UNIT 5 FLOW THROUGH SIMPLE PIPES

Structure

5.1 INTRODUCTION

A pipe is a closed conduit through which fluids can flow. The flow in a pipe is termed *pipe flow* only when the fluid completely fills the cross-section and there is no free surface of fluids. Pipes flowing partially full, with a free surface, are not having pipe flow *hydraulically*. Such pipes come under the category of *open channel flow*. The pipe running partially full behaves like an open channel. You will be introduced to the open channel flow in Unit 7.

Pipes most commonly used in engineering practice are of circular cross-section in this unit, most of the discussion is limited to pipes of circular cross-section. This is an introductory unit on pipe flow. For further details, refer to Unit 6.

Since the fluid in a pipe is in motion, it has to overcome the frictional resistance between the adjacent fluid layers and that between the fluid layers and the pipe walls. Figure 5.1 shows a pipe of uniform cross-section with its axis horizontal. As the fluid flows from point 1 to point 2, there is a loss of head due to friction. Thus, there is drop of the energy gradient line. Because the pipe is of uniform cross-section, the velocity remains constant. The hydraulic gradient line is parallel to the energy gradient line (total energy line) and at a distance of $\frac{V^2}{2g}$, where V is the velocity in the pipe. The loss of head between two points is represented by h_f.

Besides the loss of head due to friction, there are losses due to shock resistance. The shock resistance in pipe flow occurs whenever there is a disturbance in the normal flow. The disturbance may occur due to changes in the cross-section, bends, obstruction etc. In long pipes, the losses due to shock resistance are small compared to frictional losses and may be neglected. The losses due to shock resistance are usually called the *minor losses* or *secondary losses* (refer Unit 4).

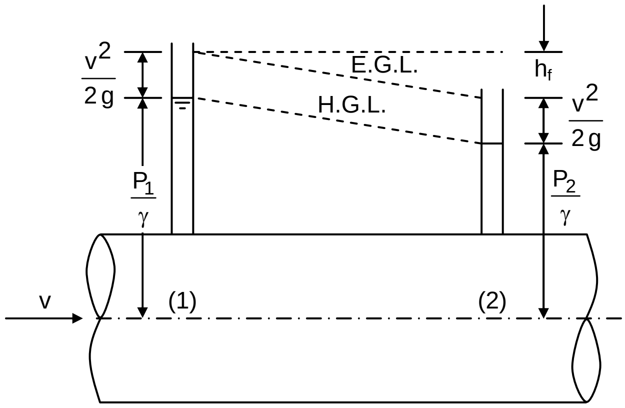


Figure 5.1

Objectives

After studying this unit, you should be able to

- explain Reynold's experiment on flow through pipes and conceptualise, its outcome pertaining to Reynold's Number,
- discuss the use and application of Darcy-Weisbach equation, and
- handle the problems of flow through simple pipes more efficiently.

5.2 REYNOLD'S EXPERIMENT ON FLOW THROUGH PIPES

Reynolds estimated the loss of head in a pipe by measuring the difference of pressure over a known length l of the pipe. Figure 5.2 shows the apparatus used by Reynolds for this purpose. The velocity of water in the pipe was determined by measuring the volume of water collected in the tank over a known period of time. By changing the velocity of flow, the corresponding values of the loss of head h_f were obtained.

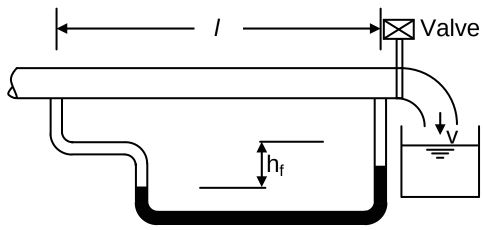
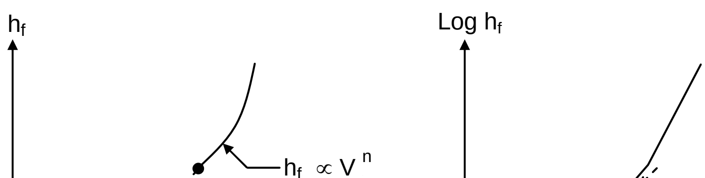


Figure 5.2

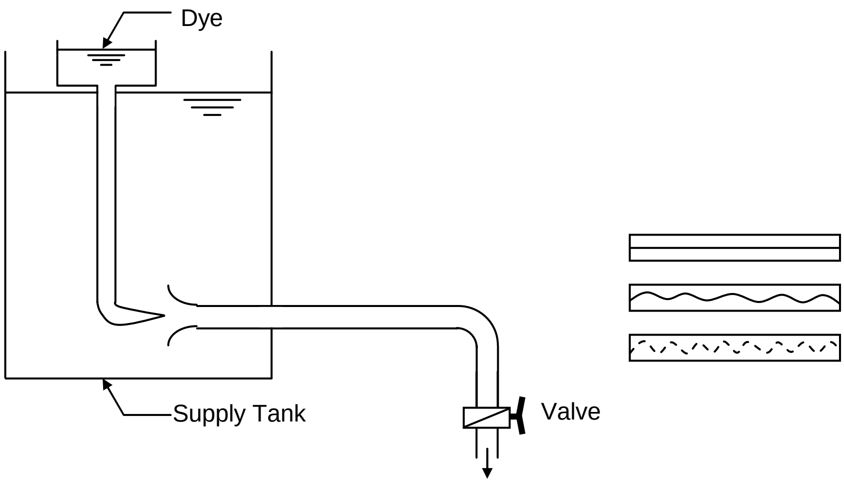
Reynolds obtained a plot between V and h_f (Figure 5.3(a)). The curve obtained is not a continuous one. At low velocities, the curve is a straight line, indicating that the loss of head is directly proportional to the velocity. The flow is laminar. At higher velocities, the flow becomes turbulent and the curve is parabolic. In this range, the loss of head (h_f) varies as V^n, where the value of n lies between 1.75 to 2.0. In the intermediate region, there is a transition zone. This is shown by a dotted line. The loss of head in this region varies in an erratic manner. Three regions of flow become more distinct when h_f and V are plotted on logarithmic scale (Figure 5.3(b)). All the regions are represented by three straight lines. The laminar region is represented by the lower line, and the turbulent region by the upper line. The intermediate line represents the transition in which the flow is neither laminar nor turbulent.



(a) **(b)**

Figure 5.3

A visual picture of the flow phenomena as the velocity is increased was first obtained by Reynolds with the apparatus shown in Figure 5.4(a). The apparatus consists of a supply tank, a dye container and a transparent glass tube with a valve. After the water level in the supply tank comes to rest, the outlet valve is gradually opened. A fine thread of dye appears in the glass tube, indicating that the flow is laminar. As the rate of flow is gradually increased, the thread of dye suddenly breaks up and mixes with the surrounding water. The flow becomes ill-defined. When the velocity is increased further, the dye mixes completely with the water and the whole of water becomes coloured. The flow becomes turbulent. Three stages are shown in Figure 5.4(b), in which the top sketch shows the laminar flow, and the bottom sketch shows the turbulent flow.



(a) **(b)**

Figure 5.4

When the velocity is gradually decreased, the flow changes back from turbulent to laminar. There is again a fine thread of dye at low velocity. It is found that the velocity at which the flow changes from laminar to turbulent is higher than the velocity at which the flow changes from turbulent to laminar. The velocity at which the flow changes from laminar to turbulent is called the *upper critical velocity*. The velocity at which the flow changes from turbulent to laminar is called the *lower critical velocity*. Points B and A represent the upper and the lower critical velocity respectively (Figure 5.5).

Flow in a pipe will be laminar or turbulent, depending upon the Reynolds number (N_R). The Reynolds number corresponding to the upper critical velocity does not have a fixed value. This depends upon initial disturbance, the shape of entry to the pipe and several other factors. Ekman was able to obtain laminar flow even up to $N_R = 5 \times 10^4$. However, the practical value of the Reynolds number corresponding to the upper critical velocity is between 2,700 and 4,000. The

Reynolds number corresponding to the lower critical velocity is more or less fixed. It is about 2,000. (**Note :** Some authors take this value as 2,100.)

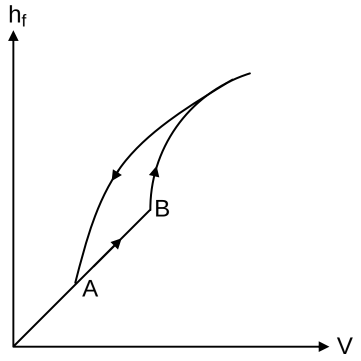


Figure 5.5

The Reynolds number corresponding to the lower critical velocity is much more important than that corresponding to the upper critical velocity. The Reynolds number corresponding to the lower critical velocity indicates that the flow will always be laminar if the Reynolds number is less than this value. The flow above this value of Reynolds number may be laminar or turbulent depending upon whether the velocity is increasing or decreasing and upon other flow conditions. When the Reynolds number is between 2000 and 4000, the transition stage exists. The flow is generally turbulent when the Reynolds number is greater than 4000. It is to be noted that above values of N_R are for pipe, with diameter D as the characteristic length (i.e. $N_R = \frac{\rho VD}{\mu}$).

5.3 DARCY-WEISBACH EQUATION

Froude estimated the frictional resistance of different surfaces by moving wooden boards in water. He towed several thin boards, made of wood, in a tank containing water, by connecting them to a side carriage. The carriage was made to run on rails at the sides of the tank. The carriage was hauled at various speeds by means of a wire rope. The force required to tow the wooden boards was measured. Boards of different length and coated with different material were used. Froude came to the following conclusions :

(a) The frictional resistance of the boards varies as the square of the velocity.

(b) The frictional resistance depends upon the nature of the surface.

(c) The frictional resistance per unit area of the surface decreases as the length of the board increases but is constant for large lengths. This is due to the fact that the effect of ends is more pronounced in small length but is negligible when the board is very long.

A formula for the frictional resistance in a pipe may be obtained on the assumption that above results hold good for the interior surface of the pipe. Let us consider a uniform horizontal pipe through which a liquid flows with a velocity V. Let the cross-sectional area of the pipe be A and the pressure be p_1 and p_2 at two points at a distance L apart (Figure 5.6). If f' is the frictional resistance per unit area at unit velocity, the total frictional resistance over the length L is given by

$$F = f' \times \text{surface area} \times V^2$$

$$= f' \times (PL) \times V^2 \qquad \ldots \text{(a)}$$

where P is the wetted perimeter.

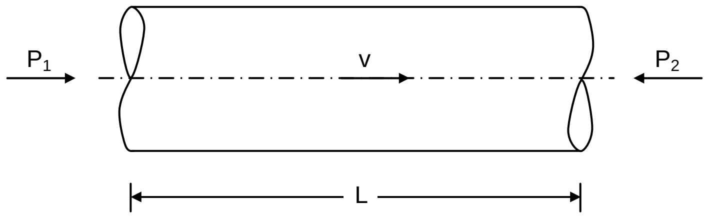


Figure 5.6

The wetted perimeter (P) of a conduit is the length of the curve of intersection of its wetted surface with the cross-section of the pipe. Thus, for a circular pipe running full, the wetted perimeter is equal to the circumference πD, where D is the diameter of the pipe.

The pressure force acting at the ends of the pipe is given by

$$F = (p_1 - p_2)\left(\frac{\pi D^2}{4}\right) \qquad \ldots \text{(b)}$$

Since the fluid is moving at a constant velocity, the acceleration is zero. According to Newton's second law of motion, the net force on the fluid must be zero. Therefore,

$$\frac{(p_1 - p_2)}{\gamma}\left(\frac{\pi D^2}{4}\right) = \frac{f'}{\gamma} \times (PL) \times V^2$$

or $$\frac{(p_1 - p_2)}{\gamma} = \frac{f'}{\gamma} \times \left(\frac{4PL}{\pi D^2}\right) V^2$$

$$= \frac{f'}{\gamma} \times \left(\frac{4 \times \pi D \times L}{\pi D^2}\right) V^2 = \frac{4f'}{\gamma} \times \left(\frac{L}{D}\right) V^2$$

or $$h_f = f\left(\frac{L}{D}\right)\frac{V^2}{2g} \qquad \ldots \text{(5.1)}$$

where $$f = \frac{4f'}{\gamma}(2g)$$

Eq. (5.1) is known as Darcy-Weisbach equation. In this equation, f is a dimensionless coefficient known as the *friction factor.*

[**Note :** In some texts, especially in the British books, the Darcy-Weisbach equation is written as $h_f = 4f'\left(\frac{L}{D}\right)\frac{V^2}{2g}$. *It is obvious that f in the standard equation* (*Eq.5.1*) *has been replaced by 4f'.* Unless otherwise mentioned, the Darcy-Weisbach equation in the form of Eq. (5.1) shall be used.]

The Darcy-Weisbach equation is a general equation which holds good for all types of flow, provided a proper valve of f is chosen. The details of the estimation of the value of f for laminar and turbulent flow are outside the scope. For the present, it may be assumed that the value of f is given for the pipe under consideration.

Example 5.1

Find the loss of head due to friction in a pipe carrying water. The pipe is 300 m long and 15 cm in diameter. The discharge through the pipe is 0.04 cumecs. Take $f = 0.04$.

Solution

$$V = \frac{Q}{A} = \frac{0.04}{\frac{\pi}{4} \times 0.15 \times 0.15} = 2.265 \text{ m/sec}$$

From Eq. (5.1), $h_f = f \dfrac{L}{D} \dfrac{V^2}{2g}$

$$= 0.04 \times \frac{300}{0.15} \times \frac{(2.265)^2}{2 \times 9.81} = 20.92 \text{ m of water}$$

SAQ 1

A pipeline 22.5 cm diameter and 1580 m long has an up slope of 1 in 200 for the first 790 m and an up slope of 1 in 100 for the next 790 m. If the pressure at the upper end of the pipeline is 107.91 kN/m^2 and that at the lower end is 53.96 kN/m^2, determine the discharge. Take Darcy's $f = 0.04$.

5.4 DEFINITIONS

Wetted Perimeter, Area of Flow, Hydraulic Radius

Before taking up the empirical formulae, we must define certain parameters. The wetted perimeter was defined in the preceding section. It is the curve of intersection of the wetted surface with the cross-section of the pipe. Another important parameter is the hydraulic mean depth (m) or the hydraulic radius (R). The hydraulic radius is the ratio of the area of cross-section of the flow to the wetted perimeter, i.e. $R = A/P$.

For a circular pipe running full, $R = \dfrac{\frac{\pi}{4} D^2}{\pi D} = \dfrac{D}{4}$

Energy Gradient Line

Total head (which is also equal to total energy per unit weight) with respect to any arbitrary datum is the sum of the elevation (potential) head, pressure head and the velocity head, i.e.

$$\text{Total head} = Z + \frac{p}{\gamma} + \frac{V^2}{2g}$$

As the fluid flows along the pipe, there is a loss of head (energy) and the total energy decreases in the direction of flow. If the total energy at various points along the axis of the pipe is plotted and joined by a line, the line so obtained is called *energy gradient line*, abbreviated as EGL. This line is also known as the *total energy line* (TEL).

Hydraulic Gradient Line

The sum of the potential head and the pressure head ($Z + p/\gamma$) at any point is called the piezometric head. If a line is drawn joining the piezometric levels at various points, the line so obtained is the *hydraulic gradient line* (HGL). If we insert piezometers, the liquid level in the piezometers would rise up to the level of HGL. The vertical intercept between the HGL and the pipe axis is equal to the pressure head (p/γ).

It is to be noted that EGL (TEL) always drops in the direction of flow because of loss of head, whereas HGL may rise or fall depending upon the pressure changes. Moreover, HGL is always below the EGL (TEL) and the vertical intercept between the two is equal to the velocity head ($V^2/2g$).

[**Note :** In case a pump is provided on the pipe, the total energy line will rise at that point.]

In the empirical formulae that follow, we shall be using the term 'slope of the energy gradient line'. The slope of the energy gradient line (S) is given by

$$S = \frac{h_f}{L}$$

where h_f is the loss of head due to friction in metres of the fluid and L is the length of pipe in meters. For a pipe of uniform cross-section, the slope of the hydraulic gradient line is equal to the slope of energy gradient line. *It must be noted that there is no relation whatsoever between the slope of the energy gradient line and the slope of the axis of the pipe*.

5.5 EMPIRICAL FORMULAE

A general formula for the loss of head due to friction in a pipe may be written as

$$h_f = K \frac{L}{D^c} V^b \qquad \text{. . . (a)}$$

where K, c and b are constants, L= length of the pipe, D = diameter of the pipe.

Eq. (a) may be written in terms of the hydraulic radius (R).

$$h_f = K_1 \frac{L}{R^a} V^b \qquad \text{. . . (b)}$$

in which D has been expressed in terms of R as such that $D^c = R^a$.

(**Note :** The hydraulic radius R is not equal to the radius of the pipe (r). The reader must note this carefully.)

Eq. (b) may be transformed as

$$V^b = K_2 \; R^{-a} \left(\frac{h_f}{L} \right)$$

Denoting $\frac{h_f}{L}$ by the slope of the energy gradient line S,

$$V^b = K_2 \; R^{-a} \; (S)$$

or $$V^b = kR^{-a/b}\,(S)^{1/b}$$

or $$V = kR^x\,S^y \qquad \text{. . . (c)}$$

where k, x and y are constants to be determined experimentally.

Eq. (c) is an important form of the general formula for the loss of head in pipes. Several investigators performed experiments on the pipe flow and gave their own values of the constants k, x and y. Chezy's formula is given below:

Chezy's Formula

On the basis of results obtained from experiments, Chezy gave the following formula :

$$V = C\,\sqrt{RS} \qquad \text{. . . (5.2)}$$

in which C is a coefficient known as Chezy's coefficient.

The relation between Chezy's and Darcy's f can be obtained as under.

From Eq. (5.1),

$$h_f = f\left(\frac{L}{D}\right)\frac{V^2}{2g}$$

or $$V^2 = \frac{2g}{f}\,(D)\left(\frac{h_f}{L}\right)$$

But $R = D/4$ or $D = 4R$.

Therefore, $$V^2 = \frac{8g}{f}\left(\frac{D}{4}\right)\left(\frac{h_f}{L}\right) = \frac{8g}{f}\,(R)\,(S)$$

$$V = \sqrt{\frac{8g}{f}}\,\sqrt{RS} = C\sqrt{RS}$$

Thus, $$C = \sqrt{\frac{8g}{f}} \qquad \text{. . . (5.3)}$$

Eq. (5.3) gives the relation between the Chezy C and the Darcy f. Since g, the acceleration due to gravity, has dimension [L/T^2], the Chezy coefficient C is not a dimensionless coefficient, although f is dimensionless. *The value of C ranges from 55 to 75 in most of the pipe used in practice.*

The Darcy-Weisbach equation is very popular. These days the trend is towards a rational approach, in which the value of the friction factor f' is estimated, considering the flow characteristics, such as the Reynolds number and the roughness of the pipe surface. In this rational approach, the value of f can be estimated accurately if the properties of the fluid and the type of pipe surface are known. The empirical formulae are not used as extensively these days as they were used some time ago.

Example 5.2

Find the loss of head due to friction in a pipe 8 cm diameter and 30 m long if the mean velocity of flow is 2 m/sec. Use Chezy's formula. Take C = 55.

Solution

From Chezy's formula, $V = C\sqrt{RS}$

or $$V = C\sqrt{\left(\frac{D}{4}\right)\times\left(\frac{h_f}{L}\right)}$$

or $$V^2 = C^2\left(\frac{D}{4}\right)\times\left(\frac{h_f}{L}\right)$$

or $$(2)^2 = (55)^2\left(\frac{0.08}{4}\right)\left(\frac{h_f}{30}\right)$$

or $$h_f = 1.98 \text{ m}$$

SAQ 2

A town having a population of 1 lakh is to be supplied water from a reservoir 4750 m away. It is stipulated that one half of the daily supply of 130 litres per head is required in 8 hours. What must be the size of the pipe to furnish the water supply if the head available is 12 m. Take Chezy's $C = 43$.

5.6 TRANSMISSION OF POWER

Power is transmitted through pipes by means of liquid under pressure. This method of transmission is commonly used for hydraulic turbines. The power supplied at the exit of the pipe is proportional to the quantity of liquid and the head at the point. The maximum power is transmitted by a pipe when the frictional loss is a certain fraction of the total head.

Let h_f and H be the loss of head due to friction and the head supplied at inlet respectively.

Let V be the velocity in pipe, and D and L be the diameter and length of the pipe respectively.

Head available at the exit, $h = H - h_f$

$$= H - f\,\frac{L}{D}\,\frac{V^2}{2g}$$

Available power, $P = W\,h$ Watts

or $$P = \frac{W\,h}{1000} \text{ kW} \qquad \ldots (5.4)$$

where W is weight of liquid passing per second in Newtons (N/s) and h is the available head in meters.

If W is in kN/s, P the power

$$P = Wh \text{ kW} \qquad \ldots (b)$$

Eq. (5.4) for P can be written as

$$P = \left(\frac{\pi}{4}\right) D^2 V \gamma \left(H - f \frac{L}{D} \frac{V^2}{2g} \right) \times \frac{1}{1000}$$

or $$P = \frac{\pi D^2 \gamma}{4000} \left(HV - f \frac{L}{D} \frac{V^3}{2g} \right)$$

The power will be maximum when

$$\frac{d}{dV} \left(HV - f \frac{L}{D} \frac{V^3}{2g} \right) = 0$$

or $$H - 3f \frac{L}{D} \frac{V^2}{2g} = 0$$

or $$H = 3f \frac{L}{D} \frac{V^2}{2g} = 3 \left[f \frac{L}{D} \frac{V^2}{2g} \right] = 3h_f$$

or $$h_f = \frac{H}{3}$$

Thus the power transmitted through a pipe is a maximum when the head loss due to friction is one-third of the head supplied,

From Eq. (5.4), Maximum power,

$$P_{\max} = \frac{W\left(H - \frac{H}{3}\right)}{1000} = \frac{2WH}{3000} \text{ kW}$$

Efficiency of transmission,

$$\eta = \frac{\text{Output}}{\text{Input}} = \frac{\frac{2WH}{3000}}{\frac{WH}{1000}} = \frac{2}{3} \text{ (= 66.67\%)}$$

Thus for the power to be a maximum, the efficiency is only 66.67%. In practice, however, power is seldom transmitted at the maximum power conditions as derived above. The condition for the maximum power requires a head loss of one-third of the total head, which is not permitted in practice. The reduced head at the exit would necessitate a bigger and costlier machine to use the power than that would be necessary if the loss of head is small. Moreover, there are difficulties in regulation if the machine is working at the maximum power conditions.

Horse Power

Sometimes, the power is expressed in (metric) horse power (hp)

1 hp = 735.75 N-m/s

Thus 1 hp = 0.736 kW

or 1kW = 1.36 hp.

Example 5.3

A hydraulic machine is supplied with water through a pipe 1000 m long. Gauges fitted to the supply pipe show pressure of 5886 kN/m^2 at the upstream end and a pressure of 5395.5 kN/m^2 at the machine. If the power supplied to the machine is 44.145 kW (60 hp), determine the diameter of the supply pipe. Take f = 0.03.

Solution

Power supplied, from Eq. (5.4),

$$\frac{Wh}{1000} = 44.145 \text{ kW} \qquad \ldots \text{(a)}$$

where $$W = \left(\frac{\pi D^2}{4}\right) V \gamma = \frac{\pi D^2}{4} \times V \times 1000 \times 9.81 \text{ N/s}$$

Also $$h = \frac{5395.5}{9.81} = 550 \text{ m}$$

Substituting these values in Eq. (a),

$$9.81 \times \left(\frac{\pi}{4}\right) D^2 V \times 550 = 44.145$$

or $$D^2 V = 1.04 \times 10^{-2}$$

or $$V = 1.04 \times \frac{10^{-2}}{D^2} \qquad \ldots \text{(b)}$$

Head loss due to friction,

$$h_f = f \frac{L}{D} \frac{V^2}{2g}$$

or $$\frac{5886 - 5395.5}{9.81} = 0.03 \times \frac{1000}{D} \times \frac{V^2}{2 \times 9.81}$$

or $$\frac{V^2}{D} = 32.7 \qquad \ldots \text{(c)}$$

From Eqs. (b) and (c),

$$\frac{(1.04 \times 10^{-2})^2}{D^5} = 32.7$$

or $$D = 0.08 \text{ m} = 80 \text{ mm}$$

SAQ 3

One hundred litres per second of water is to be pumped into a pipe 25 cm diameter and 5 km long. If the static lift from the sump to the supply point is 16 m, what will be the power required? Assume the overall efficiency of the pump as 70% and f = 0.02.

5.7 PIPE DISCHARGING FROM A RESERVOIR

Figure 5.7 shows a pipe of uniform cross-section leading from a reservoir and discharging free into atmosphere. As the fluid enters the pipe, there is a loss of head at the entrance (h_L). Consequently, there is a drop in the energy gradient line at point A. The hydraulic gradient line is at a distance of $V^2/2g$ below the energy gradient line. As the liquid flows from A to B, there is a loss of head due to friction. The loss due to friction occurs uniformly along the pipe. Loss due to friction in the entire length is h_f. The loss of head at the exit is $V^2/2g$.

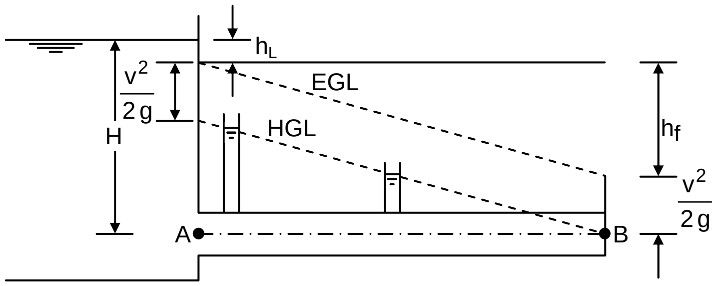


Figure 5.7

Applying Bernoulli's equation to points A and B in Figure 5.7,

$$H = h_L + h_f + \frac{V^2}{2g} \qquad \ldots (5.5)$$

If the entrance to the pipe is flush with the reservoir,

$$h_L = 0.5\,\frac{V^2}{2g},$$

and Eq. (5.5) becomes

$$H = 0.5\,\frac{V^2}{2g} + h_f + \frac{V^2}{2g}$$

Substituting the value of

$$h_f = f\,\frac{L}{D}\,\frac{V^2}{2g},$$

$$H = \frac{V^2}{2g}\left[1.50 + f\left(\frac{L}{D}\right)\right] \qquad \ldots (5.6)$$

For long pipes the term $f\left(\frac{L}{D}\right)$ is very large compared to 1.50. In such cases, the loss of head at the entrance and exit may be neglected. When the length of pipe is greater than 1000 D, only the frictional loss need be considered.

Example 5.4

Water is discharged from a large reservoir to atmosphere through a 10 cm diameter and 500 m long pipe. Find the discharge if the outlet is 15 m below the free surface of water in the reservoir. Assume the entry to the pipe as sharp. Take $f = 0.04$.

Solution

From Eq. (5.6),

$$H = \frac{V^2}{2g}\left[1.50 + f\left(\frac{L}{D}\right)\right]$$

or $$15 = \frac{V^2}{2g}\left(1.50 + 0.04 \times \frac{500}{0.10}\right)$$

or $$V = 1.21 \text{ m/sec}$$

Discharge, $$Q = AV$$

or $$Q = \frac{\pi}{4} \times 0.1 \times 0.1 \times 1.21 = 0.0095 \text{ cumecs}.$$

5.8 PIPE CONNECTING TWO RESERVOIRS

Figure 5.8 shows a pipe of uniform cross-section connecting two reservoirs with liquid surfaces at different elevations. The liquid flows from the higher reservoir to the lower reservoir. At point *A,* there is a loss of head at entrance (h_L). The loss of head due to friction h_f takes place throughout the length of the pipe. At the exit, there is a loss of head of $V^2/2g$. If H is the difference of liquid levels in the two reservoirs,

$$H = h_L + h_f + \frac{V^2}{2g}$$

$$H = 0.5\,\frac{V^2}{2g} + f\,\frac{L}{D}\frac{V^2}{2g} + \frac{V^2}{2g}$$

or $$H = \frac{V^2}{2g}\left[1.50 + f\,\frac{L}{D}\right] \qquad \text{. . . (5.7)}$$

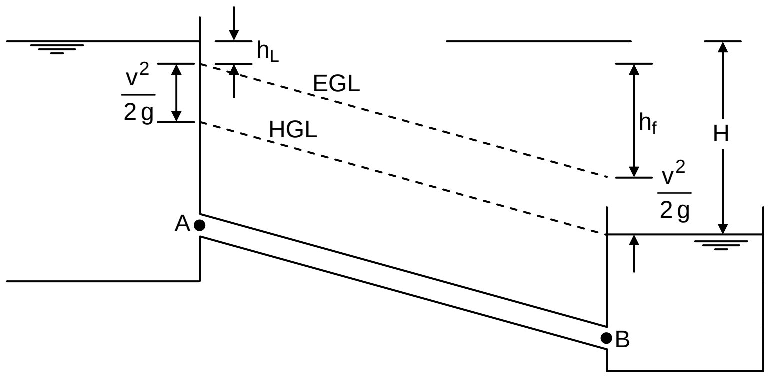


Figure 5.8

It may be noted that the velocity V is independent of the location of either end of pipe so long as the difference of liquid levels in two reservoirs is constant.

5.9 TIME OF EMPTYING A TANK THROUGH A LONG PIPE

Figure 5.9 shows a tank to which a long pipe of length L and diameter D is connected. Let the surface area of the liquid surface be A and the head of water above the outlet of the pipe be initially H_1.

Let at any instant t the liquid level be H above the outlet of the pipe when the velocity through the pipe is V. If the liquid drops by a small amount dH in time dt, then from the continuity equation, the volume of the liquid flowing out from the tank is equal to the volume of the liquid flowing out of the pipe.

Thus, $$- A\,dH = V\left(\frac{\pi D^2}{4}\right) dt \qquad \text{. . . (a)}$$

The negative sign indicates that the liquid level decreases with an increase in time. V is the velocity through the pipe.

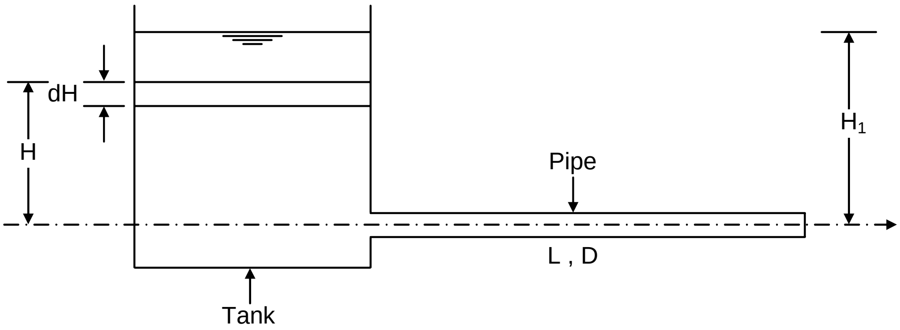


Figure 5.9

Now H = Loss of head at entry + frictional loss + loss of head at exit

or $$H = \frac{0.5V^2}{2g} + f\,\frac{L}{D}\,\frac{V^2}{2g} + \frac{V^2}{2g}$$

or $$V = \sqrt{\frac{2gH}{1.5 + f\,\dfrac{L}{D}}} \qquad \text{. . . (b)}$$

Substituting the value of V from Eq. (b) in Eq. (a),

$$- A\,dH = \frac{\pi D^2}{4}\left(\sqrt{\frac{2gH}{\left(1.5 + f\,\dfrac{L}{D}\right)}}\right) dt$$

or $$dt = - \frac{4A\sqrt{1.5 + f\,\dfrac{L}{D}}}{\pi D^2\sqrt{2g}}\,\frac{dH}{\sqrt{H}}$$

The time required for the liquid level to fall from H_1 to H_2, assuming A as constant, is given by

$$t = - \frac{4A\sqrt{1.5 + f\,\dfrac{L}{D}}}{\pi D^2\sqrt{2g}}\int_{H_1}^{H_2}\frac{dH}{\sqrt{H}}$$

$$t = - \frac{8A\sqrt{1.5 + f\,\dfrac{L}{D}}}{\pi D^2\sqrt{2g}}\,[H_2^{1/2} - H_1^{1/2}]$$

or $$t = \frac{8A\sqrt{1.5 + f\,\dfrac{L}{D}}}{\pi D^2\sqrt{2g}}\,[H_1^{1/2} - H_2^{1/2}]$$

If the tank is to be completely emptied, $H_2 = 0$.

Therefore, $$t = \frac{8A\sqrt{1.5 + f\,\dfrac{L}{D}}}{\pi D^2\sqrt{2g}}\,H_1^{1/2} \qquad \text{. . . (5.7)}$$

Example 5.5

Determine the time required to empty a vertical tank of diameter 1 m by a pipe 100 m long and 0.15 m diameter if the initial head of water above the outlet of the pipe is 9 m. Assume $f = 0.03$.

Solution

From Eq. (5.7),

$$t = \frac{8 \times \frac{\pi}{4} \times (1)^2 \sqrt{1.5 + 0.03 \times \frac{100}{0.15}}}{\pi \times (0.15)^2 \times \sqrt{2 \times 9.81}} \times (9)^{1/2}$$

$$= 279.1 \text{ s} = 4 \text{ min } 39.1 \text{ seconds.}$$

SAQ 4

A vertical cylindrical tank 4.8 m diameter has a pipe at its bottom. The pipe is vertical and 90 m long and of 225 mm diameter. Find the time taken to lower the water level in the tank from 2.7 m to 1.2 m above its bottom. Assume $f = 0.04$. Neglect loss of head at entrance.

5.10 TIME OF EMPTYING A TANK TO ANOTHER TANK THROUGH A PIPE

Figure 5.10 shows two tanks connected with a long pipe of diameter D and length L. The liquid flows from the higher tank A to lower tank B through the pipe. Let H_1 be the initial difference of the liquid levels in the two tanks. The time required to reduce the difference of the liquid levels to H_2 can be computed using the same procedure as discussed in the preceding section remembering that the head causing flow is the difference of levels in the two tanks.

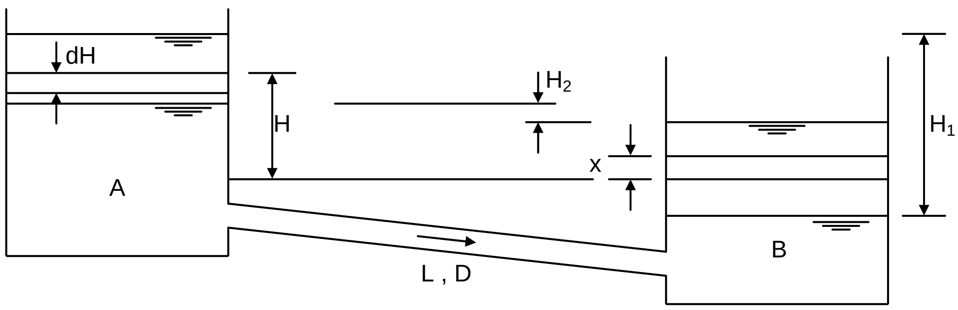


Figure 5.10

Let us consider the instant when the difference of the liquid levels in the two tanks is H. In time dt, the liquid level in the tank A drops by an amount dh and rises by an amount x given by

$$A_1\, dh = A_2\, x$$

where A_1 and A_2 are the cross-sectional areas of the tanks A and B, respectively.

Thus, $$x = \frac{A_1}{A_2}\, dh$$

The difference in the head causing flow is given by

$$dH = dh + x$$

$$dH = dh + dh\left(\frac{A_1}{A_2}\right)$$

or $$dH = dh\left(1 + \frac{A_1}{A_2}\right) \quad \ldots \text{(i)}$$

From the continuity consideration,

Volume of liquid flowing from the tank = Volume of liquid flowing in the pipe

or $$-A_1\,dh = V\left(\left(\frac{\pi}{4}\right)D^2\right)dt \quad \ldots \text{(a)}$$

But $$H = 0.5\frac{V^2}{2g} + \frac{V^2}{2g} + f\,\frac{L}{D}\frac{V^2}{2g}$$

or $$V = \sqrt{\frac{2gH}{1.5 + f\,\dfrac{L}{D}}} \quad \ldots \text{(b)}$$

From Eqs. (a) and (b),

$$-A_1\,dh = \left(\frac{\pi}{4}D^2\right)\left(\sqrt{\frac{2gH}{1.5 + f\,\dfrac{L}{D}}}\right)dt$$

Substituting the value of *dh* from Eq. (i),

$$-A_1\frac{dH}{\left(1 + \dfrac{A_1}{A_2}\right)} = \frac{\pi}{4}D^2\left(\sqrt{\frac{2gH}{1.5 + f\,\dfrac{L}{D}}}\right)dt$$

or $$dt = -\frac{4A_1A_2}{(A_1 + A_2)\,\pi D^2}\left(\sqrt{\frac{1.5 + f\,\dfrac{L}{D}}{2g}}\right)\frac{dH}{\sqrt{H}}$$

The time t required for the difference of liquid levels to drop from H_1 to H_2 is obtained by integration as

$$t = -\frac{4A_1A_2\sqrt{1.5 + f\,\dfrac{L}{D}}}{\pi D^2(A_1 + A_2)\sqrt{2g}}\int_{H_1}^{H_2}\frac{dH}{\sqrt{H}}$$

$$= \frac{8A_1A_2\sqrt{1.5 + f\,\dfrac{L}{D}}}{\pi D^2(A_1 + A_2)\sqrt{2g}}\,[H_1^{1/2} - H_2^{1/2}] \quad \ldots (5.8)$$

Example 5.6

A pipe of 150 m length and 200 mm diameter connects two tanks of the free surface areas of 2 m^2 and 1 m^2. If the initial difference of liquid levels in the two tanks is 4 m, determine the time required when the difference of levels becomes zero. Take $f = 0.03$.

Solution

In this case, $A_1 = 2\ \text{m}^2$, $A_2 = 1\ \text{m}^2$,

$$L = 150\ \text{m}, \quad D = 0.2\ \text{m}, \quad H_1 = 4\ \text{m} \quad \text{and} \quad H_2 = 0$$

Substituting the values in Eq. (5.8),

$$= \frac{8 \times 2 \times 1 \sqrt{1.5 + 0.03 \times \frac{150}{0.2}}}{\pi \times (0.2)^2\ (2 + 1)\ \sqrt{2 \times 9.81}}\ (4)^{1/2}$$

$$= \frac{8 \times 2 \times 1 \times \sqrt{24} \times 2}{\pi \times 0.04 \times 3 \times 4.43}$$

$$= 93.9\ \text{s}$$

SAQ 5

Two reservoirs A and B are connected by a pipe 60 cm diameter and 300 m long. The surface areas of reservoirs A and B are respectively 9000 m^2 and 4500 m^2. Find the time taken to lower the level of the upper reservoir A by 60 cm if the original difference of levels of the two reservoir is 6 m. Assume $f = 0.03$.

5.11 SUMMARY

- The difference between pipe flow and open channel flow is explained. Pipe flow occurs in a conduit under pressure when there is no free surface in the liquid.
- Reynolds experiment is described to differentiate between the laminar flow and the turbulent flow. The importance of the Reynolds Number is discussed.
- In practice, the Reynolds Number is generally greater than 4000, and the flow is turbulent.
- The Darcy-Weisbach equation for the loss of head is derived. The importance of the friction factor f is outlined.
- The definitions of the basic parameters such as wetted perimeter, area of flow, hydraulic radius, total energy line and hydraulic gradient line are given.
- The empirical formulae for the determination of the velocity of flow are described. The Chezy formula is explained. The value of Chezy' C generally varies between 55 and 75.
- Transmission of power by pipes is discussed. The efficiency of transmission is the maximum when the loss of head is a significant portion of the head supplied. In practice, the power is generally not transmitted at the maximum efficiency condition.

- The discharge through a pipe connected to a reservoir can be determined by the application of the Bernoulli equation and Darcy-Weisbach equation.
- The discharge equation for a pipe connecting two reservoirs is derived.
- Time of emptying a tank through a long pipe is determined from the continuity consideration.
- Time of emptying a tank to another tank through a pipe depends upon the difference of liquid levels and can be determined considering the difference of liquid levels.

5.12 ANSWERS TO SAQs

SAQ 1

Taking the datum at the lower end, and applying Bernoulli's equation to the lower end (say, point 1) and the upper and (say, point 2),

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + 0 + h_f = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + \left(\frac{790}{200} + \frac{790}{100}\right)$$

(**Note :** The flow is downward, because piezometric pressure at 2 is greater.)

Substituting the values,

$$\frac{53.96}{9.81} + \frac{V^2}{2g} + h_f = \frac{107.91}{9.81} + \frac{V^2}{2g} + 11.85$$

or $\quad 5.50 + h_f = 11.0 + 11.85$

or $\quad h_f = 17.35 \text{ m}$

From Darcy-Weisbach equation,

$$h_f = f \frac{L}{D} \frac{V^2}{2g} = 0.04 \times \frac{1580}{0.225} \times \frac{V^2}{2g}$$

or $\quad 17.35 = 14.316\, V^2$

or $\quad V = 1.10 \text{ m/s}$

$$Q = \frac{\pi}{4} \times (0.225)^2 \times 1.10$$

or $\quad Q = 0.0438 \text{ m}^3/\text{s}$

SAQ 2

Total water required per day $= 0.13 \times 10^5 = 13000 \text{ m}^3$

Water required in 8 hr $= \dfrac{13000}{2} = 6500 \text{ m}^3$

Discharge $= \dfrac{6500}{8 \times 3600} = 0.226 \text{ m}^3/\text{s}$

Now from Chezy's formula,

$$Q = AC\sqrt{RS}$$

or $$0.226 = \left(\frac{\pi}{4} \times D^2\right) \times 43 \sqrt{\frac{D}{4} \times \frac{12}{4750}}$$

or $$D^{5/2} = 0.2663$$

or $$D = 0.59 \text{ m}$$

SAQ 3

$$V = \frac{Q}{A} = \frac{10^2 \times 10^{-3}}{\frac{\pi}{4}(0.25)^2} = 2.04 \text{ m/s}$$

Loss of head, $$h_f = f \frac{L}{D} \frac{V^2}{2g}$$

$$= 0.02 \times \frac{5 \times 10^3}{0.25} \times \frac{(2.04)^2}{2 \times 9.81} = 84.84 \text{ m}$$

Total head = Static Head + Loss of Head

= 16 + 84.84 = 100.84 m

Power required $= \frac{WH}{1000}$

$$= \frac{(1 \times 10^2 \times 10^{-3} \times 9810) \times 100.84}{1000} = 98.92 \text{ kW}$$

Power supplied to the pump $= \frac{98.92}{\text{Overall Efficiency}} = \frac{98.92}{0.70} = 141.32$ kW

SAQ 4

Let the depth of water at any instant be H. The head causing flow will be ($H + 90$). Now applying Bernoulli's equation,

$$H + 90 = f \frac{L}{D} \frac{V^2}{2g} + \frac{V^2}{2g}$$

$$= 0.04 \times \frac{90}{0.225} \times \frac{V^2}{2g} + \frac{V^2}{2g} = 17 \frac{V^2}{2g}$$

or $$V = \sqrt{\frac{2g(H + 90)}{17}}$$

From the continuity of flow,

$$-A\,dH = Q\,dt = aV\,dt$$

or $$-\frac{\pi}{4} \times (4.8)^2 \times dH = \left[\frac{\pi}{4} \times (0.225)^2 \times \sqrt{\frac{2g(H+90)}{17}}\right] dt$$

or $$-dt = 423.64 \frac{1}{\sqrt{H+90}} dH$$

Integrating, $$t = -423.64 \int_{2.7}^{1.2} \frac{1}{\sqrt{H+90}} dH$$

$$= 423.64 \times \frac{2}{1} \times \left[(H+90)^{1/2}\right]_{1.2}^{2.7}$$

$$= 847.27 [9.63 - 9.55] = 67.78 \text{ s}$$

SAQ 5

Final difference of level $= 6 - 0.60 \left(1 + \frac{A_1}{A_2}\right)$

$$= 6 - 0.60 \left(1 + \frac{9000}{4500}\right) = 4.20 \text{ m}$$

The time required is given by

$$t = \frac{8 A_1 A_2 \sqrt{1.5 + \frac{f_L}{D}}}{\pi D^2 (A_1 + A_2) \sqrt{2g}} [H_1^{1/2} - H_2^{1/2}]$$

$$= \frac{8 \times 9000 \times 4500 \sqrt{1.5 + \frac{0.03 \times 300}{0.60}}}{\pi \times (0.6)^2 (9000 + 4500) \times 4.43} [6^{1/2} - 4.2^{1/2}]$$

$$= 19458 (2.449 - 2.049)$$

$$= 7783 \text{ s (2 hr 9 min 43 sec)}$$