UNIT 7 FLOW THROUGH OPEN CHANNEL

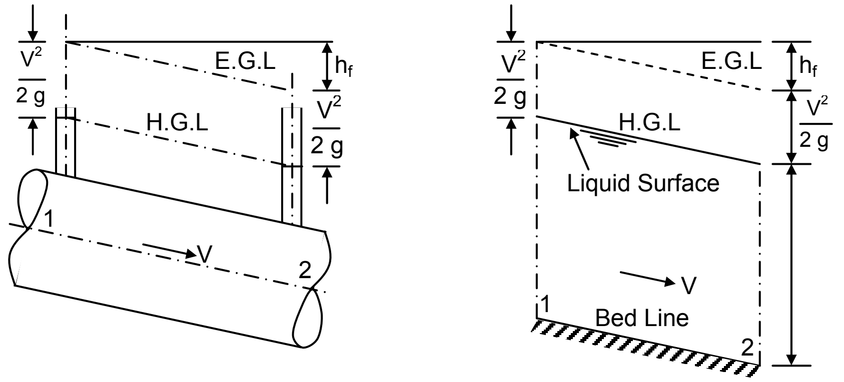
Structure

7.1 INTRODUCTION

An open channel is a duct or conduit in which the liquid flows with a free surface. This is in contrast with pipe flow in which the fluid completely fills the pipe and the flow is under pressure. The pipe flow takes place due to difference of pressure, whereas in an open channel, it is due to the slope of the channel, i.e. due to gravity. It follows that there can be flow in a pipe even with its axis horizontal. But, in an open channel, there must be some slope in the bed of the channel for flow to take place.

Two kinds of flow are compared in Figure 7.1. Figure 7.1(a) shows pipe flow. The pressure at the two sections of the pipe is indicated by the piezometers. The total energy with reference to a datum line is the sum of the elevation, pressure and velocity heads. The loss of energy that occurs when the liquid flows from section 1 to section 2 is represented by h_f. The diagram for open channel flow is shown in Figure 7.1(b). The liquid surface also represents the hydraulic gradient line as the depth of water correspond to the piezometric height. The energy gradient line is at a vertical distance $V^2/2g$ above the water surface. The loss of energy from section 1 to section 2 is h_f. It may be noted that for uniform flow in an open channel, the drop in the energy line is equal to the drop in bed. In pipe flow, there is no such relation between the drop of the energy line and the slope of the pipe axis.

It may be noted that the flow in a closed conduit is not necessarily a pipe flow. It must be classified as open channel flow if the liquid has a free surface. It follows that open channel flow is always characterized by a free surface.



(a) Pipe Flow **(b) Open Channel Flow**

Figure 7.1

Objectives

After studying this unit, you should be able to

- define open channel,
- describe different types of open channel and flow through open channel,
- handle problems based on Chezy's formula, Ganguillet Kutter formula and Manning's formula, and
- explain circular formula and conditions of maximum discharge and velocity in a circular channel.

7.2 TYPES OF OPEN CHANNELS

An open channel can be natural or artificial. Natural open channels are streams, rivers, etc. These are generally irregular in shape, alignment and roughness of the surface. Artificial open channels are built for some specific purpose, such as irrigation, water supply, water power development. They are regular in shape and alignment. The roughness of the boundary surface is also uniform.

Depending upon the shape, a channel is either prismatic or non-prismatic. A channel is said to be prismatic when the cross section is uniform and bed slope is constant. In a non-prismatic channel, either the cross section or slope or both change. It is obvious that only artificial channel can be prismatic. A prismatic channel can be of any regular shape. The most common shapes are rectangular, parabolic, triangular, trapezoidal and circular (Figure 7.2).

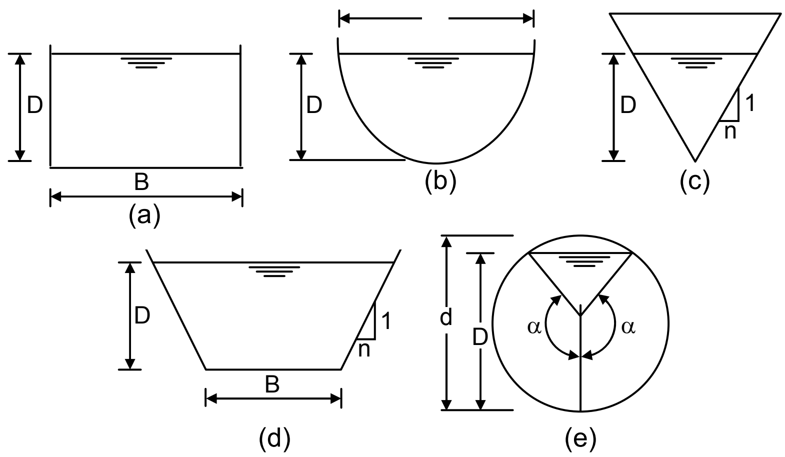


Figure 7.2

7.3 TYPES OF FLOW

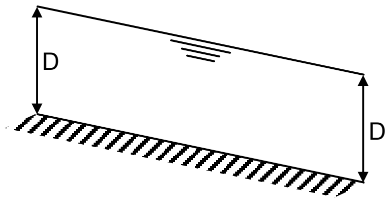
The flow in an open channel can be either uniform or non-uniform. The flow is uniform when the depth of liquid is constant (Figure 7.3(a)). If the depth varies along its length, the flow is non-uniform (Figure 7.3(b)). Non-uniform flow is also called the *varied flow.*

Non-uniform can be further divided into two types :

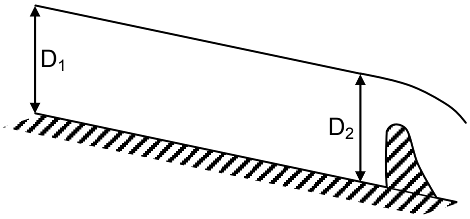
(a) Gradually varied flow, and

(b) Rapidly varied flow.

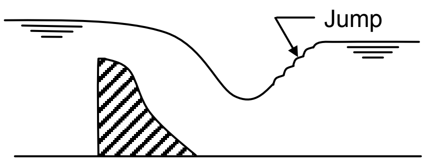
In gradually varied flow, the variation of the depth of liquid along the length is gradual (Figure 7.3(b)). In rapidly varied flow, the change in depth is sudden. For example, when water flows over an over flow dam, there is a sudden rise of water at the toe of the dam, and a hydraulic jump forms. This is a case of rapidly varied flow (Figure 7.4(c)).



(a) Uniform Flow



(b) Gradually Varied Flow



(c) Rapidly Varied Flow

Figure 7.3

The flow in an open channel can either be laminar or turbulent. In practice, however, the laminar flow occurs very rarely. The engineer is mainly concerned with the turbulent flow. The flow is turbulent when the Reynold number N_R exceeds 1000, where $N_R = \frac{\rho\, VR}{\mu}$ in which R is the hydraulic radius.

The flow in an open channel can either be steady or unsteady. The flow is steady when, at a particular section, the depth of liquid and other parameters, such as velocity, area of cross section, discharge, do not change with time. In any unsteady flow, the depth of flow and other characteristics, such as velocity, area of cross section, discharge, change with time.

The discussions in this unit will be limited to fundamentals of steady, uniform, turbulent flow in open channels.

7.4 DEFINITIONS

Before we take up the derivations of formulae for open channel flow, it will be worthwhile to define certain parameters related with it.

Wetted Perimeter (P)

The wetted perimeter is the length of the line (or curve) of inter-section of the wetted surface with a cross-section normal to the direction of flow. It follows that the wetted perimeter is equal to the length of base and sides up to the liquid surface in a rectangular channel. It must be noted that the free surface is not included in the wetted perimeter, as there is no boundary surface to cause any friction. The wetted perimeter (P) for the trapezoidal channel shown in Figure 7.2(d) is given by

$$P = B + 2\sqrt{D^2 + (nD)^2} = B + 2D\sqrt{1 + n^2}$$

where n is the side slope (n horizontal, 1 vertical), B is the base width and D is the depth.

Area of Cross Section (A)

The area of cross section is the area of the liquid surface of the channel cut when a cross-section is taken normal to the direction of flow. For the trapezoidal channel (Figure 7.2(d))

$$A = BD + 2 \times \frac{1}{2} nD \times D = BD + nD^2$$

Hydraulic Radius (R)

It is the ratio of the area of cross-section to the wetted perimeter. Thus

$$R = \frac{A}{P}$$

For the trapezoidal channel (Figure 7.2(d))

$$R = \frac{BD + nD^2}{B + 2D\sqrt{1 + n^2}}$$

For a rectangular channel (Figure 7.2(a))

$$A = B \times D;\ P = B + 2D;\ R = \frac{BD}{B + 2D}$$

Hydraulic Gradient Line (HGL)

The hydraulic gradient line is the line indicating the pressure at various sections along the channel. In case of open channels, the hydraulic gradient line coincides with the liquid surface. If a piezometer is inserted in an open channel, the liquid will rise in the tube to the level of the liquid surface.

The liquid surface slope (S_w) is the slope of the liquid surface or hydraulic gradient line (Figure 7.4).

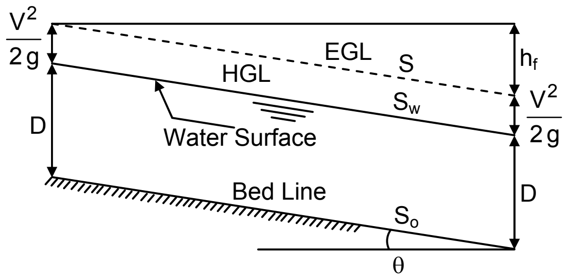


Figure 7.4

Energy Gradient Line (EGL)

The energy gradient line is the line indicating the total energy of the liquid with respect to a selected datum. The energy gradient line is above the hydraulic gradient. The vertical distance between the two lines is $\alpha V^2/2g$, where α is the velocity distribution factor that takes into account the non-uniform distribution of velocity across the section. The value of α varies from 1.1 to 1.2 for the turbulent flow in open channels. However, for simplicity, it is usually taken as unity.

Hydraulic Slope

Hydraulic slope (S) is the slope of the total energy line. It is given by

$$S = \frac{h_f}{L}$$

where L is the length of channel in which the drop of the total energy line is equal to h_f. For uniform flow, $S = S_w = S_0$; where S_w is the water surface slope and S_0 is the bed slope.

It will be assumed that the bed slope of the channel is small. The slope may be taken either the tangent or the sine of angle θ. Strictly speaking, the slope should be taken as the sine of the angle of inclination, but for small angles, it can be taken as the tangent of the angle of inclination as well.

Pressure Distribution Coefficient

When the filaments of the stream are straight and parallel, the pressure at any point at a depth d below the free surface is given by hydrostatic equation,

$$p = \gamma d \qquad \text{. . . (a)}$$

If the streamlines are curved, the flow is curvilinear. The pressure at a point at a depth d will be more than or less than that given by Eq. (a) depending upon whether the centre of curvature is upward or downward respectively. It is given by

$$p = \alpha' (\gamma d) \qquad \text{. . . (b)}$$

where α' is the pressure distribution coefficient.

For uniform flow, $\alpha' = 1$; for upward centre of curvature (concave flow), $\alpha' > 1$, and for downward centre of curvature (convex flow), $\alpha' < 1$.

As the discussions in this unit are limited to uniform flow, the pressure distribution coefficient will be taken as unity, i.e. the pressure will be assumed to be hydrostatic.

7.5 CHEZY'S FORMULA

Figure 7.5 shows the longitudinal section of open channel in which the flow is steady and uniform. Let us consider the forces acting on the liquid in the reach between two sections 1-1 and 2-2. As the flow is steady and uniform, it is neither accelerating nor decelerating. The body of the liquid between the sections must be in equilibrium. Let us assume that the pressure distribution is hydrostatic.

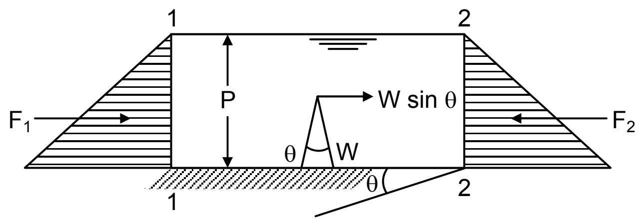


Figure 7.5

Summing up the forces acting on the mass of the liquid between the two sections in the direction of flow and equating them to zero,

$$W \sin \theta + F_1 - F_2 - \tau_0 PL = 0 \qquad \text{. . . (a)}$$

where W = weight of the liquid between sections 1-1 and 2-2 = γAL

in which γ = specific weight of liquid, A = area of cross-section

and L = length of the channel between sections 1-1 and 2-2

F_1 = hydrostatic force on the section 1-1

$$F_1 = \gamma A\bar{h} = \gamma A\left(\frac{D}{2}\right)$$

where $\bar{h}$ is the depth of the centroid $\left(= \frac{D}{2}\right)$ and D is the depth of flow.

(**Note :** The hydrostatic force is equal to the product of the area and the pressure intensity at the centroid of the area.)

F_2 = hydrostatic force on the section 2-2 $= \gamma A\bar{h} = \gamma A\left(\frac{D}{2}\right)$

and τ_0 = boundary shear stress

Substituting these values in Eq. (a)

$$(\gamma AL) \sin \theta + \gamma A\left(\frac{D}{2}\right) - \gamma A\left(\frac{D}{2}\right) - \tau_0 PL = 0$$

$$(\gamma AL) \sin \theta = \tau_0 PL$$

Substituting $\sin \theta = \frac{h_f}{L}$,

$$\frac{\gamma A h_f}{L} = \tau_0 P$$

or

$$\tau_0 = \gamma RS \qquad \text{. . . (b)}$$

where S is the hydraulic slope.

According to Chezy, the shear stress τ_0 is proportional to the square of the velocity.

Thus,

$$\tau_0 = kV^2 \qquad \text{. . . (c)}$$

where k is a constant of proportionality.

From Eqs. (b) and (c),

$$kV^2 = \gamma RS$$

or

$$V = \sqrt{\frac{\gamma}{k}}\sqrt{RS}$$

or $$V = C\sqrt{RS} \qquad \ldots (7.1)$$

where C is a coefficient equal to $\sqrt{\frac{\gamma}{k}}$ and is known as Chezy coefficient. Eq. (7.1) is the well-known Chezy's equation. The same equation is also used for pipe flow.

Many attempts had been made by various investigators to determine the value of Chezy's coefficient C as described below.

7.6 DETERMINATION OF CHEZY'S *C* BY GANGUILLET-KUTTER FORMULA

Several investigators gave their own expressions for the Chezy coefficient C. In 1869, Ganguillet and Kutter published a formula expressing the value of C in terms of slope S, hydraulic radius R and the coefficient of rugosity N, as

$$C = \frac{23 + \frac{0.00155}{S} + \frac{1}{N}}{1 + \frac{N}{\sqrt{R}}\left(23 + \frac{0.00155}{S}\right)} \qquad \ldots (7.2)$$

The coefficient N is known as Kutter's coefficient. Its value depends upon the nature of surface of the channel. Table 7.1 gives the typical values of N for various surfaces. These values are also equal to Manning's N discussed later. Once C has been estimated, Eq. (7.1) can be used for the computation of velocity V.

7.7 MANNING'S FORMULA

In 1889, Manning proposed the following formula :

$$V = \left(\frac{1}{N}\right) R^{2/3}\, S^{1/2} \qquad \ldots (7.3)$$

where V = mean velocity, R = hydraulic radius, S = slope,

N = Manning's rugosity coefficient.

Table 7.1 : Kutter's as well as Manning's Coefficients

Sl. No.	Description of Channel Surface	N
1	Well planned timber, glass or brass	0.009
2	Wood-stave flumes, finished concrete	0.010
3	Glazed tiles, vitrified sewers, concrete pipes	0.013
4	Bricks in cement mortar, ashlar	0.015
5	Rubble masonary in cement	0.025
6	Straight unlined canals in earth	0.020
7	Unlined canals in gravel, unlined canals in earth with some curves	0.0225
8	Corrugated metal flumes, unlined canal winding	0.025
9	Canals with rough stony bed or weeds	0.030
10	Winding natural streams in good conditions	0.035
11	Rivers of irregular cross-section winding	0.04 to 0.10

Owing to its simplicity and fairly good accuracy, the Manning formula is the most commonly used formula for the computations of open channel flow. In the normal range of slopes and hydraulic radii, the value of Manning's *N* is equal to Kutter's *N*. Therefore, the values of *N* given in Table 7.1 may also be used in Manning's formula. The Manning formula is much simpler than the Ganguillet-Kutter formula. For the same roughness of the channel surface, both formulae give almost the same results, except for very flat slopes

The value of *N* can also be obtained from Strickler formula,

$$N = \frac{k^{1/6}}{24}$$

where *k* is the average height of roughness projection in metres.

Comparing Manning's formula with Chezy's formula,

$$V = C\sqrt{RS} = \left(\frac{1}{N}\right) R^{2/3}\, S^{1/2}$$

$$C = \left(\frac{1}{N}\right) R^{1/6} \qquad \text{. . . (7.4)}$$

Example 7.1

The cross-section of an open channel is a trapezium with a bottom width of 4 m and side slopes 1 vertical to 2 horizontal. Calculate the discharge if the depth of water is 1.5 m and $S = \frac{1}{1600}$. Use Chezy's formula. $C = 50$.

Solution

For the trapezoidal section (see Figure 7.2 (d))

$$A = \frac{\text{(Base width + Top width)}}{2} \times \text{Depth}$$

$$= \left(\frac{4 + (4 + 2 \times 1.50 \times 2)}{2}\right) \times 1.50 = 10.5 \text{ m}^2$$

$$P = \text{Base width} + 2 \times \text{Side length}$$

$$= 4 + 2 \times 1.50\sqrt{1 + 2^2} = 10.71 \text{ m}$$

$$R = \frac{A}{P} = \frac{10.5}{10.71} = 0.98 \text{ m}$$

From Chezy's formula Eq. (7.1),

$$V = C\sqrt{RS}$$

or $$V = 50\sqrt{0.98 \times \left(\frac{1}{1600}\right)} = 1.24 \text{ m/sec}$$

Discharge = $V \times A$ = 1.24 × 10.50 = 13.02 cumecs

Example 7.2

A rectangular channel has a base width of 2.5 m and a slope of 1 in 400. Find the depth of flow if the discharge is 10 cumecs. Use Chezy's formula. $C = 50$.

Solution

Let D be the depth of flow (Figure 7.2 (a))

$$\text{Area } (A) = \text{Width} \times \text{Depth}$$

or $$A = 2.50\,D$$

Wetted perimeter (P) = Width + 2 × Depth = 2.50 + 2 × D

$$\text{Hydraulic radius } (R) = \frac{A}{P} = \frac{2.50D}{2.50 + 2D}$$

From Chezy's formula (Eq. (7.1)),

$$V = C\sqrt{RS}$$

And $$Q = VA = CA\sqrt{RS}$$

Substituting the values,

$$10 = 50 \times (2.50D)\sqrt{\frac{2.50D}{2.50 + 2D} \times \frac{1}{400}}$$

$$\frac{2.50D}{2.50 + 2D} = \left(\frac{1.6}{D}\right)^2 = \frac{2.56}{D^2}$$

or $$D^3 - 2.05D - 2.56 = 0 \qquad \text{. . . (a)}$$

Solving Eq. (a), by trial and error, D = 1.85 m

Example 7.3

A trapezoidal canal has a bottom width of 3 m and side slopes 1 vertical to 2 horizontal. If the slope of the bed is 1 in 5000 and the depth of water is 1.5 m, calculate the discharge using (a) Ganguillet-Kutter formula, (b) the Manning formula. Take N = 0.025.

Solution

(a) According to Ganguillet-Kutter formula (Eq. (7.2))

$$C = \frac{23 + \dfrac{0.00155}{S} + \dfrac{1}{N}}{1 + \dfrac{N}{\sqrt{R}}\left(23 + \dfrac{0.00155}{S}\right)}$$

$$A = (3 + 2 \times 1.5) \times 1.50 = 9 \text{ m}^2$$

$$P = 3 + 2 \times 1.50\sqrt{1 + 2^2} = 3 + 3 \times 2.24 = 9.72$$

$$R = \frac{A}{P} = \frac{9}{9.72} = 0.926$$

Thus, $$C = \frac{23 + \dfrac{0.00155}{0.0002} + \dfrac{1}{0.025}}{1 + \dfrac{0.025}{\sqrt{0.926}}\left(23 + \dfrac{0.00155}{0.0002}\right)} = 39.3$$

$$V = 39.3\sqrt{0.926 \times \frac{1}{5000}} = 0.53 \text{ m/s}$$

$$Q = 0.53 \times 9 = 4.77 \text{ cumecs}$$

(b) Using Manning's formula (Eq. (7.3))

$$V = \left(\frac{1}{N}\right) R^{2/3}\ S^{1/2}$$

$$V = \left(\frac{1}{0.025}\right) (0.926)^{2/3} \left(\frac{1}{5000}\right)^{1/2} = 0.537\ \text{m/sec}$$

$$Q = 0.537 \times 9 = 4.83 \text{ cumecs}$$

SAQ 1

(a) A rectangular channel 5.5 m wide and 1.25 depth has a bed slope of 1 in 900. Determine the discharge. Manning's $N = 0.015$.

(b) An open channel has trapezoidal section with a base width of 2 m and side slopes of 1 : 1. If the depth is 1.50, determine the required slope for a discharge of 5 m^3/s. Take $N = 0.015$.

(c) An open channel of trapezoidal section carries 1.50 m^3/s of water at a depth of 1.0 m. If the mean velocity of flow is 0.5 m/s and the side slopes of 1 V : 1 H, find the base width and bed slope. Take Chezy's $C = 60$.

(d) The cross-section of an open channel consists of a rectangle of size 1 m × 0.5 m placed over a semi-circle of diameter 1.0 m (Figure 7.7). If the bed slope is 1 in 2500, calculate the value of Chezy's C if the discharge is 0.50 m^3/s.

7.8 CIRCULAR CHANNELS

For circular channels, it is convenient to express the depth of flow (D), wetted perimeter (P) and the area of cross-section (A) in terms of the angle subtended by the free surface at the centre. Figure 7.6 shows a circular channel of diameter d with the depth of flow D. From geometry of the figure,

$$D = \frac{d}{2} + \frac{d}{2} \cos (180^{\circ} - \alpha) \qquad \text{. . . (7.5)}$$

where 2 α is the angle (in degrees) subtended by the free surface at the centre.

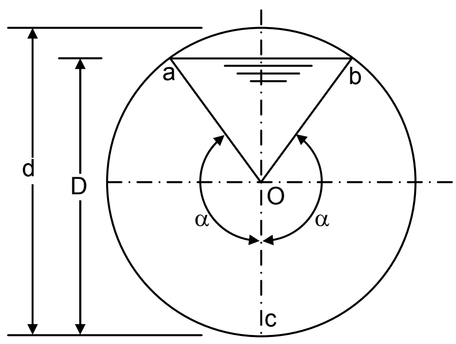


Figure 7.6

The wetted perimeter (P) is equal to

$$\frac{\pi d}{360} \times 2\alpha = \frac{\alpha \pi d}{180}$$

Also, $$P = \alpha\, d \qquad \text{. . . (7.6)}$$

where α is in radians.

The area of cross-section (A) is given by

$$A = \text{area } `acb' + \text{area } `oab'$$

$$= \frac{\pi}{4} d^2 \times \frac{1}{2\pi} \times 2\alpha + 2 \times \left(\frac{1}{2} \times \frac{d}{2} \sin \alpha \times \frac{d}{2} \cos (180^\circ - \alpha \right)$$

$$= \frac{d^2 \alpha}{4} - \frac{d^2}{4} \sin \alpha \cos \alpha$$

$$= \frac{d^2 \alpha}{4} - \frac{d^2}{8} (2 \sin \alpha \cos \alpha)$$

or

$$A = \frac{d^2 \alpha}{4} - \frac{d^2}{8} \sin 2\alpha \qquad \ldots (7.7)$$

Eqs. (7.5), (7.6) and (7.7) can be used to solve problems on circular channels.

The velocity of flow by Chezy's formula can be expressed as

$$V = C \sqrt{RS} = C \sqrt{S} \left(\frac{A}{P} \right)^{1/2}$$

$$V = C \sqrt{S} \left(\frac{\frac{d^2 \alpha}{4} - \frac{d^2}{8} \sin 2\alpha}{\alpha d} \right)^{\frac{1}{2}} \qquad \ldots (7.8)$$

The discharge Q can be expressed as

$$Q = AV = \frac{C \sqrt{S}}{(\alpha d)^{1/2}} \left(\frac{d^2}{4} \alpha - \frac{d^2}{8} \sin 2\alpha \right)^{\frac{3}{2}} \qquad \ldots (7.9)$$

For any depth of flow D, the value of α can be computed from Eq. (7.5) and then the corresponding velocity and discharge can be found from Eqs. (7.8) and (7.9), respectively.

Table 7.2 gives the values of $\frac{P}{P_f}$, $\frac{A}{A_f}$ and $\frac{R}{R_f}$ for different values of $\frac{D}{d}$ ratio,

where suffix f denotes the value for the same channel when running full. The table is quite useful for the computation of velocity and discharge at any depth D in a channel of diameter d.

Table 7.2 : Values of $\frac{P}{P_f}$, $\frac{A}{A_f}$ and $\frac{R}{R_f}$ for different $\frac{D}{d}$ ratio

D/d	***P/P$_f$***	***A/A$_f$***	***R/R$_f$***
0.10	0.205	0.052	0.254
0.20	0.295	0.142	0.482
0.30	0.369	0.252	0.684
0.40	0.436	0.374	0.857
0.50	0.500	0.500	1.000
0.60	0.564	0.626	1.110
0.70	0.631	0.748	1.185
0.80	0.705	0.858	1.217
0.90	0.795	0.948	1.192
1.00	1.00	1.00	1.000

Example 7.4

Water flows through a circular channel of diameter 600 mm at the rate of 0.142 m^3/s. If the slope of the channel is 1 in 500 and the depth of water is 450 mm, calculate Chezy's coefficient and the velocity of flow.

Solution

From Eq. (7.5),

$$D = \frac{d}{2} + \frac{d}{2} \cos (180^o - \alpha)$$

or $$0.45 = \frac{0.60}{2} + \frac{0.60}{2} \cos (180^o - \alpha)$$

or $$\cos (180° - \alpha) = 0.50 = \cos 60°$$

or $$\alpha = 120^o = \frac{2\pi}{3} \text{ radians}$$

From Eq. (7.7),

Area of flow, $$A = \frac{d^2 \alpha}{4} - \frac{d^2}{8} \sin 2\alpha$$

$$= \frac{(0.60)^2 \times \frac{2\pi}{3}}{4} - \frac{(0.60)^2}{8} \sin 240^o$$

$$= 0.188 + 0.039 = 0.227 \text{ m}^2$$

From Eq. (7.6), Wetted perimeter,

$$P = \alpha\, d$$

$$= \frac{2\pi}{3} \times 0.60 = 1.257 \text{ m}$$

Hydraulic radius, $R = \frac{0.227}{1.257} = 0.181$ m

Velocity of flow, $V = \frac{Q}{A}$

$$= \frac{0.142}{0.227} = 0.626$$

Now from Chezy's formula,

$$V = C\sqrt{RS}$$

$$0.626 = C\sqrt{0.181 \times \frac{1}{500}}$$

$$C = 32.9$$

SAQ 2

A circular sewer of 500 mm diameter has a slope of 1 in 144. Find the depth of flow for a discharge of 0.30 m^3/s. Chezy's $C = 50$.

7.9 CONDITIONS FOR MAXIMUM DISCHARGE AND MAXIMUM VELOCITY IN A CIRCULAR CHANNEL

Figure 7.6 shows the cross-section of a circular channel. Let D be the depth of liquid for the maximum discharge. The diameter of the channel is d. In circular channels, it must be noted that the cross-sectional area A and the wetted perimeter P both depend on the angle α, where 2 α is the angle subtended by the free surface at the centre. For circular sections, the angle α is the most suitable variable for the determination of conditions for the maximum discharge. For rectangular and trapezoidal channels, the depth D is most suitable parameter.

Using the Chezy's formula,

$$Q = AC\sqrt{RS}$$

or $$Q = AC\sqrt{\left(\frac{A}{P}\right)S} = C\sqrt{\left(\frac{A^3}{P}\right)S}$$

For the discharge to be a maximum, $\frac{dQ}{d\alpha} = 0$

or $$\frac{d}{d\alpha}\left(\frac{A^3}{P}\right) = 0 \quad \text{as } C \text{ and } S \text{ are constant.}$$

or $$3P\frac{dA}{d\alpha} - A\frac{dP}{d\alpha} = 0 \qquad \text{. . . (a)}$$

Substituting these values of P and A from Eqs. (7.6) and (7.7) in Eq. (a)

$$3d\alpha\frac{d}{d\alpha}\left(\frac{d^2\alpha}{4} - \frac{d^2}{8}\sin 2\alpha\right) - \left(\frac{d^2\alpha}{4} - \frac{d^2}{8}\sin 2\alpha\right)\frac{d\,(\alpha d)}{d\alpha} = 0$$

or $$3d\alpha\left(\frac{d^2}{4} - \frac{d^2}{4}\cos 2\alpha\right) - \left(\frac{d^2\alpha}{4} - \frac{d^2}{8}\sin 2\alpha\right)d = 0$$

or $$\frac{3}{4}\alpha d^3 - \frac{3}{4}d^3\alpha\cos 2\alpha - \frac{d^3\alpha}{4} + \frac{d^3}{8}\sin 2\alpha = 0$$

or $$4\alpha - 6\alpha\cos 2\alpha + \sin 2\alpha = 0$$

The solution of this equation is (by trial and error),

$$\alpha = 2.68 \text{ radians} \quad \text{or} \quad \alpha = 154^\circ$$

Therefore, the depth of flow,

$$D = \frac{d}{2} + \frac{d}{2}\cos(180^\circ - \alpha)$$

or $$D = \frac{d}{2} + (1 - \cos\alpha)$$

Substituting $\alpha = 154°$, $D = \frac{d}{2}(1 - \cos 154^\circ)$

or $$D = \frac{d}{2}(1 + 0.90) = 0.95\,d \qquad \text{. . . (7.10)}$$

Thus the maximum discharge in a circular channel occurs when the depth of flow is 0.95 times the diameter. *This condition holds good when the Chezy's formula is* *used*. If any other formula is used, the depth of flow will be different. If Manning's formula is used, the maximum discharge occurs when the depth is 0.94 d ($\alpha = 151^o$).

7.9.1 Conditions for Velocity in a Circular Channel

The condition for the maximum velocity is different from the condition for maximum discharge. The condition for the maximum velocity may be obtained as follows :

From the Chezy formula,

$$V = C\sqrt{RS} = C\sqrt{\left(\frac{A}{P}\right)S}$$

For the velocity to be a maximum, $\frac{dV}{d\alpha} = 0$

or
$$\frac{d\left(\frac{A}{P}\right)}{d\alpha} = 0$$

or
$$P\frac{dA}{d\alpha} - A\frac{dP}{d\alpha} = 0$$

Substituting the values of P and A,

$$(d\alpha)\left(\frac{d^2}{4} - \frac{d^2}{4}\cos 2\alpha\right) - \left(\frac{d^2\alpha}{4} - \frac{d^2}{8}\sin 2\alpha\right)d = 0$$

or
$$2\alpha - 2\alpha\cos 2\alpha - 2\alpha + \sin 2\alpha = 0$$

or
$$2\alpha\cos 2\alpha = \sin 2\alpha$$

or
$$\tan 2\alpha = 2\alpha \qquad \text{. . . (a)}$$

The solution of Eq. (a) is (by trial and error),

$$\alpha = 2.25 \text{ radians or } \alpha = 128.75^o$$

Depth of flow,
$$D = \frac{d}{2} - \left(\frac{d}{2}\right)\cos\alpha = \frac{d}{2}(1 - \cos\alpha)$$

$$= \frac{d}{2}(1 + 0.62) = 0.81d \qquad \text{. . . (7.12)}$$

Thus, the maximum velocity occurs when the depth of flow is 0.81 times the diameter.

The maximum velocity will occur at the same depth even if Manning's formula is used.

It may be noted that the procedure adopted for circular sections may also be used for other shapes with gradually closing tops.

Example 7.5

An open channel has a diameter of 1.68 m. If the channel slope is 1 in 5000, calculate the maximum discharge which the channel can carry. Use Chezy's formula, $C = 70$.

Solution

For the maximum discharge, α = 154°

Area of cross-section, $A = \left(\frac{d^2}{4}\right) \alpha - \left(\frac{d^2}{8}\right) \sin 2\alpha$

Substituting α = 154° = 2.68 radians,

$$A = \left(\frac{d^2}{4}\right)(2.68) - \left(\frac{d^2}{8}\right) \sin 308^{\circ} = 0.768\, d^2$$

Substituting $d = 1.68$ m, $A = 0.768\,(1.68)^2 = 2.17$ m^2

Wetted perimeter, $P = \alpha\, d = 2.68\, d = 4.50$ m

Hydraulic radius, $R = \frac{A}{P} = \frac{2.17}{4.50} = 0.482$ m

Discharge $\quad Q = AV = AC\sqrt{RS}$

or $\quad Q = 2.17 \times 70 \sqrt{0.482 \times \left(\frac{1}{5000}\right)} = 1.49$ m^3/s

Example 7.6

A circular sewer with diameter 0.5 m is laid at a slope of 1 in 225. What is the maximum velocity of flow that can occur? What would be the discharge at that velocity? Take $C = 60$.

Solution

For the maximum velocity, $\alpha = 128.75^{\circ} = 2.25$ radians

Area of flow, $A = \frac{d^2}{4}\alpha - \frac{d^2}{8}\sin 2\alpha = \frac{(0.5)^2}{4}(2.25) + \frac{(0.5)^2}{8}(0.975)$

$= 0.171$ m^2

Wetted perimeter, $P = \alpha\, d = 2.25 \times 0.50 = 1.125$ m

$$R = \frac{A}{P} = \frac{0.171}{1.125} = 0.152 \text{ m}$$

Velocity, $\quad V = C\sqrt{RS} = 60\sqrt{0.152 \times \frac{1}{225}} = 1.56$ m/sec

$$Q = AV = 0.171 \times 1.56 = 0.267 \text{ cumecs}$$

Example 7.7

For flow in open channels, derive continuity equation in differential from. Reduce it for case of a rectangular channel of constant width (Figure 7.7).

Solution

Discharge coming in high section 1 = Q

Discharge coming out through section $2 = Q + \frac{\partial Q}{\partial x}\,\delta x$

$\therefore$ Net rate of inflow $= Q - \left(Q + \dfrac{\partial Q}{\partial x}\, \delta x \right) = - \dfrac{\partial Q}{\partial x}\, \delta x$

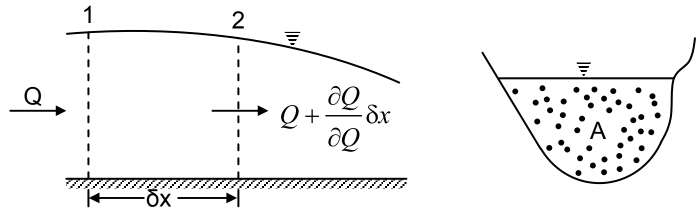


Figure 7.7 : Definition Sketch

If there is net inflow, then liquid volume within sections 1 and 2 must increase. Liquid within the two sections $= A\ \delta x$.

$\therefore$ Its rate of increase $\dfrac{\partial}{\partial t}(A\ \delta x) = \delta x\ \dfrac{\partial A}{\partial t}$

$\therefore$ Equating the two, one gets $\delta x\ \dfrac{\partial A}{\partial t} + \dfrac{\partial Q}{\partial x}\ \delta x = 0$

or
$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \qquad \text{. . . (7.13)}$$

For rectangular channel $A = By$ where B is the channel width. If B is constant, the above equation can be written as

$$\frac{\partial}{\partial t}(By) + \frac{\partial}{\partial x}(qB) = 0$$

or
$$\frac{\partial y}{\partial t} + \frac{\partial q}{\partial x} = 0$$

where q is the discharge per unit width. If $q = Uy$ is substituted in the above equation, one gets

$$\frac{\partial y}{\partial t} + U\ \frac{\partial y}{\partial x} + y\ \frac{\partial U}{\partial x} = 0 \qquad \text{. . . (7.14)}$$

If flow is steady $\dfrac{\partial A}{\partial t} = 0$ and $\dfrac{\partial A}{\partial t} + \dfrac{\partial Q}{\partial x} = 0$ reduces to $\dfrac{\partial Q}{\partial x} = 0$ or

$$Q = \text{const.} \quad \therefore\ U_1\ A_1 = U_2\ A_2$$

Example 7.8

Show that for narrow deep rectangular channels, hydraulic radius R is nearly equal to $\dfrac{B}{2}$.

Solution

$$A = By,\ P = B + 2y$$

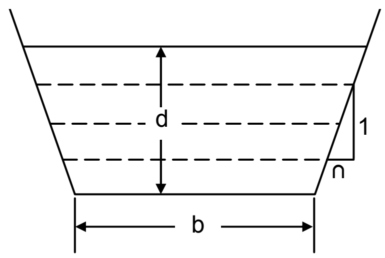
$$R = \frac{A}{P} = \frac{By}{B + 2y} = \frac{B}{\dfrac{B}{y} + 2}$$

Now as y becomes large, for small B, $\frac{B}{y} \to 0$.

Hence for large y values $R \to \frac{B}{2}$.

Example 7.9

Develop the following set of conditions for the maximum discharge through a channel of trapezoidal section for a given cross-sectional area and a fixed bed slope.



$$d \sqrt{(1+n)^2} = \frac{b + 2nd}{2}$$

and $\qquad n = \frac{1}{\sqrt{3}}$

or $\qquad w = \frac{d}{2}$

Simplify these conditions for a rectangular channel and for a triangular channel.

Solution

For a trapezoidal section as shown,

Area of flow $A = \frac{b + (b + 2nd)}{2} \cdot d = (b + nd)\, d$ $\qquad$ . . . (7.15)

Wetted perimeter $P = b + 2\sqrt{(d^2 + n^2 d^2)}$

or $\qquad P = b + 2d\sqrt{(1+n^2)}$ $\qquad$ . . . (7.16)

Eliminating b by using Eq. (7.15)

$$P = \frac{A}{d} - nd + 2d\sqrt{(1+n^2)}$$

Discharge $\qquad Q = AU = AC\sqrt{mi}$

$$= AC\sqrt{\frac{A}{P}}\, i \;\alpha\; \sqrt{\frac{1}{P}}$$

If A, C and i are otherwise fixed, as is implied in the problem.

Maximum Q corresponds to minimum P. The condition requires that

$$\frac{\partial P}{\partial d} = 0 \text{ and } \frac{\partial P}{\partial n} = 0$$

(a) $$\frac{\partial P}{\partial d} = 0 = -\frac{A}{d^2} - n + 2\sqrt{(1+n^2)}$$

Substituting $A = (b + nd)\,d$ and simplifying,

$$d\sqrt{(1+n^2)} = \frac{b+2nd}{2} \qquad \text{. . . (7.17)}$$

which shows that the sloping side equals half the top width.

Further, for this section,

$$m = \frac{A}{P} = \frac{(b+nd)\,d}{b+2d\sqrt{(1+n^2)}}$$

or $$m = \frac{(b+nd)\,d}{b+b+2nd} = \frac{d}{2} \qquad \text{. . . (7.18)}$$

implying that the hydraulic mean depth equals half the actual depth.

(b) $\frac{\partial P}{\partial n} = 0$ for constant or optimum depth, requires that

$$d + 2d\,.\,\frac{1}{2}\,(1+n^2)^{1/3} \times 2n = 0$$

or $$2n = -\sqrt{1+n^2}$$

or $$4n^2 = 1 + n^2;\ n = \frac{1}{\sqrt{3}} \qquad \text{. . . (7.19)}$$

which corresponds to a side slope of 60° with the horizontal.

An important comment here is that the ultimate maximum discharge through a trapezoidal channel demands the fulfillment of both the above conditions but in practice, only one of the two conditions may be invoked, the other being assigned by the design and feasibility considerations.

It is easy to visualize a rectangular channel as a special case of the trapezoidal channel when $n = 0$.

From conditions (7.17) and (7.18)

$$d = \frac{b}{2}$$

and $m = \frac{d}{2}$ for a rectangular channel.

Another special case of the trapezoidal channel is a triangular channel,

when $b = 0$

Then $A = nd^2$

and $$P = 2d\sqrt{1+n^2} = 2\sqrt{A}\sqrt{\frac{1+n^2}{2}}$$

or $$P = 2\sqrt{A}\sqrt{\frac{\sec^2\theta}{\tan\theta}} = \frac{2\sqrt{A}\,\sec\theta}{\sqrt{\tan\theta}}$$

For minimum P, $\frac{\partial P}{\partial \theta} = 0$

or $$\frac{\sec\theta \tan\theta}{\sqrt{\tan\theta}} - \frac{\sec^3\theta}{2(\tan\theta)^{1.5}} = 0$$

Rearranging and simplifying the terms,

$$2\tan^2\theta = \sec^2\theta$$

or $$\sin\theta = \frac{1}{\sqrt{2}}$$

and $$\theta = 45^{\circ}$$

for the maximum discharge through a triangular channel.

SAQ 3

Determine the diameter of an open channel of circular section required to carry a maximum discharge of 0.30 m^3/s. The bed slope is 1 in 2500. $N = 0.0125$.

7.10 SUMMARY

- Pipe flow and open channel flow are compared. For open channel to occur, there should be free liquid surface and slope of the bed of the channel.
- Different types of open channels are outlined. Simple theories of open channel flow are considered only for prismatic channels.
- Depending on the variation of depth, the open channel flow can be uniform or non-uniform. In this unit, only steady uniform flow has been discussed.
- Various parameters are defined. The parameters are somewhat different from those in pipe flow because there is no boundary at the free surface.
- Empirical formulae such as Chezy's formula, Ganguillet-Kutter formula and Manning's formula are discussed. In practice, generally Manning's formula is used these days.
- Circular channels are commonly used for sewers. For circular channels, it is convenient to express the various parameters in terms of the angle 2α subtended by the free surface at the centre.
- In a circular channel, the maximum discharge occurs when the depth of flow is 0.95 times the diameter if the Chezy formula is used and 0.94 times the diameter if Manning's formula is used.
- The conditions for the maximum velocity are given. The maximum velocity occurs when the depth of flow is 0.81 times the diameter.

7.11 ANSWERS TO SAQs

SAQ 1

(a) $A = 5.5 \times 1.25 = 6.875 \text{ m}^2$

$$P = 5.5 + 2 \times 1.25 = 8.0 \text{ m}$$

$$R = \frac{A}{P} = \frac{6.875}{8.0} = 0.859 \text{ m}$$

$$Q = A \times V$$

$$= 6.875 \times \left(\frac{1}{N} R^{2/3} S^{1/2} \right)$$

$$= 6.875 \times \left(\frac{1}{0.015} \times (0.859)^{2/3} \left(\frac{1}{900} \right)^{1/2} \right)$$

or $\quad Q = 13.86 \text{ m}^3/\text{s}$

(b) $\quad A = (2 + 1 \times 1.5) \times 1.5 = 5.25 \text{ m}^2$

$$P = 2 + 2 \times 1.5 \sqrt{(1)^2 + (1)^2} = 6.24 \text{ m}$$

$$R = \frac{5.25}{6.24} = 0.841 \text{ m}$$

$$Q = A \times \frac{1}{N} R^{2/3} S^{1/2}$$

or $\quad 5.0 = 5.25 \times \dfrac{1}{0.015} \times (0.841)^{2/3} (S^{1/2})$

or $\quad S^{1/2} = 0.016$

or $\quad S = 2.56 \times 10^{-4}$ (= 1 in 3906)

(c) $\quad A = \dfrac{Q}{V} = \dfrac{1.50}{0.5} = 3.0 \text{ m}^2$

Now $\quad A = (B + nD)\, D$

or $\quad 3.0 = (B + 1.0 \times 1.0) \times 1.0$

or $\quad B = 2.0$ m

$$P = B + 2D \sqrt{1 + n^2}$$

$$= 2.0 + 2 \times 1.0 \sqrt{1 + 1} = 4.83 \text{ m}$$

$$R = \frac{A}{P} = \frac{3.0}{4.83} = 0.62 \text{ m}$$

$$V = C \sqrt{RS}$$

or $\quad 0.50 = 60 \sqrt{0.62 \times S}$

or $\quad S = 1.12 \times 10^{-4}$ (= 1 in 8928)

(d) $\quad A = 1 \times 0.5 + \dfrac{\pi}{8} \times (1)^2 = 0.893 \text{ m}^2$

$$P = 2 \times 0.50 + \frac{\pi \times 1.0}{2} = 2.57 \text{ m}$$

$$R = \frac{0.893}{2.57} = 0.347 \text{ m}$$

$$Q = AC\sqrt{RS}$$

or $$0.50 = 0.893 \times C\sqrt{0.347 \times \frac{1}{2500}}$$

or $$C = 47.52$$

SAQ 2

$$A = \frac{d^2\,\alpha}{4} - \frac{d^2}{8}\sin 2\alpha$$

$$P = \alpha\, d$$

$$Q = AC\sqrt{RS}$$

or $$Q^2 = A^3\, C^2 \left(\frac{1}{P}\right) S$$

or $$(0.30)^2 = \left(\frac{d^2\,\alpha}{4} - \frac{d^2}{8}\sin 2\alpha\right)^3 \times (50)^2 \times \frac{1}{\alpha\, d} \times \frac{1}{144}$$

or $$(2\alpha - \sin 2\alpha)^3 = \frac{(0.30)^2 \times (8)^3 \times (0.5) \times 144 \times \alpha}{(0.5)^6 \times 2500} = 84.90\,\alpha$$

or $$2\alpha - \sin 2\alpha = 4.396\,\alpha^{1/3}$$

Solving by trial and error, $\alpha = 2.5$ radians ($= 143.24^{\circ}$)

Depth of flow, $$D = \frac{d}{2}(1 - \cos 143.24^{\circ})$$

$$= \frac{0.50}{2}(1 + 0.801)$$

or $$D = 0.45 \text{ m}$$

SAQ 3

$$Q = \frac{A}{N} R^{2/3}\, S^{1/2}$$

or $$0.30 = \frac{A}{0.0125}\left(\frac{A}{P}\right)^{2/3}\left(\frac{1}{2500}\right)^{1/2}$$

or $$\frac{A^{5/3}}{P^{2/3}} = 0.1875$$

Substituting values of A and P in terms of diameter d,

$$\frac{\left(\frac{d^2}{4}\alpha - \frac{d^2}{8}\sin 2\alpha\right)^{\frac{5}{3}}}{(\alpha\, d)^{\frac{2}{3}}} = 0.1857 \qquad \text{. . . (a)}$$

For the discharge to be a maximum,

$$\frac{d}{d\alpha}\left(\frac{A^{5/3}}{P^{2/3}}\right) = 0$$

$$\frac{d}{d\alpha}\left(\frac{A^{5/3}}{P^{2/3}}\right)^{\frac{3}{2}} = 0$$

$$\frac{d}{d\alpha}\left(\frac{A^{5/2}}{P}\right) = 0$$

$$P \times 2.5\ A^{3/2}\ \frac{dA}{d\alpha} - A^{5/2}\left(\frac{dP}{d\alpha}\right) = 0$$

or $$2.5P\ \frac{dA}{d\alpha} - A\ \frac{dP}{d\alpha} = 0$$

or $$2.5\ (d\,\alpha)\left(\frac{d^2}{4} - \frac{d^2}{4}\cos 2\alpha\right) - \left(\frac{d^2}{4}\alpha - \frac{d^2}{8}\sin 2\alpha\right) d = 0$$

or $$3\alpha - 5\alpha \cos 2\alpha + \sin 2\alpha = 0$$

By trial and error, $\alpha = 2.63$ radians ($= 151.0^{\circ}$)

From Eq (a),

$$\frac{\left[\frac{d^2}{4} \times 2.63 - \frac{d^2}{8}\sin(302^{\circ})\right]^{\frac{5}{3}}}{(2.63\ d)^{2/3}} = 0.1875$$

or $d = 0.80$ m.

FURTHER READING

FLUID MECHANICS

The subject matter of Fluid is concerned with the study of fluids under all conditions of rest and motion. Almost every branch of engineering requires a good understanding of the basic principles of Fluid Mechanics in dealing with the diverse problems of flow of fluids encountered in practice. In addition, engineers and those aspiring to be engineers, like you, need to master the fundamentals as well as application of Fluid Mechanics, in order to be able to effectively practice the engineering profession.

A fluid can be defined as a substance which deforms or yields continuously when shear stress is applied to it, no matter how small it is. Fluids can be sub-divided into liquids and gases. We are living in an environment of air and water, i.e. fluids and therefore, whatever we do, it is connected to some extent with the concepts of Fluid Mechanics. Fluid Mechanics is the study of forces and motions on fluids, and is based on the basic principles of conservation of mass, momentum, energy and laws of thermodynamics.

The present course introduces you to the principles of Fluid Mechanics and is presented in seven units. We start the course with Unit 1, where you will be introduced to the various fluid properties, principles of hydrostatics leading to computation of forces on plane and curves surfaces.

In Unit 2, we present a discussion on flow of fluids. In this unit, you will be introduced to the concepts of flow of fluids through Euler's and Bernoulli's equations and their applications.

Orifices are used for the measurement of flow. Flow through orifices is presented in Unit 3. the unit describes different types of orifice and other relevant details.

In Unit 4, you will study flow through mouthpiece and minor losses. The unit also discusses the measurement of discharge through as open channel by a weir, notch and a venturi flume.

There are many situations in which a fluid is transported through pipes from one place to another. Water, for example, is an extremely essential fluid for us and it has to be brought to the place of our need. Therefore, we must have knowledge of mechanics of flow through simple pipes. The concept of flow through simple pipes has been introduced in Unit 5. In Unit 6, you will be introduced to flow through complex pipes. The unit deals with flow through pipes in series, pipes in parallel, branch pipes connecting three reservoirs, siphons, etc.

A liquid flowing with a free surface exposed to a constant pressure throughout constitutes an important class of flows with numerous applications in engineering. This is known as open channel flow irrespective of the fact whether the channel is open or closed as along as its surface is exposed to constant pressure. In Unit 7, you will be introduced to the flow through open channel.

During the course of study, you will observe that the course lays emphasis on the concepts, the basic principles leading to the development of governing equations of flow. The application and utility of these principles are demonstrated through illustrative examples at the end of each section. The Self Assessment Questions (SAQ) are intended to help you in verifying whether you have grasped the concept presented and provides the needed feedback about your progress. You are advised to study the text and illustrative examples very carefully. Try to solve the SAQs on your own and verify your answers with those given at the end of each unit. This will, definitely develop your confidence in analysing and solving the practical problem.