{200} = 1.6$$

or $$I_2 = 1.6\, I_1 = 1.6 \times 5 \text{ mA} = 8 \text{ mA}$$

Hence, the current with 40 V, 400 Hz supply = 8 mA.

SAQ 2

(a) Fill in the blanks :

In a capacitor the current . . . . the voltage by . . . . degrees, while in a . . . . it is in phase with voltage. For a given applied voltage an inductor permits a . . . . current as the frequency is raised.

(b) Find the inductance of an inductor which draws a current of 1.1A when connected to 230 V, 50 Hz voltage. What current will it draw if the supply voltage is changed to 150V, 25 Hz?

3.3.2 Concept of Impedance and Admittance

We note from Equations that the ratio of $\bar{V}$ and $\bar{I}$ each of the three cases considered is a constant which is a function of only the element value and frequency and is independent of the value of the applied voltage or current. These relations are reminiscent of Ohm's law, except that now the quantities involved are complex constants. The differentiation in time domain is equivalent to multiplication by $j\omega$ in the phasor domain and integration in time domain is equivalent to division by $j\omega$ in the phasor domain. Thus if $i(t)$ transforms into $\bar{I}$, then di/dt transforms into $j\omega\bar{I}$ and $\int idt$ transforms into $\bar{I}/j\omega$. We shall now develop this theme for a more general situation. To this end, consider a 2-terminal network N comprising linear circuit elements and let v and i be its terminal voltage and current when connected in an AC circuit, as shown in Figure 3.9.

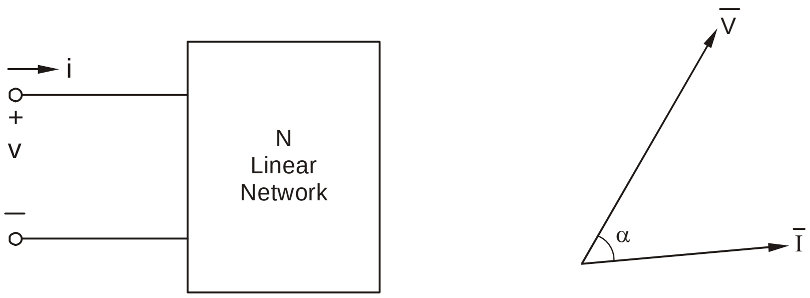


Figure 3.9 : A General Linear 2-terminal Network in an AC Circuit

The relation between v and i would, in general, be an involved differential equation with time as the independent variable. But under sinusoidal regime with an angular frequency ω, the phasors $\bar{V}$ and $\bar{I}$ would have a proportionality relationship independent of time. This proportionality constant is termed the **impedance** $\bar{Z}$ of the network N. Thus

$$\bar{Z} = \frac{\bar{V}}{\bar{I}} = Z \angle \alpha \qquad \ldots (3.9)$$

The following are the characteristics of $\bar{Z}$.

- $\bar{Z}$ is a function only of ω and the values of elements in N. It is independent of time and the value of $\bar{V}$ or $\bar{I}$. You will learn later how to compute $\bar{Z}$ for any given N.

- $\bar{Z}$ has the same dimensions as resistance and is measured in Ω.
- From the definition of $\bar{Z}$, we have

$$\bar{V} = \bar{Z}\,\bar{I} \qquad \text{. . . (3.10)}$$

which plays the same role in AC circuits as Ohm's law in DC circuits.

- The magnitude Z for the impedance is the ratio of the effective values of voltage and current. Sometimes, Z itself is referred to as the impedance instead of $\bar{Z}$.
- α is called the angle of the impedance $\bar{Z}$ and denotes the phase angle by which $\bar{V}$ leads $\bar{I}$.
- In rectangular coordinate form, $\bar{Z}$ can be expressed as

$$\bar{Z} = R + j\,X\,,$$

where the real and imaginary components R and X are respectively called the effective resistance and reactance of N, the latter may, in general, be positive or negative, but for a network N comprising no active elements (e.g., dependent sources), R is always non-negative

- $\bar{Z}$ is to be viewed purely as a complex number and cannot be associated with any sinusoidal signal as its phasor.

Note : We should distinguish between a physical element and its circuit parameter. But expressions like a resistor having a resistance of 20 Ω is connected in parallel with a capacitor of 1 μF capacitance are not only inconvenient but may even sound pedantic. Hence, you often find in literature an element itself being referred to by its circuit parameter – resistance, inductance or capacitance. Thus, the previous expression would usually be rewritten "a 20 Ω resistance is connected in parallel with a 1 μF capacitance". We may call the two-terminal network N itself as the impedance $\bar{Z}$.

As a dual concept to impedance, we define the admittance $\bar{Y}$ of the network N as

$$\bar{Y} = \frac{\bar{I}}{\bar{V}} = Y \angle \beta \qquad \text{. . . (3.11)}$$

Obviously $\bar{Y}$ is reciprocal to $\bar{Z}$, is measured in Siemens and enables determination of current for a given applied voltage by

$$\bar{I} = \bar{Y}\,\bar{V} \qquad \text{. . . (3.12)}$$

also, $Y = Z^{-1}$ and $\beta = -\alpha$.

In rectangular coordinate form, $\bar{Y}$ may be expressed as

$$\bar{Y} = G + jB,$$

where G and B are referred to as the effective conductance and susceptance of N. Let us now discuss the behaviour of single elements.

From Equation and the definition of $\bar{Z}$, we deduce the following expressions for the impedance.

Element	Resistance (*R*)	Inductance (*L*)	Capacitance (*C*)
Impedance $\bar{Z}$	R	$j\omega L = jX_L$	$1/j\omega C = -j/\omega C = -jX_C$

The impedance of a resistor is purely real, equal to its resistance R and has no imaginary component. It is in anticipation of this that we have used the symbol R for the real part of $\bar{Z}$. On the other hand, inductors and capacitors have purely imaginary impedances, inductive reactance X_L being positive and capacitive reactance $(-X_C)$ being negative.

How does one calculate the impedance of a network with several elements? To obtain a clue, let us consider a series combination of several sub-networks with impedances $\bar{Z}_1, \bar{Z}_2, \ldots, \bar{Z}_n$ as shown in Figure 3.10.

When an AC current having $\bar{I}$ for its phasor passes through the series combination, we have

$$\bar{V}_1 = \bar{Z}_1\,\bar{I}, \bar{V}_2 = \bar{Z}_2\,\bar{I}, \ldots, \bar{V}_n = \bar{Z}_n\,\bar{I}$$

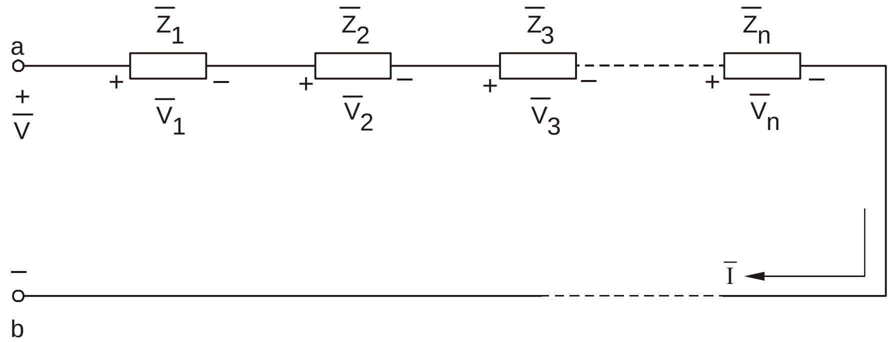


Figure 3.10 : Series Combination of Impedances

Using KVL in phasor form, the terminal voltage of the combination is

$$\bar{V} = \bar{V}_1 + \bar{V}_2 + \ldots + \bar{V}_n = \bar{Z}_1\,\bar{I} + \ldots + \bar{Z}_n\,\bar{I} = (\bar{Z}_1 + \bar{Z}_2 + \ldots + \bar{Z}_n)\,\bar{I}$$

Thus, the equivalent impedance of the series combination is

$$\bar{Z}_s = \bar{Z}_1 + \bar{Z}_2 + \ldots + \bar{Z}_n \qquad \ldots (3.13)$$

In a similar fashion, it is left to the reader to show using KCL that the equivalent impedance and admittance of the parallel combination shown in Figure 3.11 are given by

$$\frac{1}{Z_p} = \frac{1}{Z_1} + \frac{1}{Z_2} + \ldots + \frac{1}{Z_n} \qquad \ldots (3.14(a))$$

or

$$\bar{Y}_P = \bar{Y}_1 + \bar{Y}_2 + \ldots + \bar{Y}_n \qquad \ldots (3.14(b))$$

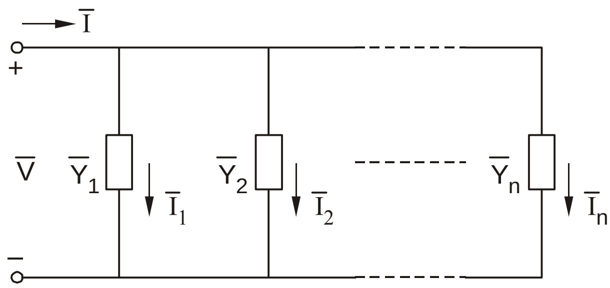


Figure 3.11: Parallel Combination of Admittances

Eqs. (3.13) and (3.14) are similar to rules for combining resistors in series and in parallel. We will encounter a similar pattern in future. All the techniques that we have learnt for DC circuit analysis can be applied to AC circuits as well with certain modifications. The techniques include finding series-parallel equivalents, current and voltage division rules, star-delta conversions, loop circuit and node voltage analyses etc. The departure from DC methods are :

- KVL and KCL relations are expressed in phasor form.
- Impedance $\overline{Z}$ (or admittance $\overline{Y}$) is used in place of resistance (or conductance),
- Terminal relations $\overline{V} = \overline{Z}\ \overline{I}$ and $\overline{I} = \overline{Y}\ \overline{V}$ are used instead of $V = RI$ and $I = GV$ in DC domain.

This is the power and beauty of the phasor concept. Essentially, the same methods and formulations applicable to circuits with constant currents and voltages are made applicable to circuits with currents and voltages varying in time sinusoidally. The price we have to pay for this simplification, namely dealing with complex numbers, is indeed less taxing than the alternative of working in time domain with trigonometric functions and carrying out differentiation and integration operations thereof.

Example 3.4

Find the impedance of the element combinations shown in Figure 3.12, taking the frequency to be 400 Hz.

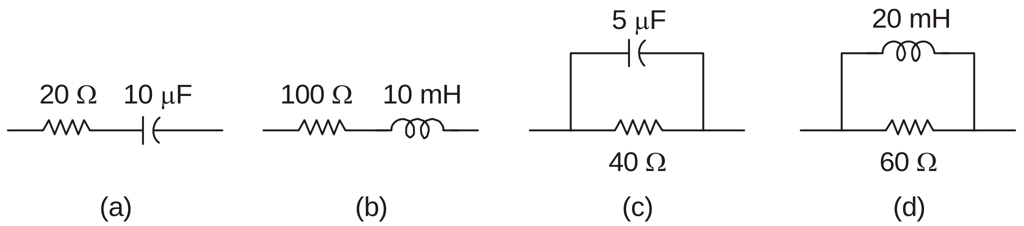


Figure 3.12

Solution

(a) The impedance of the series combination is the sum of the two impedances.

$$\overline{Z}_s = \overline{Z}_1 + \overline{Z}_2 = 20 + \frac{1}{j\ 2\pi \times 400 \times 10 \times 10^{-6}}$$

$$= 20 - j\ 39.8\ \Omega$$

(b) $\overline{Z}_s = \overline{Z}_1 + \overline{Z}_2 = 100 + j\ 2\pi \times 400 \times 100 \times 10^{-3}$

$$= 100 + j\ 251\ \Omega$$

(c) **Alternative I**

$$\overline{Z}_1 = 40\ \Omega;\ \overline{Z}_2 = \frac{1}{j\ 2\pi \times 400 \times 5 \times 10^{-6}} = -j\ 79.6\ \Omega$$

The two impedances being in parallel,

$$\overline{Z}_r = \frac{\overline{Z}_1\,\overline{Z}_2}{\overline{Z}_1 + \overline{Z}_2} = \frac{40(-j\,79.6)}{40 - j79.6} = \frac{3184\ \angle -90^{\circ}}{89.1\angle -63.3^{\circ}}$$

$$= 35.7\angle -26.7^{\circ} = 31.9 - j\ 16.0\ \Omega$$

Alternative II

$$\bar{Y}_1 = \frac{1}{40} = 0.02\ S\,;\ \bar{Y}_2 = j\omega C = j\ 2\pi \times 400 \times 5 \times 10^{-6} = j\ 0.0126\ S.$$

The two admittances being in parallel,

$$\bar{Y}_P = \bar{Y}_1 + \bar{Y}_2 = 0.025 + j\ 0.0126 = 0.028\ \angle\ 26.7^o$$

Then

$$\bar{Z}_P = (\bar{Y}_P)^{-1} = (0.028\angle 26.7^o)^{-1} = 35.7\ \angle -26.7^o = 31.9 - j16.0\Omega$$

(d) $$Z_P = \frac{3018\ \angle 90^o}{78.3\angle\ 40^o} = 38.5\ \angle\ 50^o = 24.7 + j\ 29.5\ \Omega$$

Example 3.5

When the element combination in Figure 3.12(a) is connected to 200 V, 400 Hz supply, what would be the current drawn? What would be the voltage across the resistance and capacitance?

Solution

As the phase of the supply voltage is not specified, we need to compute only the RMS value of the current.

$$Z = (20^2 + 39.8^2)^{1/2} = 44.54\ \Omega$$

$$I = V/Z = 200 / 44.54 = 4.49\ A$$

$$V_R = IR = 4.49 \times 20 = 89.8\ V$$

$$V_C = (I / \omega C) = 4.49 \times 39.8 = 178.7V$$

Example 3.6

If a voltage of 200 V, 400 Hz is applied across the element combination in Figure 3.12(c), find the total current taken by the combination.

Solution

Since the phase of the supply voltage has not been specified, let us take it as 0^o. That is, we are taking the applied voltage phasor as the so-called **reference phasor.**

$$\bar{V} = 200\ \angle 0^o$$

Now $$\bar{I}_R = 200\ \angle 0^o\ /\ R = 200/40 = 5$$

and $$\bar{I}_C = \bar{V}\ j\omega C = (200\ \angle\ 0^o)\ (0.0126\ \angle 90^o) = j\ 2.52$$

$$\bar{I} = \bar{I}_R + \bar{I}_C = 5 + j\ 2.52 \Rightarrow I = (5^2 + 2.52^2)^{1/2} = 5.6\,A$$

The phasor diagram showing the relative positions of the different phasors is given in Figure 3.13.

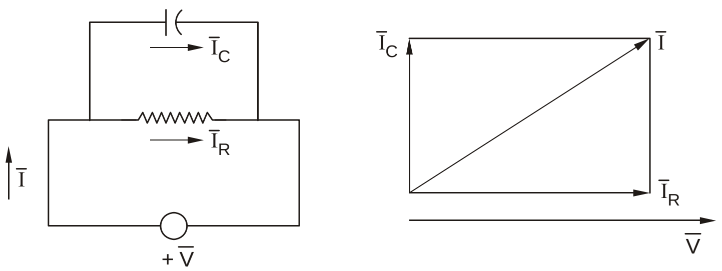


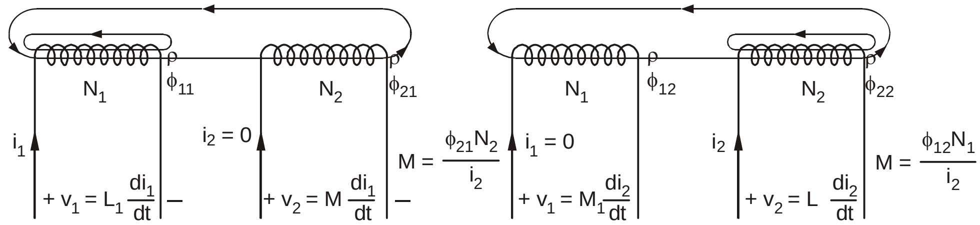
Figure 3.13

If $\overline{V}$ has any other phase angle say α^o it only means that the entire figure will rotate by α^o. There will be no change in the magnitudes of the voltages and currents and in their phase differences. This is an important point to note. *When no contrary information is specified, we are at liberty to arbitrarily assume one convenient quantity as the reference phasor*.

3.3.3 Mutual Inductance

The property of an inductance coil is to set up a magnetic field when carrying a current. Some of the magnetic flux lines so set up may also link with another coil (inductor) in its proximity. In such an event, the two coils are said to be magnetically coupled. When the current in the first coil changes, not only do its self flux linkages change but also the flux linkages (called mutual flux linkages) produced by it in the second coil. Consequently, there is a voltage induced in the second coil due to a change of current in the first coil. The action described above is reciprocal in that a change of current in the second coil would also induce a proportionate voltage in the first coil. **The induced voltage in each coil is produced due to per unit rate of change of current in the other and is defined to be the mutual inductance *M* between the coils.** *M* is another passive circuit parameter like *R*, *L* and *C* that we discussed in Unit 1 and arise whenever two inductors are coupled magnetically. It is measured in the same units as *L* viz., henrys. The principle of mutually induced voltage forms the basis of transformer action.

Figure 3.14 shows two coils of N_1 and N_2 turns having inductances L_1 and L_2 (called self inductances) coupled through a mutual inductance M. The self and mutually induced voltages when either coil carries a current are illustrated therein. ϕ_2 represents that part of the flux produced by coil 1 which is linked with coil 2 and vice-versa.



(a) When Coil 1 Only is Carrying Current **(b) When Coil 2 Only is Carrying Current**

Figure 3.14 : Self and Mutually Induced Voltage

In the event when the two carry currents i_1 and i_2 simultaneously as in Figures 3.14(a) and (b) seen together each coil has both self and mutually induced voltage components. The terminal voltages of the coils are then given by

$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt} \qquad \ldots (3.15)$$

SAQ 3

(a) "If a sinusoidal voltage $v(t)$ applied to a circuit element delivers a current $i(t)$, then the impedance of the element is $v(t) / i(t)$". Comment on the above statement.

(b) Fill up the following table

Element	Resistance (*R*)	Inductance (*L*)	Capacitance (*C*)
Admittance $\overline{Y}$			

SAQ 4

(a) In Example 3.5, V_R and V_C do not add up to the magnitude of the supply voltage. Is this not a violation of KVL?

(b) If a current $i\,(t) = \sqrt{2}\,\sin\,(400\,t + 30^{o})$ mA passes through the element combination in Figure 3.12(b), find an expression for the voltage across the combination.

SAQ 5

(a) A resistance of 20 Ω and an impedance 40 + *j* 60Ω are connected across an AC supply source as shown in Figure 3.15. If the voltage across the resistor is 50 V, find the source voltage. Draw a phasor diagram.

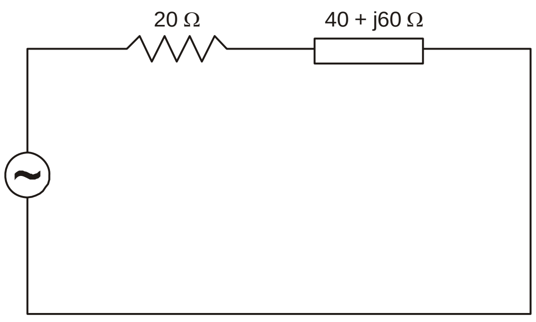


Figure 3.15

(b) A fluorescent lamp may be considered to be a pure resistance. A 40 W lamp is designed to operate at a voltage of 130 V at 50 Hz. This lamp, in series with a choke coil (which may be considered a pure inductor), is connected across 220 V, 50 Hz supply. Calculate the required value of inductance of the choke coil.

3.4 CONCEPTS RELATING TO POWER

Since AC circuits have periodically varying voltages and currents, the power delivered to an element or a section of a circuit is also a periodically varying quantity $p(t)$. A meaningful measure of power in such situations is the average power over a cycle. The term power when used in the context of an AC circuit without any additional qualification means this average value and is denoted by the symbol P. In a DC circuit, the power delivered to a 2-terminal network is equal to VI, the product of the terminal voltage and current. In what follows, we develop the corresponding formula applicable to AC circuits.

3.4.1 Power, Apparent Power and Power Factor

Consider the circuit given in Figure 3.16, where sinusoidal voltage $v = \sqrt{2}\ V \sin(\omega t + \theta)$ supplies power to a 2-terminal network N having an equivalent impedance $\bar{Z} = Z\angle\alpha = R + jX$. In this context, N is also referred to as a load and $\bar{Z}$ as the load impedance.

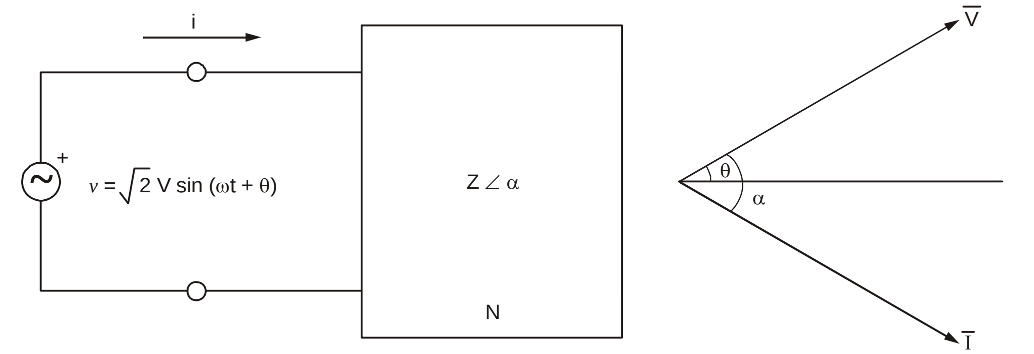


Figure 3.16

The phasor $\bar{I}$ of the resultant current I in this circuit is

$$\bar{I} = \left[\frac{V\angle\theta}{Z\angle\alpha}\right] = \left(\frac{V}{Z}\right)\angle(\theta - \alpha)$$

Thus $$I = \sqrt{2}\left(\frac{V}{Z}\right)\sin(\omega t + \theta - \alpha) = \sqrt{2}\ I\sin(\omega t + \theta - \alpha)$$

The instantaneous power supplied by the source to the load is

$$P(t) = \upsilon i = \sqrt{2}\ V\sin(\omega t + \theta)\ .\ \sqrt{2}\ I\sin(\omega t + \theta - \alpha)$$

$$= 2V\,I\sin(\omega t + \theta)\sin(\omega t + \theta - \alpha)$$

$$= V\,I\,[\cos\alpha - \cos(2\omega t + 2\theta - \alpha)]$$

The variation of $p(t)$ is shown in Figure 3.17.

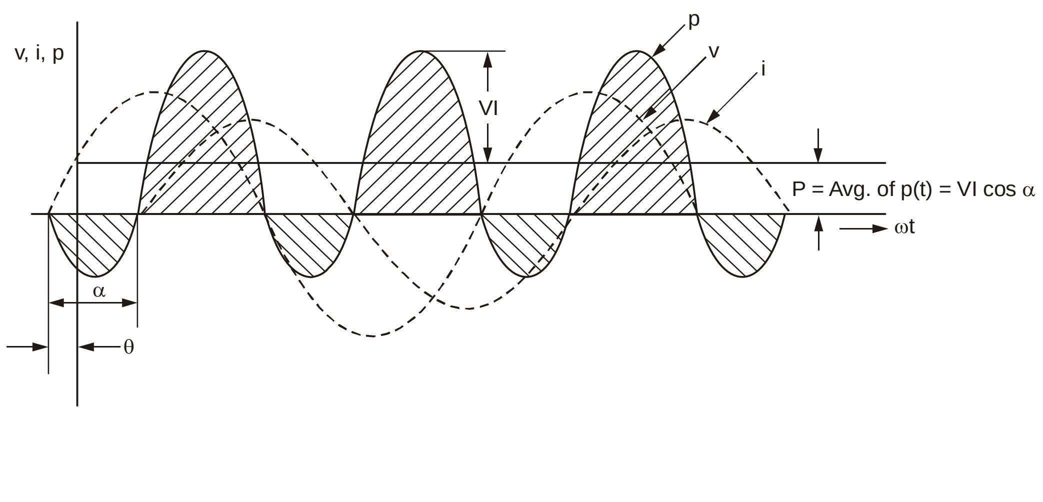


Figure 3.17 : Instantaneous Power $p(t)$ Delivered to N

It is seen from Figure 3.17 (hatched portion) that $p(t)$ consists of a constant component $V I \cos \alpha$ second component of peak value $V I$ varying sinusoidally at a frequency 2ω. As the average of the second component over a period of the input voltage or current is zero, the average of $p(t)$ is equal to the first component itself. Thus,

$$P = V I \cos \alpha \text{ Watts} \qquad \ldots (3.16)$$

The above formula is of great significance. It indicates that the power received by a load is not merely the product of the RMS values of its terminal voltage and current but includes an additional multiplicative factor $\cos \alpha$, called the power factor of the load. Power factor is the cosine of the impedance angle α and hence is a property of the concerned load. Formally, power factor (p.f.) may be defined as

$$\text{p.f.} = \frac{\text{Power deliver to load}}{\text{Product of effective values of terminal voltage and current of the load}}$$

$$\text{p.f.} = \frac{P}{VI} \qquad \ldots (3.17)$$

The power factor is said to be of the *leading* type if $\overline{I}$ leads $\overline{V}$ (i.e., $X < 0$) and of the lagging type if $\overline{I}$ lags $\overline{V}$ (i.e. $X > 0$).

In contrast to power P, the product VI is termed apparent power S and is indicated in units of volt-amperes (abbreviated VA). Though dimensionally 1 VA equals 1 Watt, two different names of the unit are adopted to emphasize the distinct between apparent power S and P. Apparent power is an important parameter in the specifications of electrical equipment, as the size and cost of many electrical machines depend on their VA rating rather than wattage rating. For instance, a 500 kVA distribution transformer is rated in terms of its ability to handle S up to 500 kVA level which determines maximum current at rated voltage rather than power P it can deliver to a load which in dependent on load power factor.

Table 3.1 gives the particular forms of the relations discussed above for special categories of loads. Note that a pure inductor and capacitor have zero p. f. since they are only energy storage elements and not energy dissipating elements.

Table 3.1

Load	**Z (Ω)**	**α**	**Apparent Power (S) (VA)**	**Power Factor**	**Power (P) (W)**
Resistor	R	0^o	$VI = I^2 R = V^2/R$	1.0 (unity)	$VI = I^2 R = (V^2/R)$
Inductor	$j \omega L$	90^o	$VI = I^2 \omega L = (V^2/\omega L)$	Zero (lagging)	Zero
Capacitor	$1/j \omega C$	-90^o	$VI = I^2 / \omega C = (V^2 \omega C)$	Zero (leading)	Zero

3.5 THREE-PHASE CIRCUITS

A sinusoidal voltage source with 2 terminals having a single voltage output is termed a single-phase source. Circuits incorporating such sources are called single phase (1-phase) circuits and formed the subject of our study uptil now in this unit. In contrast, a poly-phase system contains sources each of which has several voltage outputs with a fixed phase difference between them. The three-

phase
(3-phase) system is the most common example of a poly-phase system.

The generation and transmission of electrical energy and its utilization in bulk form is effected through 3-phase systems. You will learn about the precise nature of a 3-phase system and the advantages it provides relative to a single-phase system.

3.5.1 Nature of a 3-Phase System

A single phase AC generator consists of a rotating magnet driven by a prime mover and a winding embedded in the stationary part of the machine called the stator. Figure 3.18 shows an elementary form of the generator with a single turn coil AA' on the stator. As the magnet rotates, the flux lines linking with the coil undergo a periodic variation and hence induce a periodic emf in the latter. The frequency of this emf is fixed by the speed of rotation of the magnet. Special steps are taken in the design and construction of the machine to make the waveform of the induced voltage sinusoidal. Thus, *the coil functions as a single-phase AC voltage source* with terminals A_1 and A_2, to which a load may be connected.

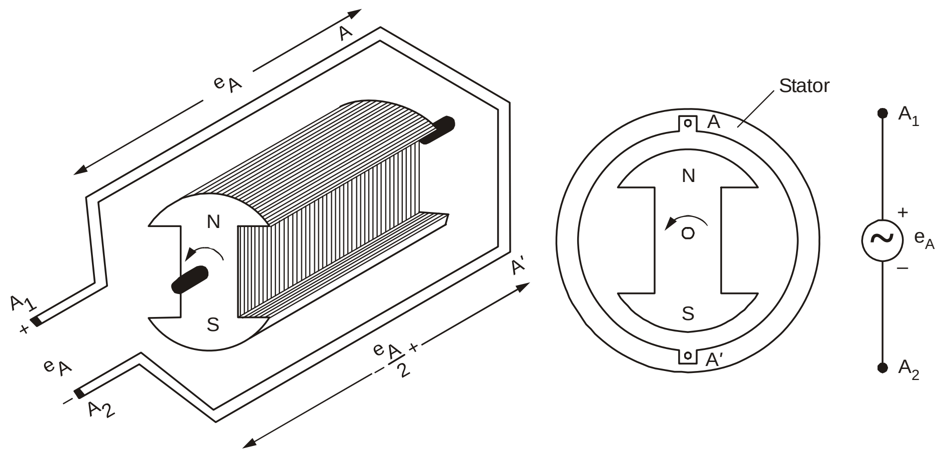


Figure 3.18 : Elementary Single Phase Generator and its Circuit Representation

Figure 3.19 illustrates the construction of an elementary 3-phase generator. Here, we have 3 identical coils AA', BB', CC' placed on the stator with a displacement of 120° from one another. The three emfs e_A, e_B, e_C generated in the coils, therefore, have the same RMS value but have a phase difference of 120° from one another as shown in Figures 3.20(a) and (b).

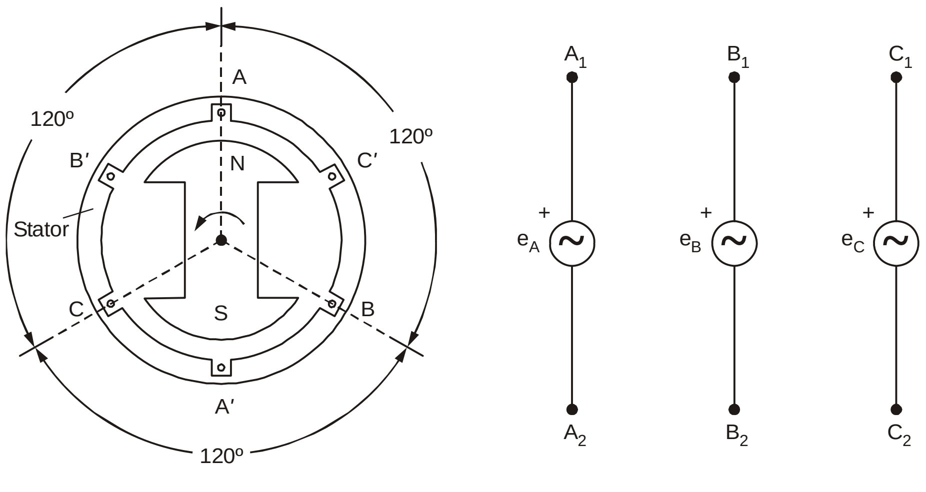
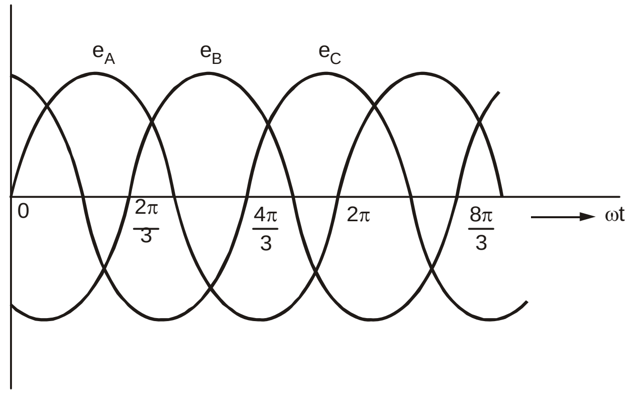
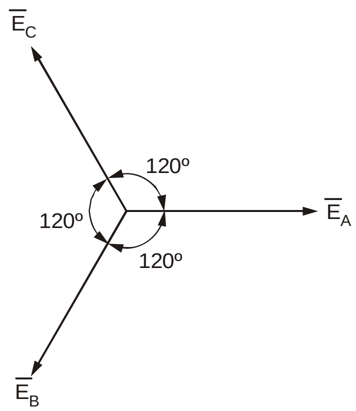


Figure 3.19 : Elementary 3-phase Generator and its Circuit Representation



(a) Waveforms



(b) Phasors

Figure 3.20 : Voltages Produced in a 3-phase Generator

3-phase generator can, therefore, be viewed as a composite unit comprising 3-single-phase voltage sources with a fixed phase difference of 120^o between any two of them. In practice, it is rare for a 3-phase generator to have all the six terminals brought out. The three coils are connected either in star or delta and only 3 or 4 terminals are brought out, as we shall see later.

A 3-phase system contains 3-phase sources besides 3-phase load impedances and feeder lines interconnecting them.

The three individual sections which constitute a 3-phase source besides 3-phase arrangement are referred to as Phase A, Phase B, and Phase C respectively. Another common practice is to label them as R (red), Y (yellow) and B (blue) phases. We shall follow the former convention in our work.

3.5.2 Merits of a 3-Phase System

Let us now look at the advantages provided by 3-phase systems relative to single-phase systems.

A 3-phase AC generator utilises the available space on the stator more effectively than 1-phase generator and has 50% more kVA rating for the same physical size. All commercial power stations, therefore, employ 3-phase generators as they cost less than single-phase generators for the same kVA rating.

The cost of electrical transmission and distribution lines used to carry bulk power from generating stations to receiving substations and distribute power from the substations to different load centres depends substantially on the volume of conducting material (usually aluminium) required for constructing these lines. It turns out that a 3-phase arrangement for transmission and distribution requires less conductor material for the lines and is, therefore, less expensive than a 1-phase system for handling the same amount of power at a given system voltage.

In power utilization, a 3-phase motor develops essentially a constant output torque whereas a single-phase motor can inherently provide only a pulsating torque. Not only is the three-phase motor consequently quieter in operation but it also provides better starting characteristics, higher efficiency of power conversion from electrical to mechanical form and better p. f. It is also, in general, cheaper than a 1-phase motor of the same power rating.

The foregoing economic and technical advantages have led to the universal adoption of the 3-phase system for the generation, transmission and utilization of bulk power. Small electrical loads, of typically less than 3 kW power rating, are, however, designed and built for single-phase operation. These include electric lights, fans, heaters and small motors needed for various domestic appliances, machines tools, pump sets and the like. The benefits that may stem from 3-phase operation of these loads are not commensurate with the additional cost of manufacturing them to be suitable for 3-phase use and of running the additional cost of manufacturing them to be suitable for 3-phase use and of running 3-phase lines to each individual item. In practice, these small loads are fed from single-phase supplies available from a 3-phase distribution system.

SAQ 6

(a) Distinguish between a 3-phase generator and a single-phase generator.

(b) Fill up the blanks :

A 3-phase generator has more . . . . than 1-phase generator of the same physical size. A 3-phase transmission line employs less . . . . than a 1-phase transmission line for the same power transmitted and the same system voltage. The torque developed by a 3-phase motor is . . . . while the torque developed by a 1-phase motor is . . . .

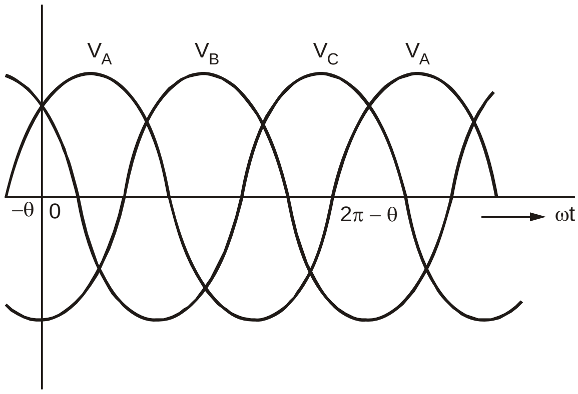
3.5.3 Characteristics of a 3-phase System

After having been acquainted with the nature of 3-phase systems and their advantages, you will study in this section the characteristics of 3-phase sources, loads and associated systems in greater detail.

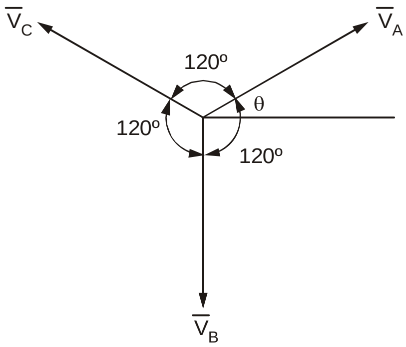
Balanced Sets of Voltages and Currents

At any section of the circuit representing a 3-phase system, there exist three voltages and three currents which constitute the variables of interest. **Three such voltages or currents are said to form a balanced set if they have**

equal effective values and if the phase difference between any two is 120°. A balanced set of three voltages $\overline{V}_A, \overline{V}_B$ and $\overline{V}_C$ is depicted in Figure 3.21.



(a) Waveforms



(b) Phasors

Figure 3.21 : A Balanced Set of Three Voltages with ABC Phase Sequence

Note that similar events in the three waveforms (e.g., positive peak values) occur in the sequence *ABCABC* . . . for this reason, the three voltages are said to have the *ABC* **phase sequence.** (We could as well have called it the *BCA* or *CAB* phase sequence but, by convention, choose the natural alphabetical order). Referring to the phasor diagram in Figure 3.21(b), if one were to imagine the three phasors to rotate in the anticlockwise direction, they sweep past a stationary point in the sequence *ABC*. This alternative way of judging the phase sequence from a phasor diagram would be useful when the waveforms are not explicitly plotted.

There exists a second possible phase sequence for a balanced voltage set, as depicted in Figure 3.22. Here, similar events in the three signals occur in the sequence *ACBACB* . . . This sequence is called the ***ACB* phase sequence** (it could as well have been called *CBA* or *BAC* phase sequence). Note that the three related phasors now sweep past a stationary observer in the order *ACB*.

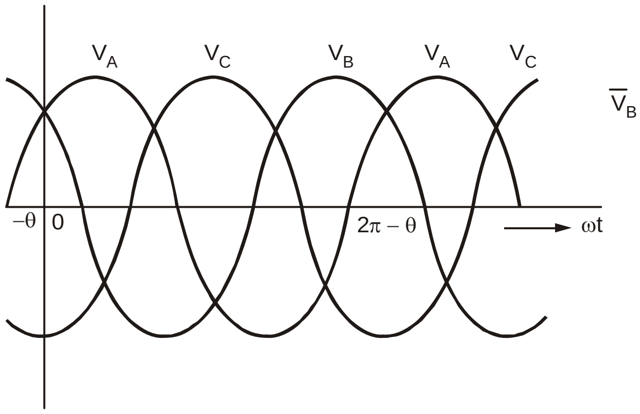
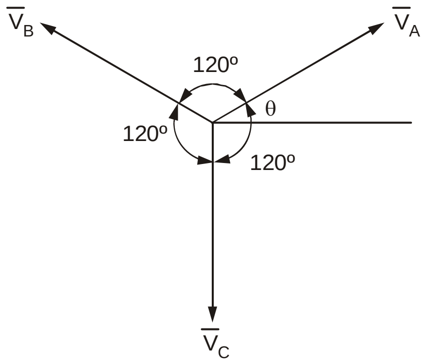


Figure 3.22(a) : Waveforms



(b) Phasors

Figure 3.22 : A Balanced Set of Three Voltages with ACB Phase Sequence

In normal practice, the individual phases are so labelled as to correspond to the *ABC* phase sequence. We shall assume this to be the phase sequence in all our further work unless the contrary is specifically indicated. What has been discussed above with respect to a set of balanced voltages holds equally well with respect to a set of 3 currents. A set of balanced 3-phase voltages or currents would then have the following expressions (with *ABC* phase sequence assumed).

$$v_A = \sqrt{2}\, V \sin (\omega t + \theta)$$

$$v_B = \sqrt{2}\, V \sin \left(\omega t + \theta - \frac{2\pi}{3} \right) \qquad \ldots (3.18)$$

$$v_C = \sqrt{2}\, V \sin \left(\omega t + \theta - \frac{2\pi}{3} \right)$$

$$i_A = \sqrt{2}\, V \sin (\omega t + \beta)$$

$$i_B = \sqrt{2}\, V \sin \left(\omega t + \beta - \frac{2\pi}{3} \right) \qquad \ldots (3.19)$$

$$i_C = \sqrt{2}\, V \sin \left(\omega t + \beta - \frac{2\pi}{3} \right)$$

The corresponding phasors would be

$$\overline{V}_A = V \angle \theta ; \overline{V}_B = V \ (\theta - 2\pi/3); \ \ \overline{V}_C = V \angle \ (\theta + 2\pi/3) \qquad \ldots (3.20)$$

$$\overline{I}_A = I \angle \beta ; \overline{I}_B = I \, (\beta - 2\pi/3); \quad \overline{I}_C = I \angle (\beta + 2\pi/3) \qquad \text{. . . (3.21)}$$

The following properties of balanced voltages (or currents) are noteworthy :

With *ABC* phase sequence, v_A leads v_B by 120°, v_B leads v_C by 120° and v_C leads v_A by 120°. With ACB phase sequence, v_A leads v_C by 120°, v_C leads v_B by 120° and v_B leads v_A by 120°.

If the set of voltages (or currents) is known to be balanced and the phase sequence is fixed, the data pertaining to one voltage or current would suffice to deduce the other two. For example, if it is known that $\overline{V}_B = 100 \angle 30^o$ and that the phase sequence is ABC, it follows that $\overline{V}_A = 100 \angle 150^o$ and $\overline{V}_C = 100 \angle -90^o$.

The sum of three balanced quantities is identically zero in time domain.

$$v_A + v_B + v_C = 0 \qquad \text{. . . (3.22)}$$

$$i_A + i_B + i_C = 0 \qquad \text{. . . (3.23)}$$

The above results can be proved through manipulation of the trigonometric expressions in Eqs. (3.18) and (3.19). They can also be verified by observing that the ordinates of the three pertinent waveforms like those in Figure 3.22(a) add up to zero at every instant of time.

The equivalent results in phasor domain are

$$\overline{V}_A + \overline{V}_B + \overline{V}_C = 0 \qquad \text{. . . (3.24)}$$

$$\overline{I}_A + \overline{I}_B + \overline{I}_C = 0 \qquad \text{. . . (3.25)}$$

To check the validity of Eq. (3.24) refer to Figure 3.23(a) since $\overline{V}_A$ and $\overline{V}_B$ have equal magnitudes and are 120° apart, their resultant $\overline{V}_R = \overline{V}_A + \overline{V}_B$ is at 60° from $\overline{V}_A$ and has same magnitude. $\overline{V}_A + \overline{V}_B$ is, therefore, equal and opposite to $\overline{V}_C$. Hence $\overline{V}_R + \overline{V}_C = \overline{V}_A + \overline{V}_B + \overline{V}_C = 0$. If drawn from end to end, the three directed line segments $\overline{V}_A$, $\overline{V}_A$ and $\overline{V}_C$ add up to zero (Figure 3.23(b)).

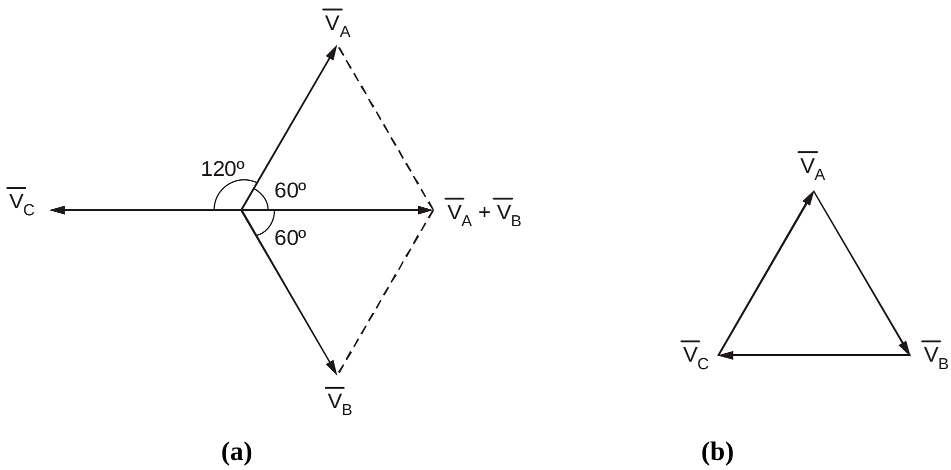


Figure 3.23 : Balanced Set of 2-Phase Voltages Add up to Zero

Finally, you should note that three voltages/currents are unbalanced if their effective values are not equal or their phase differences are not 120° or both. Figure 3.24 gives examples of sets of unbalanced voltages.

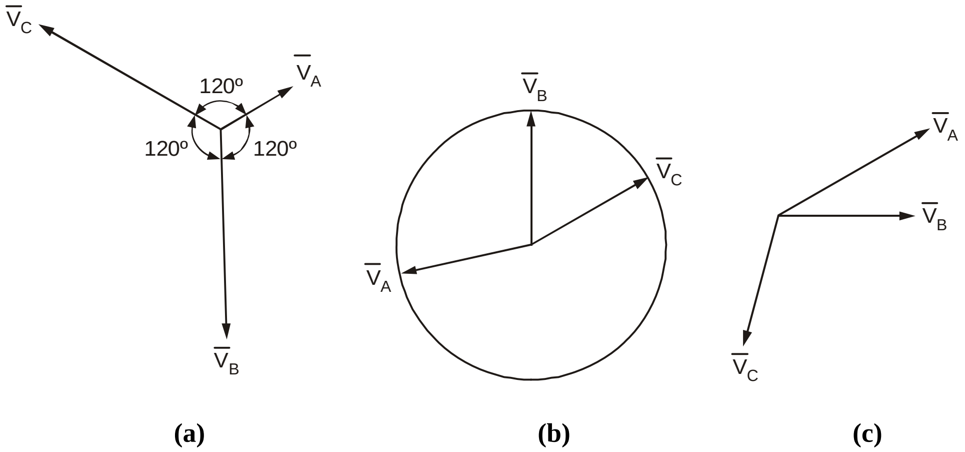


Figure 3.24 : Examples of Sets of Unbalanced Voltages

Example 3.7

At a certain section in a 3-phase circuit, $\bar{V}_B = 120 \angle 60^o$ and $\bar{I}_C = 4 \angle 180^o$. If the voltages and currents are balanced and the phase sequence is *ABC*, deduce $\bar{V}_A$, $\bar{V}_B$, $\bar{I}_A$ and $\bar{I}_B$.

Solution

With *ABC* sequence, $\bar{V}_B$ leads $\bar{V}_C$ by 120°. Thus,

$$\bar{V}_A = 120 \angle 180^o ; \bar{V}_C = 120 \angle -60^o$$

$\bar{I}_C$ leads $\bar{I}_A$ by 120° and lags $\bar{I}_B$ by 120°

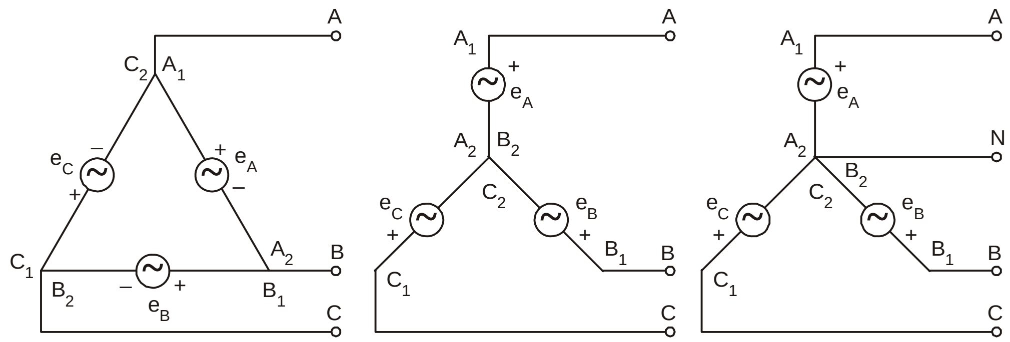
$$\Rightarrow \bar{I}_A = 4 \angle 60^o ; \bar{I}_B = 4 \angle -60^o .$$

3.5.4 Star and Delta Connections

You would recall that a 3-phase generator essentially consists of 3 single-phase sources, having output voltages say, e_A, e_B and e_C. **A balanced 3-phase source is** one in which these three voltages form a balanced set. All commercial 3-phase generators are built in this manner. The three single-phase sources are connected internally either in *delta* or in *star* as shown in Figure 3.25 and terminals brought out for connection to external loads.

Note the symmetrical way of connecting the three single-phase sources. In the delta connection, A_2 is connected to B_1, B_2 is connected to C_1 and C_2 is connected to A_1. In the star connection, A_2, B_2 and C_2 are joined together. Such an orderly method of connections is needed to ensure the balanced condition of the voltages available between the terminals *A*, *B* and *C* of the 3-phase generator.

In the delta connection, the effective emf of the three series connected sources around the closed circuit is $e_A + e_B + e_C$. If the three voltages do not add up to zero there would be a large circulating current in the delta even with no load connected to terminals *A*, *B*, *C* and this is clearly an undesirable situation. However, for a balanced source this contingency does not arise as $e_A + e_B + e_C = 0$. It is this fact which makes the delta connection of a 3-phase source feasible.

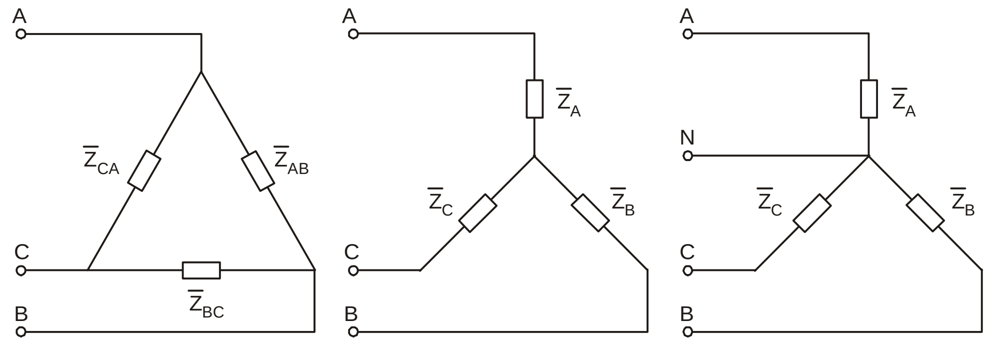


(a) (b) (c)

Figure 3.25 : Source Connections (a) Delta Connection; (b) 3-wire Star Connection; and (c) 4-wire Star Connection

In the star connection, the common terminal of the 3 sources (star point) is called the **neutral point**. Here, there exist two possible arrangements. Where a separate terminal is not provided for the neutral point as in Figure 3.25(b), the generator forms part of what is known as a **3-wire 3-phase system**. On the other hand, the arrangement shown in Figure 3.25(c) permits connection of the generator in a **4-wire 3-phase system**. The terminals *A*, *B*, *C* are called the line terminals and *N* is called the neutral terminal.

A 3-phase load generally comprises three impedances in a configuration suitable for connection in a 3-phase circuit. Similar to the connections in a 3-phase generator, here, also we have 3 possible connections as shown in Figure 3.26. Notice that the configuration in Figure 3.26 is suitable for connection only in a 4-wire 3-phase system.



(a) (b) (c)

Figure 3.26 : 3-phase Connections (a) Delta; (b) 3-wire Star; and (c) 4-wire Star

A 3-phase load is balanced if the three complex impedances are equal, i.e.

$$\overline{Z}_A = \overline{Z}_B = \overline{Z}_C \text{ for a star-connected load}$$

and $$\overline{Z}_{AB} = \overline{Z}_{BC} = \overline{Z}_{CA} \text{ for a delta-connected load}$$

Not only the magnitudes but also the angles of the impedances should be equal for a 3-phase load to be balanced. We can, therefore, use a common symbol $\overline{Z}_Y$ for the three impedances in star and $\overline{Z}_A$ for the three impedances in delta.

For balanced loads, the formulas for star-delta, and vice-versa, conversion is

$$\overline{Z}_Y = \left(\frac{1}{3}\right) \overline{Z}_\Delta \quad \text{. . . (3.28)}$$

A 3-phase circuit is formed through the interconnection of 3-phase sources and 3-phase loads. If all the sources are balanced and have the same phase sequence and all the loads are also balanced, then the 3-phase circuit is said to be balanced. A characteristic of a balanced 3-phase circuit is that the voltages and currents at any arbitrary location are balanced. In our study, we shall be concerned only with *balanced systems.*

SAQ 7

(a) Fill u

The ________________________ [1] connection of sources/impedances is suitable either for 3-wire or for 4-wire three-phase systems but the ____________________ [2] connection of sources/impedances is suitable only for ________________________ [3] three-phase systems.

(b) State if the following assertions are true or false.

(i) Three impedances $\bar{Z}_A$, $\bar{Z}_B$ and $\bar{Z}_C$ form a balanced 3-phase load if $\bar{Z}_A + \bar{Z}_B + \bar{Z}_C = 0$.

(ii) The neutral point is not available in a 3-phase delta-connected source.

Example 3.8

A balanced 3-phase load is formed by three impedances of 60 + *j*90 ohms each, connected in delta. If this load is equivalent to a star-connected load having $\bar{Z}_Y$ in each leg of the star, calculate $\bar{Z}_Y$.

Solution

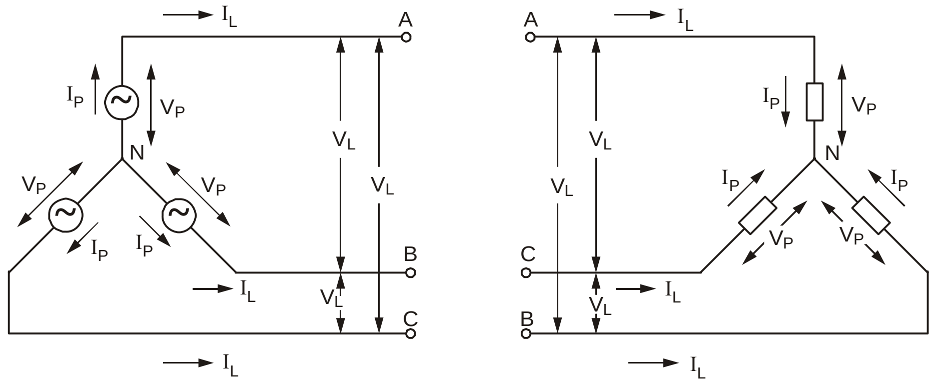
$$\bar{Z}_Y = \frac{1}{3}\,\bar{Z}_\Delta = \frac{1}{3}\,(60 + j90) = 20 + j30\ \Omega$$

Relations between Line and Phase Quantities

In 3-phase circuits, one distinguishes between *line voltages* and *currents* on one hand and *phase voltages and currents* on the other. Phase quantities are the internal voltages or currents associated with the single phase sources constituting a 3-phase source or the three impedances constituting the 3-phase load. Line quantities, on the other hand, are those which can be measured at the 3-external terminals. These are the voltages between and the current in the external supply lines connected to the terminals.

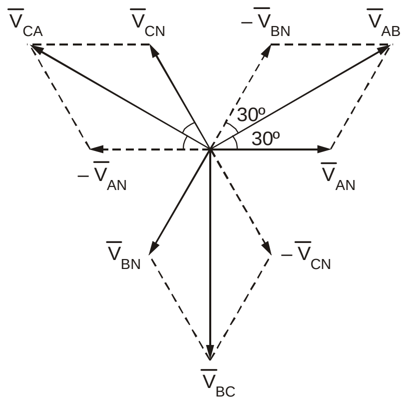
Star Connection

The line and phase quantities for star connected balanced 3-phase system are shown in Figure 3.27.



(a) **(b)**

Figure 3.27



(c)

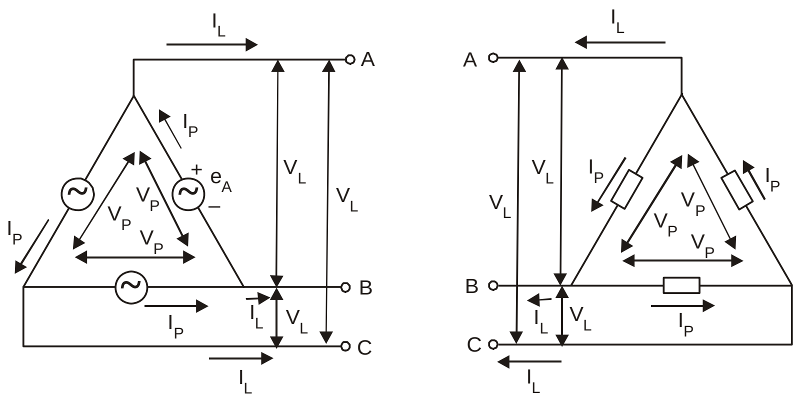
Figure 3.27 : Phase and Line Quantities in a Star-connected (a) Source; (b) Load; and (c) Deduction of Line Voltages from Phase Voltages in a Balanced Star Configuration

A balanced star configuration has the following important characteristics :

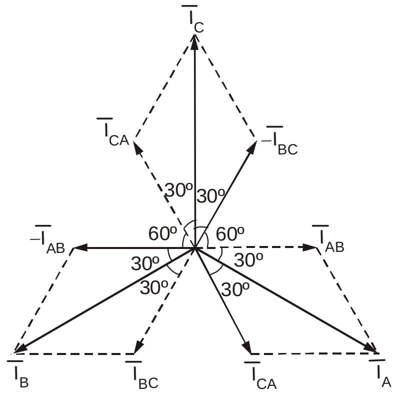
- Phase currents and line currents have the same effective value ($I_L = I_P$).
- Line voltages have $\sqrt{3}$ times the effective value of phase voltage ($V_L = \sqrt{3}\ V_P$).
- The line voltages and phase voltages have the same phase sequence and the set of line voltages phasors is displaced by 30^o from the set of phase voltage phasors.

Delta Connection

Line and phase quantities for delta connected source and load are shown in Figure 3.28.



(a) **(b)**



(c)

Figure 3.28 : Phase and Line Quantities in a Delta-Connected (a) Source; (b) Load and (c) Deduction of Line Currents from Phase Currents in a Balanced Delta Configuration

A balanced delta configuration has the following important characteristics :

- Phase voltages and line voltages have the same effective value ($V_L = V_P$).
- Line currents have $\sqrt{3}$ times the effective value of phase currents ($I_L = \sqrt{3} I_P$).
- The line currents and phase currents have the same phase sequence and the set of line current phasors is displaced by 30° from the set of phase current phasors.

SAQ 8

(a) State if the following assertions are true or false.

(i) Three currents in a 3-phase system are balanced if their phasors are equal.

(ii) In a set of balanced 3-phase voltages with *ABC* phase sequence $\bar{V}_A$ leads the other two voltages $\bar{V}_B$ and $\bar{V}_C$.

(iii) If $i_A + i_B + i_C = 0$, then i_A, i_B and i_C form a balanced set of 3-phase currents.

(b) Taking v_A and i_A as in Eqs. (3.18) and (3.19), write the expressions for the other quantities if phase sequence is *ACB.*

(c) Taking following to be balanced sets of voltages/currents with ABC phase sequence, fill the blanks

(i) $v_A = \sqrt{2} \times \ldots \sin(\omega t + \ldots)$;

$v_B = \sqrt{2} \times 100 \sin(\omega t + \ldots)$

$v_C = \sqrt{2} \times \ldots \sin(\omega t + 45^o)$.

(ii) $\bar{I}_A = \ldots \angle \ldots ; \bar{I}_B = \ldots \angle 60^o ; \bar{I}_C = 4 \angle \ldots$

3.6 SUMMARY

In this unit, the main emphasis was on the study of single-phase AC circuits and the concepts of three phase systems.

Sinusoidal voltages and currents called AC voltages and currents play a key role in the theory and practice of electrical engineering. Here, you have learnt how to calculate response of R, L, C elements to sinusoidal excitation and also how to calculate power.

You have also been introduced to the basic features of a 3-phase voltages and currents.

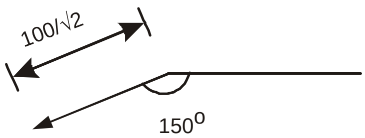
In the next unit, the electrical machines and power distribution will be discussed.

3.7 ANSWERS TO SAQs

SAQ 1

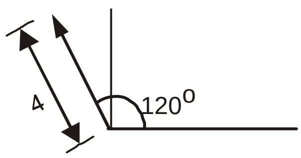
(a) $-100 \sin (\omega t + 30^{o})$

$= 100 \sin (\omega t - 150^{o})$



(b) $4\sqrt{2} \cos (\omega t + 30^{o})$

$= 4\sqrt{2} \cos (\omega t + 120^{o})$



SAQ 2

(a) (i) leads

(i) 90

(ii) resistor

(iii) smaller

(b) $230 = (2\pi \times 50)\, L \times 1.1 \Rightarrow L = 0.666$ H

Changed current $= 1.1 \times \frac{150}{230} \times \frac{50}{25} = 1.435\text{A}$

Also, $I = \frac{V}{\omega L} = \frac{150}{2\pi \times 50 \times 0.666} = 1.435$ A

SAQ 3

(a) The statement is false. It is the ratio of *phasors* of $v(t)$ and $i(t)$, which is equal to the impedance.

(b) R^{-1}, $1/j\omega L$, $j\omega C$

SAQ 4

(a) No. v_R and v_c have a phase difference of 90^o since one is in phase with the current and the other lags the current by 90^o. The magnitude of their phasor sum is $(V_R^2 + V_c^2)^{\frac{1}{2}} = (89.8^2 + 178.7^2)^{\frac{1}{2}}$, which is indeed the RMS value of the supply voltage v_s. KVL implies that $v_s = v_R + v_c$ or equivalently $\overline{V}_S = \overline{V}_R + \overline{V}_C$ and not $V_s = V_R + V_c$.

(b) $\bar{I} = 10^{-3} \angle 30^o ; \bar{Z} = 100 + j251 = 270 \angle 68.3^o$

$\bar{V} = \overline{ZI} = [270 \angle 68.3^o][10^{-3} \angle 30^o] = 0.27 \angle 98.3^o$

Hence, $v(t) = 0.27\sqrt{2} \sin (400t + 98.3^o)$

SAQ 5

(a) We have $\overline{V}_S = \overline{V}_R + \overline{V}_Z$. Let us take $\overline{V}_R$ as reference.

$\overline{V}_R = 50 \angle 0^o$

$\bar{I} = (50/20) \angle 0^o = 2.5 \angle 0^o = 2.5$

$\overline{V}_Z = \overline{ZI} = (40 + j60)\, 2.5 = 100 + j150$

$\overline{V}_S = \overline{V}_R + \overline{V}_Z = 50 + 100 + j150 = 150 + j150$

$\overline{V}_S = \sqrt{150^2 + 150^2} = 212 \text{ V}$

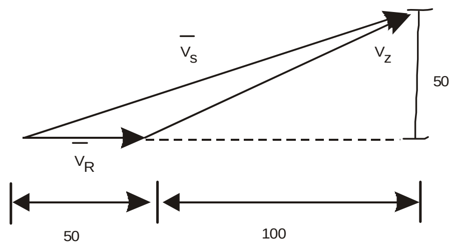


Figure for Answers to SAQ 5(a)

(b) We know power, P, in a resistor equals $P = \frac{V_{\text{eff}}^2}{R}$.

Here, $V_{\text{eff}} = 130 \text{ V}$.

$R_{\text{lamp}} = 130^2/40 = 422.5\ \Omega$

$I_{\text{lamp}} = 130/422.5 = 0.308$ A

Impedance of the combination = 220/0.308 = 714.3 Ω

Thus, we must have

$R^2_{\text{lamp}} + \omega^2 L^2 = 714.3^2$

$$L = \frac{(714.3^2 - 422.5)^{\frac{1}{2}}}{100\pi} = 1.83 \text{ H}.$$

SAQ 6

(a) A single-phase generator has two terminals and produces a single output voltage between the two terminals. A 3-phase generator is a composite unit comprising 3-single-phase generators, each generator producing a voltage which has fixed phase difference with the other two. The three 1-phase generators are connected internally in star or delta and the resulting 3-phase generator has 3 or 4 external terminals.

(b) (i) kVA rating

(ii) conductor material

(iii) constant

(iv) pulsating

SAQ 7

(a) Star[1], Delta[2], and 3-wire[3]

(b) (i) False

(ii) True

SAQ 8

(a) (i) False

(ii) False. $\bar{V}_C$ leads $\bar{V}_A$ by 120^o (angle of lag/lead is limited to 180^o).

(iii) False. The converse of the statement above Eq. (3.23) is not necessarily true. For example, i_A, i_B and i_C with $i_A = -\, i_B$ and $i_C = 0$ do not form a balanced set.

(b)
$$v_B = \sqrt{2}\, V \sin\left(\omega t + \theta + \frac{2\pi}{3}\right)$$

$$v_C = \sqrt{2}\, V \sin\left(\omega t + \theta - \frac{2\pi}{3}\right)$$

$$i_B = \sqrt{2}\, I \sin\left(\omega t + \beta + \frac{2\pi}{3}\right)$$

$$i_B = \sqrt{2}\, I \sin\left(\omega t + \beta - \frac{2\pi}{3}\right)$$

(c) (i) 100, 285^o, 165^o, 100.

(ii) 4, 180^o, 4, -60^o.