UNIT 2 FLOW OF FLUIDS

Structure

2.1 INTRODUCTION

This unit introduces you to the basic concepts of flow of fluids. Various methods of the flow visualisation are given. The streamline and streamtubes are defined. Different types of flow, depending upon the variation of space and time, are described.

Two basic equations of fluid mechanics, viz. the continuity equation and the Bernoulli's equation are applied to various fluid flow problems.

Basic equations are introduced in this unit. Application of these equation to flow-measuring devices are discussed.

Further applications of the basic equations shall be discussed in the subsequent units.

Objectives

After studying this unit, you should be able to

- explain the various types of fluid flow,
- describe continuity, Euler's and Bernoulli's equation along with their areas of applications in day-to-day life, and
- discuss the use and applications of orifice meter and different types of venturimeter.

2.2 VISUALIZATION OF THE FLOW PATTERN

It is a matter of common experience to observe the path taken by smoke emerging from a chimney on a windy day. The smoke gives a visual picture of the movement of the wind. To observe the path taken by a fluid particle in a laboratory, it is common practice to use droplets of oil or other shiny materials such as aluminum. Sometimes a visible agent, such as soot, silt, is also used to produce a visual picture of the flow phenomenon.

The motion of a fluid at any point is described in terms of velocity, which is defined as the rate of change of distance per unit time. The velocity differs from the speed in which no attention is paid to the change of direction. Velocity is a vector quantity as it has both magnitude and direction. It is commonly represented by an arrow. The length of the arrow is proportional to the magnitude of the velocity and the orientation of the arrow indicates the direction. If Δs is the distance travelled by a particle along a path in time Δt, the velocity 'V' is given by (Refer Figure 2.1).

$$V = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} \qquad \text{. . . (2.1)}$$

The direction of velocity is along the tangent to the path of particle at that point.

Pathline

The path followed by a fluid particle is called the *pathline*. A pathline shows the direction of a particular particle as it moves ahead. In general, this is a curve in three-dimensional space. However, if the conditions are such that the flow is two-dimensional, the curve becomes two-dimensional. Figure 2.2 shows a pathline. Velocities at different points on the pathline are also shown. At a particular instant, the velocity is V_1 which changes to V_2 after sometime, and then to V_3.

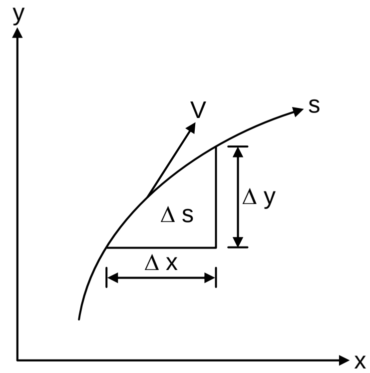


Figure 2.1

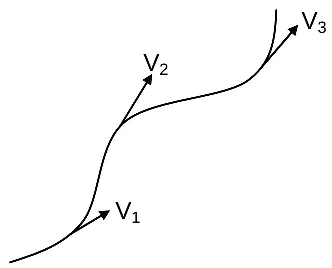
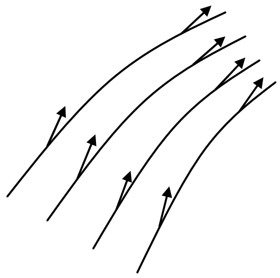


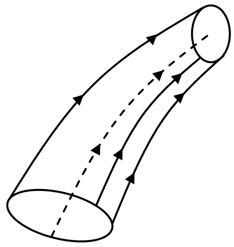
Figure 2.2

Streamline

The above method of representing flow phenomenon is not satisfactory as the flow pattern becomes very confusing when pathlines are drawn for a number of particles. A more satisfactory representation of flow as a whole at any instant may be obtained by sketching a series of curves in such a way that the velocity vectors for different points lying on the curves would meet the curves tangentially. These curves are known as streamlines. Thus, a *streamline may be defined as an imaginary line within the flow such that the tangent at any point on it indicates the velocity at that point.* It may be noted that a pathline gives the path of one particular particle at successive instants of time, whereas a streamline indicates the directions of a number of particles at the same instant. Streamlines are shown in Figure 2.3(a).



(a) Stream Line



(b) Stream Tube

Figure 2.3

In steady flow of real fluids where there are no velocity fluctuations, or in an ideal frictionless fluid, a pathline also becomes a streamline because particles move tangentially to streamlines. In unsteady flow, pathlines and streamlines are different (for definitions of steady and unsteady flow, refer Section 2.3).

Streakline

A streakline is a curve that *gives an instantaneous picture* of the location of fluid particles which had passed through a given point some earlier time. The path taken by smoke coming out of a chimney is an example of a streakline.

Streamtube

As the direction of velocity is always tangential to the streamline, it is obvious that there cannot be any movement of fluid across the streamline. The same reasoning holds if we consider a number of streamlines. A streamtube is a fluid mass bounded by a group of streamlines (Figure 2.3(b)). Because a streamtube is completely bounded by streamlines on all sides (except at ends), there can be no velocity normal to its sides. The fluid can enter or leave the streamtube only at the ends. The concept of streamlines and streamtubes is extremely useful in problems of fluid flow.

2.3 TYPES OF FLOW

There are different types of flow as explained below :

Laminar Flow and Turbulent Flow

Laminar Flow

Flow is said to be laminar when the paths taken by the individual particles do not cross one another. For example, in a pile flow, the fluid moves in the form of concentric cylinders sliding one within the other like the tubes of a telescope. These cylinders look like laminae rolled up into tubes. That is the reason why it is called laminar flow. In laminar flow the pathlines, which are also streamlines, are parallel. The streamtube is of constant cross-section and has the sides parallel to the walls of the conduit (Figure 2.4).

Laminar flow occurs in viscous fluids or in fluid phenomena in which viscosity plays a predominant role. This is the case when the

Reynold's number (N_R) of the flow is low. The flow of thick oil through a small tube is an example of laminar flow.

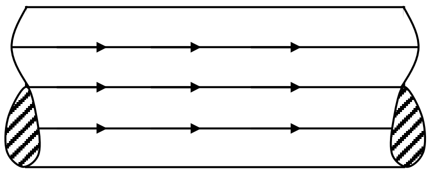
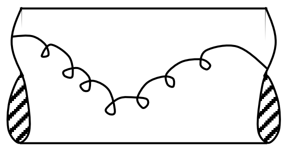
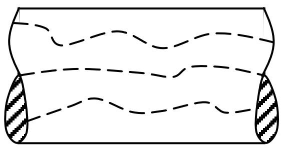


Figure 2.4 : Laminar Flow

Turbulent Flow

Flow is said to be turbulent when its pathlines are irregular curves crossing one another. The fluid particles occupy successively different transverse positions. The paths are neither parallel nor fixed. A particle may be at the centre of the conduit at an instant and near the wall at the next instant. Figure 2.5(a) shows the erratic path followed by a single particle during an interval of time. Figure 2.5(b) shows the paths taken by a number of particles at any instant.





(a) **(b)**

Figure 2.5

In turbulent flow, there is an irregular, chaotic motion of fluid particles. However, it has been observed in practice that at any fixed point in turbulent flow, the instantaneous velocity fluctuates about a mean value. In the study of turbulent flow, it is convenient to split the velocity into a temporal mean value $\vec{v}$ which is the average velocity at that point over a long period of time and a fluctuating component of the velocity v' which fluctuates about the mean value. The streamline concept, although not strictly applicable in turbulent flow, can still be used. In turbulent flow, the streamlines indicate the temporal mean velocity. The general fluid motion is taken as the criterion of flow and the fluctuations of the velocity are neglected.

The flow in rivers at the time of floods is turbulent. Turbulent flow is more common of the two types of flow. Generally, the flow in nature is turbulent.

Steady and Unsteady Flow

Steady Flow

The flow is said to be steady when the flow characteristics, such as velocity, density, pressure, temperature, do not change with time. A flow is steady when the rate of change of these characteristics is zero. For example, if V is the velocity at any point, the flow will be steady if

$$\frac{dV}{dt} = 0$$

Water flowing through a tap at a constant rate is an example of steady flow.

Unsteady Flow

The flow is unsteady if the velocity and other hydraulic characteristics change with respect to time. Mathematically, $\frac{dV}{dt} \neq 0$.

If the water is flowing at a changing rate, as is the case when the tap is just opened, the flow is unsteady. Similarly, in the case of a canal when the regulator is just opened, the flow is unsteady. After sometime the flow in the canal becomes constant and the flow becomes steady.

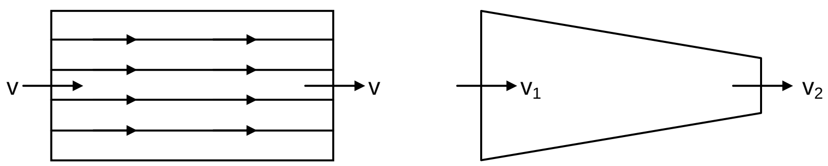
A truly steady flow is possible only when the flow is laminar. However, even in the case of turbulent flow when mean velocity and mean values of other characteristics do not change with time, the flow is termed steady.

Uniform and Non-uniform Flow

Uniform Flow

The flow is said to be uniform when the velocity and other characteristics are constant in a particular reach. In other words, a flow will be uniform if the rate of change of these characteristics with respect to distance along the path is zero. For example, if V is the velocity at any point, the flow will be uniform if $\frac{dV}{dS} = 0$; where S is the distance measured from some fixed point on the path of flow. In other words, the velocity is constant in the reach.

A liquid flowing through a long straight pipe of uniform diameter at constant rate is an example of uniform flow, as the velocity is the same at all sections in reach as shown in Figure 2.6(a).



(a) Uniform Flow **(b) Non-uniform Flow**

Figure 2.6

Non-uniform Flow

The flow is non-uniform when the flow characteristics change at various points along the path. For example, if V is the velocity at any point, the flow will be non-uniform if $\frac{dV}{dS} \neq 0$.

If the diameter of the pipe changes, i.e., the pipe is either converging or diverging, the velocity at different sections in the reach is not constant (Figure 2.6(b)) and the flow is non-uniform.

The reader should note that flow through a long uniform pipe, when the flow (discharge) is constant, is an example of uniform steady flow. If the pipe is non-uniform in diameter but the flow is constant, the flow is non-uniform steady flow. The varying flow, through a long uniform pipe is unsteady uniform flow. This type of flow very rarely occurs in practice. If the flow is changing with time and the pipe is also non-uniform, the flow is unsteady non-uniform flow. *Unless otherwise mentioned, the flow will be assumed to be steady and uniform.*

Incompressible and Compressible Flow

Incompressible Flow

A fluid is said to be incompressible if it cannot be compressed easily. The density of an incompressible fluid is almost constant. Incompressible fluid flow is flow of incompressible fluids.

Compressible Flow

A fluid is compressible if it can be easily compressed. Compressible fluids have variable density.

In reality, there is no fluid which can be classified as perfectly incompressible. The term is used for the fluids in which the density changes are negligibly small. Almost all the liquids at ordinary temperature and pressure are incompressible. But when the pressure variations are very large, as in the case of water hammer in pipes, the liquids behave like compressible fluids.

Almost all the gases behave like compressible fluids. However, when the pressure changes are very small, a gas may behave like an incompressible fluid. For example, in the case of flow of air in a ventilating system, the air is treated as an incompressible fluid. Thus the same fluid may be considered as compressible in one phenomenon and incompressible in another phenomenon. *Unless otherwise mentioned, the fluid shall be assumed to be incompressible.*

Ideal and Real Fluids

Ideal Fluids

An ideal fluid is one which has zero viscosity. Since there is no viscosity, there is no shear stress between adjacent fluid layers and between the fluid layers and the boundary. *Only normal stresses can exist in an ideal fluid.* In reality, there is no fluid which is ideal. However, in certain cases the fluid is assumed to be ideal. The assumption of an ideal fluid allows a fluid to be treated as an aggregation of small particles which support forces normal to their surfaces but slide over one another without any resistance. The assumption simplifies the treatment of flow phenomena. This enables mathematical or graphical methods to be adopted in the solution of flow problems. In general terms, an ideal fluid has no viscosity, no surface tension, no vaporization and it is incompressible.

Real Fluids

A real fluid is one which possesses viscosity. As soon as motion takes place, shearing stresses come into existence in real fluids. These stresses oppose the sliding of one layer over the other. *Thus a real fluid is characterized by its frictional resistance when it is in motion.*

Irrotational and Rotational Flow

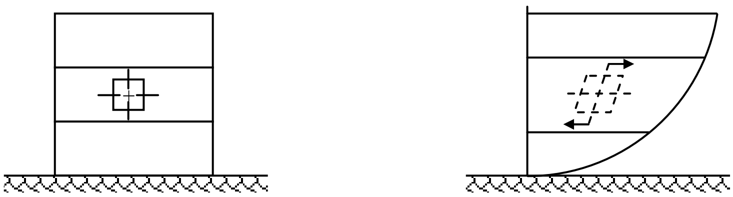
Irrotational Flow

A flow is said to be irrotational if the fluid elements do not rotate about their own mass centres.

Rotational Flow

In rotational flow, the fluid elements rotate about their mass centers.

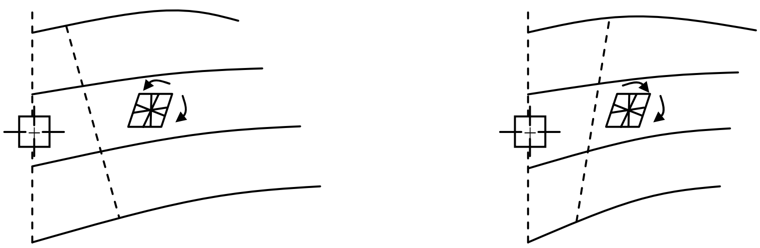
Figure 2.7(a) shows the velocity distribution adjacent to the straight boundary. As the velocity distribution is uniform, the fluid element shown does not rotate about its mass centre. Figure 2.7(b) shows the velocity distribution which is non-uniform. The velocity reduces as the boundary is approached. The fluid element shown suffers an angular deformation, and rotation occurs about an axis which passes through the mass centre and is normal to the plane of paper.



(a) Irrotational Flow **(b) Rotational Flow**

Figure 2.7

The flow is also irrotational if the average of the angular velocities of two mutually perpendicular axes of the element is zero. In this case, the element deforms in such a way that the clockwise rotation of the horizontal line is equal to the counter-clockwise rotation of the vertical line (Figure 2.8(a)). It may be noted that the velocity decreases as the distance from the centre of curvature increases as shown by the dotted line. On the other hand, if the velocity increases with the distance from the centre of curvature, the flow is rotational. In this case, the element rotates in such a manner that both the horizontal line and vertical lines rotate in the same direction as shown in Figure 2.8(b).



(a) Irrotational Flow **(b) Rotational Flow**

Figure 2.8

One-dimensional, Two-dimensional and Three-dimensional Flow

One-dimensional Flow

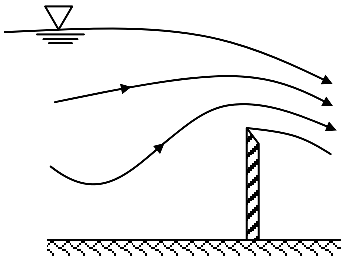
One-dimensional flow is represented by one dimension. Since a streamline has only one dimension (i.e., length), the flow along a streamline is one-dimensional. *The flow is also called one-dimensional if there is no variation of pressure, velocity, etc., in the direction normal to the direction of flow.*

Two-dimensional Flow

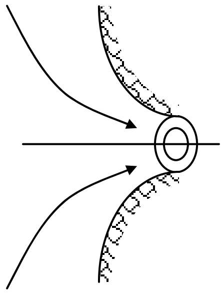
If the flow is such that all streamlines are *plane curves* and are *identical in parallel planes*, it is called a two-dimensional flow. In this type of flow, the velocity vector has components in two directions, the component in the third direction is zero. Figure 2.9(a) shows the flow over a long weir. This is an example of a two-dimensional flow.

Three-dimensional Flow

In three-dimensional flow, the streamlines are space curves. The velocity at any point has components in three dimensions. Figure 2.9(b) shows an example of three-dimensional flow through a converging pipe. The streamlines are in fact stream surfaces. The streamtubes are of annular cross section. This type of flow is very difficult to analyze.



(a) Two-dimensional Flow



(b) Three-dimensional Flow

Figure 2.9

Representation of Actual Flow as One-dimensional Flow

If a three-dimensional flow is axisymmetric, it may be treated as one-dimensional flow. In this case, one dimension is taken along the central streamline of the flow. The velocity, pressure and elevation along the streamline are taken as the average values on the section normal to the streamline. For example, the flow in a pipe is actually three-dimensional, but it may be considered as one-dimensional taking the length of the pipe as one dimension. The velocity, pressure, etc., along the length are taken as the average values at various section. *For correct representation of a three-dimensional flow by a one-dimensional flow, it is essential that the streamlines be straight and parallel*. This method of considering a three-dimensional flow as a one-dimensional flow is very useful in flow problems. Because only one dimension is considered, the analysis is greatly simplified.

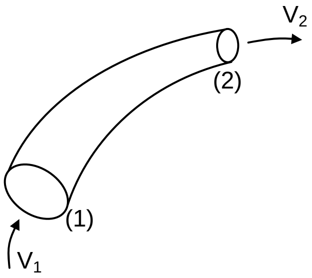
2.4 CONTINUITY EQUATION

The continuity equation is based on the principle of conservation of mass. Let us consider a small streamtube shown in Figure 2.10(a). If the cross-sectional area of the tube is small, the velocity at the centre of the tube will be the average velocity over the section. Let dA_1 and dA_2 be the cross-sectional areas of the tube at sections 1 and 2, respectively. If v_1 and v_2 are the average velocities and ρ_1 and ρ_2 are the mass densities at these two sections, then

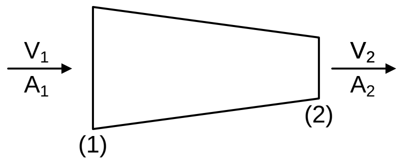
Volume of fluid passing at section 1 per unit time = $v_1 dA_1$

Mass of fluid passing at section 1 per unit time = $\rho_1 v_1\ dA_1$

Similarly, mass of fluid passing at section 2 per unit time = $\rho_2 v_2\ dA_2$



(a)



(b)

Figure 2.10

According to the law of conservation of mass, the mass of fluid entering section 1 must be equal to mass of fluid leaving section 2, if there is no storage of mass in the tube between sections 1 and 2. No storage of mass is possible in steady flow. If any mass were stored in the tube, the mass would go on changing with time, which is impossible in steady flow. Therefore,

$$\rho_1\ v_1\ dA_1 = \rho_2\ v_2\ dA_2 = \text{constant} \qquad \ldots \text{(a)}$$

In words, the mass of fluid per unit time passing through any section of a streamtube is constant.

It may be noted that velocities v_1 and v_2 are normal to the respective area dA_1 and dA_2.

Let us now consider a conduit shown in Figure 2.10(b). The conduit may be considered as an assemblage of a number of small streamtubes. The mass flow rate in the conduit may be obtained by integration of Eq. (a),

$$\int_{A_1} \rho_1\ v_1\ dA_1 = \int_{A_2} \rho_2\ v_2\ dA_2 \qquad \ldots \text{(b)}$$

where v_1 and v_2 are the velocities normal to the small areas dA_1 and dA_2 .

If the mass density is constant over cross-sections, Eq. (b) becomes

$$\rho_1 \int_{A_1} v_1\ dA_1 = \rho_2 \int_{A_2} v_2\ dA_2$$

or

$$\rho_1\ V_1\ A_1 = \rho_2\ V_2\ A_2 \qquad \ldots (2.1)$$

in which V_1 and V_2 are the mean velocities over the entire sections 1 and 2, respectively, and A_1 and A_2 are the cross-sectional areas of the conduit at these sections. *The quantity ρVA is known as mass rate of flow or mass flow rate* (M). It is measured in kg/s.

If the fluid is incompressible, the mass density is constant, and $\rho_1 = \rho_2$. Therefore,

$$V_1 A_1 = V_2 A_2 \qquad \ldots (2.2)$$

The product $V \times A$ is known as discharge. *Discharge is the volume rate of flow at any section.* It is expressed in cubic metres per second (cumecs) or litres per second.

1 cumec = 1 m^3/s = 1000 litres per second

Discharge is usually represented by the letter Q.

Eq. (2.2) indicates that for a steady incompressible flow, velocity is inversely proportional to area of cross-section. In other words, where the cross-sectional area decreases, the velocity increases and vice-versa.

For compressible fluids, Eq. (2.1) is sometimes written in a slightly modified form. Multiplying both sides of Eq. (2.1) by '*g*'

$$\rho_1 g V_1 A_1 = \rho_2 g V_2 A_2 \quad \text{or} \quad \gamma_1 V_1 A_1 = \gamma_2 V_2 A_2$$

The quantity γVA is known as the *weight rate of flow or weight flow rate (G).* It is measured in N/s or kN/s.

2.5 MEAN VELOCITY

Mean velocity is the average velocity over the section. It is also defined as the ratio of discharge Q to the cross-sectional area A of the conduit. If the velocity distribution over the cross-section is uniform, mean velocity is also equal to the velocity at any point. If the velocity distribution is non-uniform, mean velocity may be computed as follows.

Let us consider an elementary area dA in the cross-section of the pipe (Figure 2.11). Discharge dQ through this area is given by

$$dQ = vdA$$

where v is the velocity over the elementary area and is normal to it. Total discharge Q may be obtained by integration,

$$Q = \int vdA \qquad \text{. . . (a)}$$

From the definition of mean velocity, the discharge must be equal to the product of the mean velocity (V) and the cross-sectional area A, i.e.

$$Q = VA \qquad \text{. . . (b)}$$

From Eqs. (a) and (b),

$$VA = \int vdA$$

or

$$V = \frac{1}{A}\int vdA \qquad \text{. . . (2.4)}$$

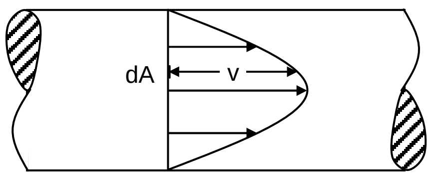


Figure 2.11

The mean velocity may be obtained from Eq. (2.4) if the velocity distribution over the section is known.

The concept of mean velocity is quite useful in flow problems.

Example 2.1

Two and a half cumecs of water (2500 litres/second) flows through a 50 cm diameter pipe. Calculate the mean velocity. If the diameter is reduced to 25 cm, what would be the mean velocity?

Solution

$$V = \frac{Q}{A} = \frac{2.50}{(\pi/4) \times (0.50)^2} = 12.73 \text{ m/s}$$

In the second case, $$V = \frac{2.50}{(\pi/4) \times (0.25)^2} = 50.93 \text{ m/s} = 50.93 \text{ m/s}$$

Example 2.2

The velocity distribution in a circular pipe of radius R is given by

$$v = V_{\max}\left(1 - \frac{r^2}{R^2}\right)$$

where v is the velocity at radius r and $V_{\max}$ is the velocity at the centre. Calculate the mean velocity.

Solution

From Eq. (2.4),

$$V = \frac{1}{A}\int v\,dA \qquad \text{. . . (a)}$$

Let us consider a small annular ring of thickness dr at a radius 'r'. Therefore,

$$dA = 2\pi\, r\, dr$$

Substituting the values of v and dA in Eq.(a),

$$V = \frac{1}{\pi R^2}\int_0^R V_{\max}\left[1 - \frac{r^2}{R^2}\right] 2\pi\, r\, dr$$

or $$V = \frac{2\pi\, V_{\max}}{\pi R^2}\left[\frac{r^2}{2} - \frac{r^4}{4R^2}\right]_0^R$$

$$= \frac{2\pi\, V_{\max}}{\pi R^2}\left[\frac{R^2}{4}\right] = \frac{V_{\max}}{2}$$

The mean velocity is one-half of the maximum velocity. This type of velocity distribution occurs in laminar flow.

Example 2.3

Oil flows through a pipe of 45 cm diameter at point A to 30 cm diameter at point B. At point B, it bifurcates into two branches consisting of pipes of 20 cm and 15 cm diameters (Figure 2.12). If the velocity at A is 2 m/sec, calculate the discharge at A and the velocities at B and C. The velocity at D is 4 m/sec.

Solution

(Refer Figure 2.12)

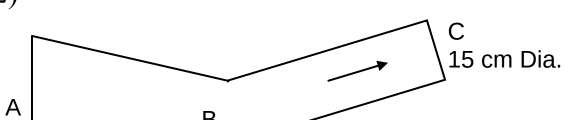


Figure 2.12

Discharge at $A = A \times V = \pi/4 \times (0.45)^2 \times 2 = 0.318$ cumecs.

From the continuity of flow,

Discharge at B = Discharge at A = 0.318 cumecs

$$\text{Velocity at } B = \frac{0.318}{(\pi/4)\,(0.30)^2} = 4.5 \text{ m/sec}$$

Discharge at *A* = Discharge at *C* + Discharge at *D*

If V_1 is the velocity at C,

$$0.318 = (\pi/4)\,(0.15)^2\, V_1 + (\pi/4)\,(0.20)^2 \times 4$$

$$= 0.0177\, V + 0.126$$

or $$V_1 = 0.192 / 0.0177 = 10.85 \text{ m/s.}$$

SAQ 1

(a) A pipe of 20 cm diameters carries water with a mean velocity of 3 m/s. Calculate the discharge.

(b) If the pipe is bifurcated into two pipes of 10 cm diameter each, find the velocity in each pipe.

2.6 GENERAL ENERGY EQUATION

A dynamic equation describing fluid motion may be obtained by applying Newton's second law to a fluid particle. Applying the law in x-direction,

$$\Sigma F_x = m a_x \qquad \ldots \text{(a)}$$

where ΣF_x is the resultant net force acting on the particle, m is the mass and a_x is the acceleration.

The forces may be due to gravity $(F_g)_x$, pressure $(F_p)_x$, viscosity $(F_v)_x$, turbulence $(F_t)_x$, and compressibility (elasticity) $(F_e)_x$. However, when the volume changes are small, the last force may be neglected. Thus

$$(F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x = m a_x \qquad \ldots \text{(b)}$$

Similar equations can be written for y and z directions.

When the values of various quantities are substituted in Eq. (b), the equations obtained are known as Reynolds equations. For low Reynolds number, the force due to turbulence can be neglected. Thus,

$$(F_g)_x + (F_p)_x + (F_v)_x = m a_x \qquad \ldots \text{(c)}$$

If the flow is assumed to be ideal, the force due to viscosity is omitted. Thus,

$$(F_g)_x + (F_p)_x = ma_x \qquad \ldots \text{(d)}$$

The energy equations which take into account only the gravity forces and the pressure forces are known as Euler's equations. The Euler equation for one-dimensional flow is derived in the following section.

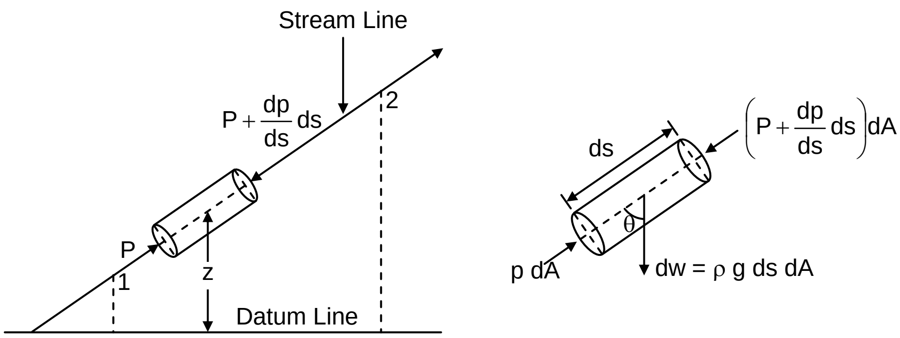
2.7 EULER'S EQUATION

Let us consider a streamtube of differential cross-section dA shown in Figure 2.13(a). The external forces acting on the free body of the fluid element, shown in Figure 2.13(b), are :

(i) The pressure force due to pressure 'p' at one end and $\left(p + \frac{dp}{ds} ds\right)$ at the other end.

(ii) The component of the fluid weight in the direction of flow.

Weight of the fluid element = $\rho \, g \, ds \, dA$



(a) **(b)**

Figure 2.13

Component of the weight in the direction of flow $- (\rho g \, ds \, dA) \cos \theta$.

Applying Newton's second law of motion to the element,

$$\sum F = \text{Mass} \times \text{Acceleration}$$

or $$p\,dA - \left(p + \frac{\partial p}{\partial s} ds\right) dA - (\rho g \, ds \, dA) \cos \theta = (\rho \, ds \, dA) \, a_t$$

where a_t is tangential acceleration given by

$$a_t = V \frac{dV}{ds}$$

Taking $\cos \theta = \frac{dz}{ds}$,

$$- \frac{\partial p}{\partial s} ds \; dA - \rho g \; ds \; dA \left(\frac{dz}{ds}\right) = \rho \; ds \; dA \left(V \frac{dV}{ds}\right)$$

Dividing both sides by $\rho \; ds \; dA$,

$$\frac{1}{\rho} \frac{\partial p}{\rho \, \partial s} + g \left(\frac{dz}{ds}\right) + \left(V \frac{dV}{ds}\right) = 0 \qquad \ldots \text{(a)}$$

Putting $V\dfrac{dV}{ds}=\dfrac{d}{ds}\left(\dfrac{V^2}{2}\right)$, Eq. (a) can be written as

$$\frac{1}{\rho}\frac{\partial p}{\rho\,\partial s}+g\left(\frac{dz}{ds}\right)+\frac{d}{ds}\left(\frac{V^2}{2}\right)=0 \qquad \text{. . . (2.5)}$$

Eq. (2.5) is the Euler's equation for one-dimensional flow.

2.8 DERIVATION OF BERNOULLI'S EQUATION FROM EULER'S EQUATION

Integrating Eq. (2.5),

$$\frac{p}{\rho}+gz+\frac{V^2}{2}=\text{constant} \qquad \text{. . . (b)}$$

Each term of Eq. (b) represents energy per unit mass (N-m/kg).

For incompressible fluids, Eq. (2.5) is written in a slightly modified form which is obtained by dividing both sides of Eq. (2.5) by g

$$\frac{dp}{\gamma\,ds}+\frac{dz}{ds}+\frac{d}{ds}\left(\frac{V^2}{2g}\right)=0 \qquad \text{. . . (c)}$$

Integrating Eq. (c) with respect to 's',

$$\frac{p}{\gamma}+z+\frac{V^2}{2g}=\text{constant} \qquad \text{. . . (2.6)}$$

Each term of Eq. (2.6) represents the energy per unit weight (N-m/N).

Eq. (2.6) is the well-known *Bernoulli equation*. It may be mentioned that the following assumptions have been made in the derivation of Bernoulli's equation :

(a) The fluid is ideal and incompressible.

(b) The flow is steady and continuous.

(c) The flow is along the streamline, i.e. it is one-dimensional.

(d) The velocity is uniform over the section and is equal to the mean velocity.

(e) The only forces acting on the fluid are the gravity forces and the pressure forces.

The accuracy of the results obtained by the application of the Bernoulli equation to a particular problem will depend upon the extent to which these assumptions are justified.

In Bernoulli's equation, the terms p/γ, z and $V^2/2g$ are respectively the pressure head, elevation (or datum) head and velocity head. All these terms have the dimension of length [L] and may be expressed in metres of the fluid. *Hence the Bernoulli equation states that in an ideal, incompressible fluid when the flow is steady and continuous, the sum of the pressure head, elevation head and velocity head is constant along a streamline*. The Bernoulli equation may be visualized by means of an apparatus shown in Figure 2.14. It is a conduit, which first converges and then diverges. As the liquid flows from A to B, the velocity head increases and the pressure head decreases. The velocity head at B may be calculated if the diameter at B is known. The pressure head can be measured by means of a piezometer. As the liquid flows from B to C, the velocity head decreases and the pressure head increases. It is observed that the total head remains practically constant.

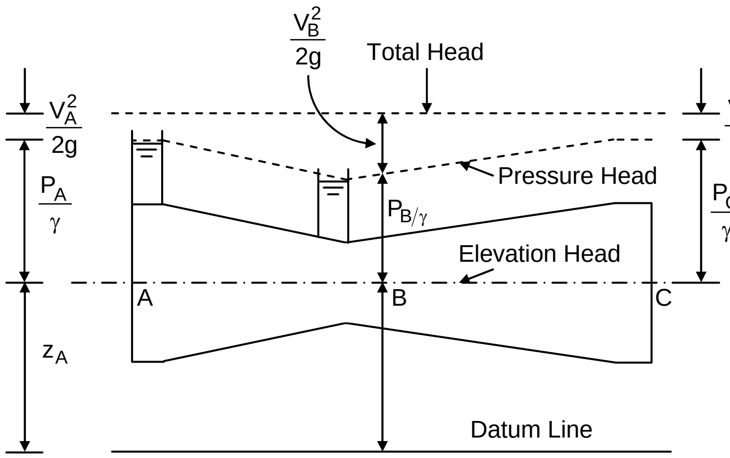


Figure 2.14

2.9 BERNOULLI'S EQUATION AS AN ENERGY EQUATION

In the foregoing section, the Bernoulli equation was derived from Euler's equation. The Bernoulli equation may also be derived from the energy consideration. The reader is familiar with the kinetic energy and potential energy. The kinetic energy of the unit weight of liquid is V^2 / 2g and the potential energy is z where V is the velocity of the fluid and z is the vertical distance of the point above the datum line.

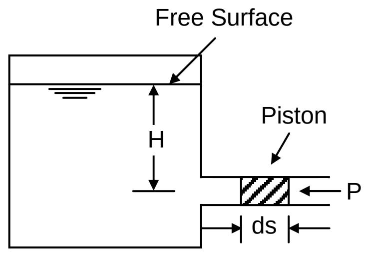


Figure 2.15

The pressure energy differs from kinetic energy and potential energy. An independent fluid mass can have kinetic energy and potential energy, but it will have pressure energy only when it is in *contact with other masses of fluid.* For getting insight into the pressure energy, let us consider the vessel shown in Figure 2.15. On one side of the vessel, there is a very small horizontal cylinder with a piston. Because the pressure intensity (p) at the piston is γH, where H is the head and γ is the specific weight, a force $P = pa$ must be applied to the piston to hold it in position where a is the cross-sectional area of the piston. If the piston is now permitted to move through a small ds, the work done against the force P is $P.ds$. To do this work, the liquid of weight ($\gamma.a.ds$) has entered the cylinder. The energy given up by this liquid must be equal to the work done. Hence the energy lost per unit weight of the liquid is $\frac{P\,ds}{\gamma\,a\,ds} = \frac{p\,a\,ds}{\gamma\,a\,ds} = \frac{p}{\gamma}$. The energy per unit weight expressed as p/γ is termed the *pressure energy* of the liquid.

Derivation of the Bernoulli Equation

Let us consider the flow through the streamtube shown in Figure 2.16. The fluid enters the tube at the end *A* and leaves at the end *B* in a small interval *dt* of time, the weight of the fluid entering the tube at *A* is equal to $(\gamma\ ds_1\ dA_1)$ where dA_1 is the differential area of the tube at *A*. In the same interval of time, the weight of the liquid leaving the tube at *B* is $(\gamma\ ds_2\ dA_2)$ where dA_2 is the differential area of the tube at *B*. The movement of the fluid between *AA* to $B'B'$ is equivalent to moving the fluid between *AA* and $A'A'$ to the new position between *BB* and $B'B'$. The remainder of the fluid (between $A'A'$ and *BB*) may be looked upon as stationary.

The work done associated with the displacement of the fluid mentioned above may be computed from the products of the forces $p_1\ dA_1$ and $p_2\ dA_2$ and the displacements ds_1 and ds_2.

$$\text{Work done per unit weight at } A = \frac{p_1\ dA_1\ ds_1}{\gamma\ ds_1\ dA_1} = \frac{p_1}{\gamma}$$

$$\text{Work done per unit weight at } B = \frac{p_2\ dA_2\ ds_2}{\gamma\ ds_2\ dA_2} = \frac{p_2}{\gamma}$$

The law of conservation of energy states that energy entering the control volume + work done = energy leaving the control volume.

or $$\left(\frac{V_1^2}{2g} + z_1\right) + \left(\frac{p_1}{\gamma} - \frac{p_2}{\gamma}\right) = \left[\frac{V_2^2}{2g} + z_2\right]$$

where subscripts 1 and 2 refer to ends *A* and *B* respectively.

Simplifying, $$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

or $$\frac{p}{\gamma} + \frac{V^2}{2g} + z = \text{constant} \qquad \text{[same as Eq. (2.6)]}$$

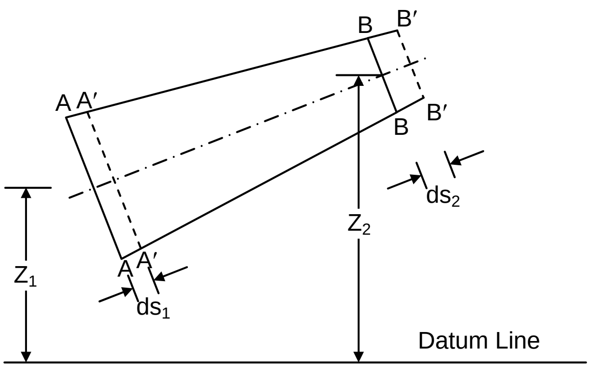


Figure 2.16

This is the Bernoulli equation as derived before. *The equation states that the sum of the pressure, potential and kinetic energy per unit weight remains constant.* Energy has the dimension of [*FL*]. The energy per unit weight has the dimension [L]. Therefore, the energy per unit weight has the unit of N-m per N or simply metre.

2.10 BERNOULLI'S EQUATIONS FOR REAL FLUIDS

The Bernoulli equation was derived on the assumption that the fluid is ideal. However, the equation may be used even for real fluids, provided it is modified to take into account the frictional resistance caused due to viscosity. The modified equation for real fluid is given below

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + H_L \qquad \text{. . . (2.7)}$$

where H_L is the loss of energy (or head) from section 1 to 2.

Further, in real fluids, the velocity is not uniform over the cross-section, and consequently the velocity head expressed in terms of the mean velocity is not correct. To take into account the non-uniformity of velocity, the velocity head calculated on the basis of the mean velocity must be multiplied by a correction factor known as the *kinetic energy correction factor.*

Let us consider an element of fluid of differential area dA (Figure 2.11). Let the velocity of flow at the element be v. The discharge through the element is given by $dQ = v\ dA$.

Kinetic energy of the fluid = $(\gamma v\, dA)\, \dfrac{v^2}{2g}$

Total kinetic energy at the section = $\dfrac{\gamma}{2g} \displaystyle\int v^3\, dA$ . . . (a)

Total kinetic energy on the basis of mean velocity = $\alpha\, (\gamma\, A V)\, \dfrac{V^2}{2g}$ . . . (b)

From Eqs. (a) and (b),

$$\alpha = \frac{\int v^3\, dA}{V^3 A} = \frac{1}{A} \int \left(\frac{v}{V}\right)^3 dA \qquad \text{. . . (2.8)}$$

The value of the energy correction factor α may be obtained from Eq. (2.8) if the expression for the velocity distribution across the section is known. The value of α is always greater than unity. For laminar flow in pipes, its value is 2, whereas for turbulent flow in pipes, its value ranges from 1.01 to 1.10, depending upon the velocity distribution. The value of α is usually assumed to be unity, unless mentioned otherwise.

The modified form of the Bernoulli equation, taking into account the energy correction factor α, is

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + H_L \qquad \text{. . . (2.9)}$$

2.11 APPLICATION OF THE BERNOULLI EQUATION TO SIMPLE PROBLEMS

Figure 2.17 shows a typical example of steady flow of an ideal fluid from a large reservoir through a system of pipes varying in size and terminating in a nozzle. As the fluid is assumed to be ideal, there is no loss of head due to friction and Eq. (2.6) is, therefore, applicable. In dealing with problems based on the application of the Bernoulli equation, it is convenient to plot the energy gradient line and the hydraulic gradient line.

The energy gradient line (EGL) [also known as the total energy line (TEL)] is the line which represents the total head above the datum line. *The energy gradient line shows the variation of the total energy along the axis of the conduit.* It is, therefore, a graphical representation of the total head or total energy with respect to the datum line.

The *hydraulic gradient line* (*also known as the pressure gradient line*) *shows the variation of pressure head in the conduit.* The hydraulic gradient line joins the points to which the liquid would rise in the piezometers inserted at these points. It is also known as piezometric head line. This line is a graphical representation of the piezometric head (i.e., the sum of the pressure and elevation head) with respect to any selected datum. The hydraulic gradient line is always below the total energy line, the vertical distance between the two being equal to the velocity head at that point.

In Figure 2.17, the energy gradient line is horizontal, as there is no loss of energy. Because the surface area of the reservoir is very large, the velocity in reservoir will be very small and the velocity head may be neglected. Thus, the energy gradient line and the hydraulic gradient line coincide with the free surface of liquid in reservoir. If the discharge is known, velocities at various sections may be obtained from the continuity equation and the velocity head calculated. The hydraulic gradient line is plotted below the energy gradient, keeping the vertical intercept between the two equal to the corresponding velocity head.

The vertical intercept between the hydraulic gradient line and the centerline of the pipe is the pressure head at that section. When the hydraulic gradient line is above the centerline of the pipe, the pressure head is positive. On the other hand, if the hydraulic gradient line is below the centerline of the pipe, the pressure head is negative. The pipeline in which the hydraulic gradient line is below the centerline of the pipe is known as a siphon or syphon.

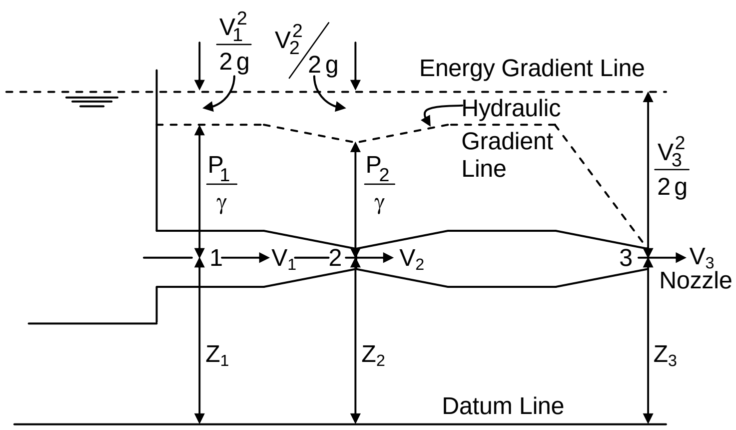


Figure 2.17

At the end point 3, there is a nozzle with a very small cross-sectional area. The velocity and hence the velocity head is very large at point 3. The pressure at this point reduces to the atmospheric pressure.

(**Note :** Vertical scale in Figure 2.17 is exaggerated.)

The foregoing treatment is based on the assumption that the fluid is ideal, and there is no loss of energy (or head) due to friction. In real fluids, there will be a loss of head due to friction, and the energy gradient line will slope downward from the reservoir to the nozzle. In addition to the frictional loss, there will be losses due to sudden changes in cross-section of the pipe. These losses would cause sudden drops in the energy gradient line at those points.

Example 2.4

Oil of specific gravity 0.75 flows through a 15 cm diameter pipe under a pressure of 98.1 kN/m^2. If the datum is 3 m below the centre of the pipe and the total energy with respect to the datum is 20N-m/N, calculate the discharge.

Solution

Total energy per N of oil with respect to the datum

= Pressure Energy + Kinetic Energy + Elevation Energy

or $$H = \frac{p}{\gamma} + \frac{V^2}{2g} + z$$

or $$20 = \frac{98.1}{9.81 \times 0.75} + \frac{V^2}{2g} + 3 = 13.33 + \frac{V^2}{2g} + 3$$

or $V = 8.48$ m/sec

Discharge $Q = VA$

$= 8.48 \times \pi/4 \times 0.15 \times 0.15 = 0.15$ cumecs.

Example 2.5

In the pipe shown in Figure 2.18, 0.5 cumecs of water flows from point A to B. The diameters of the pipe at A and B are respectively 30 cm and 60 cm.

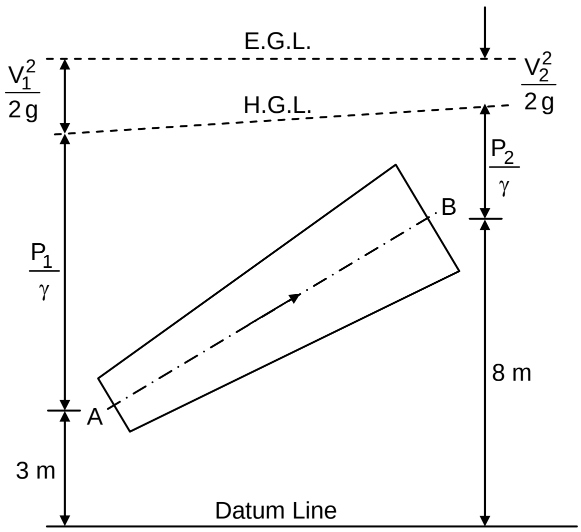


Figure 2.18

If the pressure head at A is 7 m of water, find the pressure head at B. Neglect losses.

Solution

The energy gradient line (abbreviated as EGL) is horizontal. The hydraulic gradient line (HGL) slopes upward as the velocity head decreases from *A* to *B*.

$$V_1 = \text{velocity at } A = \frac{Q}{A} = \frac{0.50}{\pi/4 \times 0.30^2} = 7.07 \text{ m/sec}$$

$$V_2 = \text{velocity at } B = \frac{Q}{A} = \frac{0.50}{\pi/4 \times 0.60^2} = 1.77 \text{ m/sec}$$

Applying Bernoulli's equation to points *A* and *B*

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

or $$7 + \frac{(7.07)^2}{19.62} + 3 = \frac{p_2}{\gamma} + \frac{(1.77)^2}{19.62} + 8$$

$$7 + 2.55 + 3 = \frac{p_2}{\gamma} + 0.16 + 8$$

or $$\frac{p_2}{\gamma} = 4.39 \text{ m of water}$$

or $$= 4.39 \times 9.81 = 43.07 \text{ kN/m}^2$$

Example 2.6

Figure 2.19 shows a pipe of 8 cm diameter working as a syphon.

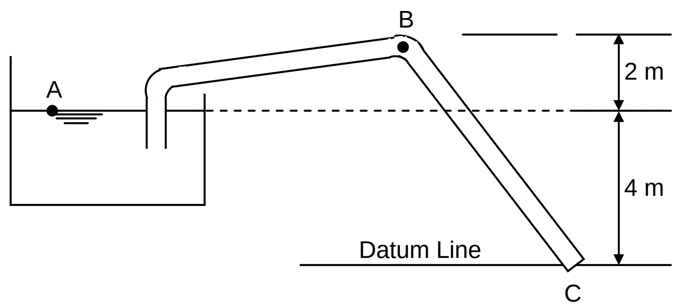


Figure 2.19

Find the velocity of flow, discharge, and the absolute pressure at B if the atmospheric pressure is equivalent to 10 m of water. Neglect losses.

Solution

Taking the datum line at the level of point *C*, and applying Bernoulli's equation to points *A* and *C*,

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_C}{\gamma} + \frac{V_C^2}{2g} + z_C$$

The pressure at points *A* and *C* is atmospheric, which is taken as zero. Since the area of the vessel is very large, the velocity V_A may be taken as zero.

Thus $$0 + 0 + 4 = 0 + \frac{V_C^2}{2g} + 0$$

or $$V_C = \sqrt{4 \times 2 \times 9.81} = 8.86 \text{ m/sec}$$

Discharge $$Q = AV$$

$$= \frac{\pi}{4} \times 0.08 \times 0.08 \times 8.86 = 0.0445 \text{ cumecs}$$

Pressure at B can be obtained by applying Bernoulli's equation to points A and B, taking $V_B = V_C$.

$$z_A + \frac{p_A}{\gamma} + \frac{V_A^2}{2g} = z_B + \frac{p_B}{\gamma} + \frac{V_B^2}{2g}$$

$$4 + 0 + 0 = 6 + \frac{p_B}{\gamma} + \frac{(8.86)^2}{19.62}$$

or $$\frac{p_B}{\gamma} = -6 \text{ m (vacuum)}$$

Absolute pressure at B = Atmospheric pressure – Vacuum pressure

= 10 – 6 = 4 m of water

= 4 × 9.81 = 39.24 kN/m^2 absolute.

SAQ 2

An inclined pipe carrying water gradually changes in diameter from 15 cm at A to 40 cm at B at a height of 4.5 m above A. If the pressures at A and B are respectively 68.67 kN/m^2 and 49.05 kN/m^2 and the discharge is 0.150 m^3 /s, determine

(a) the direction of flow, and

(b) the head loss between A and B.

2.12 HORIZONTAL VENTURIMETER

The venturimeter was invented by Herschel in 1887, but he named it in honour of Venturi who did pioneering work in flow through convergent tubes. A venturimeter consists of a converging cone, a throat section and a diverging cone, all combined in one unit (Figure 2.20). As the flow takes place in the converging cone, the velocity increases, and there is a fall in the pressure, according to the Bernoulli equation. The velocity and hence discharge can be calculated from the measurement of difference of pressure at the two ends of the converging cone, i.e. at the inlet and the throat.

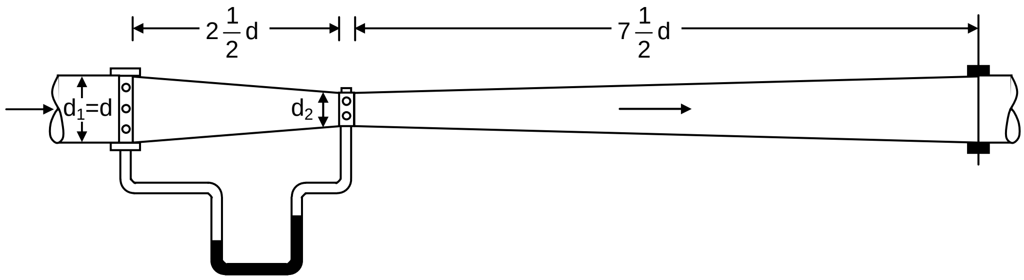


Figure 2.20 : Venturimeter

In the converging cone, as the fluid is being accelerated, there is no appreciable loss of energy. Therefore, the converging cone may have a sharp angle of convergence (upto 20°). In the diverging cone, there is deceleration of flow with considerable loss of energy due to formation of eddies. In order to avoid separation and consequent excessive energy loss, it is essential to keep the angle of divergence very small. The most efficient conversion of the kinetic energy to the pressure energy occurs when the angle of divergence is about 7°. The usual lengths of the converging and diverging cones are 2.5 *d* and 7.5 *d* respectively where *d* is the diameter of the pipe.

When the pressure difference at the inlet and the throat is small and the fluid flowing in the venturimeter is a liquid, piezometers may be used to measure the difference of pressure. Usually the difference of pressure is measured by means of a differential manometer. Pressure chambers, which are hollow rings, are generally fitted at the inlet and the throat. The pressure chambers are also called the piezometric rings. These chambers communicate with the manometer through a number of holes in the periphery of pipe. The pressure connections are taken from the pressure chambers for accurate measurement of pressure.

For satisfactory working of the venturimeter, the flow must be fully established before it enters the converging cone. The flow is fully established if the meter is preceded by a straight and uniform length of pipe equal to 30 *d* or more. Rotational motion occurs in curved pipes. Straightening vanes are placed in the curved pipe to reduce the rotational motion of the fluid before it enters the meter.

Working Principle

As the fluid passes from point 1 (inlet) to point 2 (throat), there is an increase in the velocity and corresponding fall in pressure (Figure 2.21).

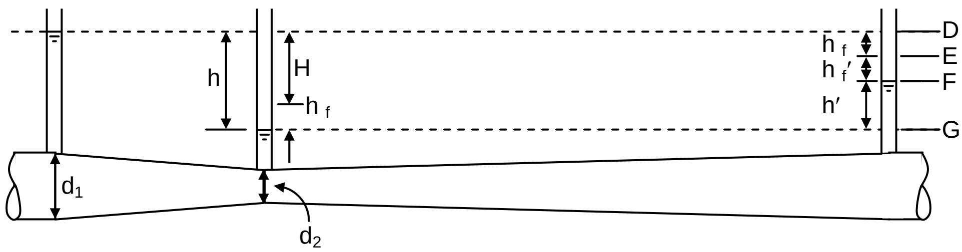


Figure 2.21

Applying Bernoulli’s equation to points 1 and 2, with datum at the axis, considering horizontal venturimeter (for inclined venturimeter, ref. Section 2.13)

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + 0 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + 0$$

or
$$\frac{p_1 - p_2}{\gamma} = \frac{V_2^2 - V_1^2}{2g}$$

Representing the theoretical pressure difference by *H*,

$$\frac{V_2^2 - V_1^2}{2g} = H \qquad \text{. . . (a)}$$

From the continuity equation, $a_1 V_1 = a_2 V_2$, where a_1 is the area at inlet and a_2 at throat.

or $$V_1 = \left(\frac{a_2}{a_1}\right) V_2$$

Substituting this value of V_1 in Eq. (a),

$$\frac{V_2^2 - \left(\frac{a_2}{a_1}\right)^2 V_2^2}{2g} = H$$

or $$V_2 = \sqrt{\frac{2gH}{1 - \left(\frac{a_2}{a_1}\right)^2}}$$

Because of the loss of head from section 1 to 2, the actual measured difference of head (h) will be greater than the thoeretical difference (H). Thus

$$V_2 = C\sqrt{\frac{2gh}{1 - \left(\frac{a_2}{a_1}\right)^2}} \qquad \text{. . . (b)}$$

where C is the coefficient of the meter.

Discharge, $$Q = a_2 V_2$$

or $$Q = Ca_2\sqrt{\frac{2gh}{1 - \left(\frac{a_2}{a_1}\right)^2}} \qquad \text{. . . (2.10)}$$

$$= C_{a_1}\sqrt{\frac{2gh}{\left(\frac{a_1}{a_2}\right)^2 - 1}}$$

or $$Q = C\frac{a_1\sqrt{2g}}{\sqrt{\left(\frac{a_1}{a_2}\right)^2 - 1}}\sqrt{h} \qquad \text{. . . (2.11)}$$

For a given venturimeter, the quantity

$$\frac{a_1\sqrt{2g}}{\sqrt{\left(\frac{a_1}{a_2}\right)^2 - 1}}$$

is constant and may be represented by k. Thus, Eq. (2.11) may be written as

$$Q = Ck\sqrt{h} \qquad \text{. . . (2.12)}$$

The coefficient C may be omitted if we make allowance for the loss of head (h_f) due to friction.

Thus $$V_2 = C\sqrt{\frac{2g(h - h_f)}{1 - \left(\frac{a_2}{a_1}\right)^2}} \qquad \text{. . . (c)}$$

Equating two values of V_2 from Eqs. (b) and (c),

$$C\sqrt{\frac{2gh}{1-\left(\frac{a_2}{a_1}\right)^2}} = \sqrt{\frac{2g(h-h_f)}{1-\left(\frac{a_2}{a_1}\right)^2}}$$

or $$C^2 h = h - h_f$$

or $$h_f = h(1 - C^2) \qquad \text{. . . (2.13)}$$

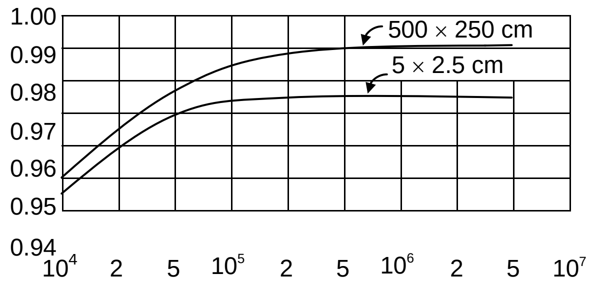
Eq. (2.13) gives the loss of head in the converging cone.

Substituting the value of h from Eq. (b),

$$h_f = \left(\frac{1}{C^2} - 1\right)\left[1 - \left(\frac{a_2}{a_1}\right)^2\right]\frac{V_2^2}{2g} \qquad \text{. . . (2.14)}$$

The ratio of the throat diameter (d_2) to the diameter of the pipe (d_1) ranges from 1/4 to 3/4, usually the ratio is kept 1/2. The smaller the ratio, the greater would be the difference of pressure, and hence the more accurate would be the measurement.

But the large pressure difference produces a very low pressure at the throat which may cause separation of gases and cavitation. The lowest pressure, for water flowing in the venturimeter, is about 20.6 kN/m^2 (2.1 m of water). This fixes the limit of reduction of the throat diameter.



$$N_R = \frac{\rho_2 v_2 d_2}{\mu_2}$$

Figure 2.22

The value of the coefficient C varies from 0.97 to 0.99 provided the flow is occurring at a high Reynold number. Increased roughening of the surface, with the passage of time, reduces the coefficient. Figure 2.22 shows the variation of C with the Reynold number (N_R) and the size of the venturimeter. The diagram is valid for ratio $d_2/d_1 = 1/2$, but is reasonably valid for smaller ratios as well. For larger ratios, however, the value of C decreases slightly. As is evident, the coefficient increases with the size, for the same d_1/d_2 ratio.

Example 2.7

Water flows through a horizontal venturimeter, 30 cm × 15 cm diameter, at the rate of 0.039 cumecs. If the difference of pressure is 0.25 m of water, calculate the coefficient of the venturimeter.

Solution

From Eq. (2.11),

$$Q = C \frac{a_1}{\sqrt{\left(\frac{a_1}{a_2}\right)^2 - 1}} \sqrt{2gh}$$

Now $$\frac{a_1}{a_2} = \frac{0.30^2}{0.15^2} = 4$$

Substituting these values in equation,

$$0.039 = \frac{C \times \pi \times 0.30^2}{4\sqrt{16-1}} \sqrt{2 \times 9.81 \times 0.25}$$

or $$C = 0.965.$$

2.13 INCLINED VENTURIMETER

Upward Flow

Figure 2.23 shows a venturimeter with its axis inclined. Let us first consider the case when the flow is upward. In this case the throat is at a higher level than the inlet.

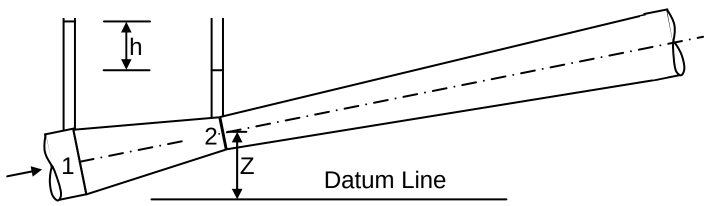


Figure 2.23 : Inclined Venturimeter

Applying Bernoulli's equation to points 1 and 2,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + 0 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + Z$$

or $$\frac{V_2^2 - V_1^2}{2g} = \left(\frac{p_1 - p_2}{\gamma}\right) - Z \qquad \text{. . . (a)}$$

From the continuity equation,

$$a_1 V_1 = a_2 V_2 \text{ or } V_1 = \left(\frac{a_2}{a_1}\right) V_2$$

Substituting the value of V_1 in Eq. (a),

$$\frac{V_2^2}{2g}\left[1 - \left(\frac{a_2}{a_1}\right)^2\right] = \left(\frac{p_1 - p_2}{\gamma}\right) - Z$$

where $\left(\frac{p_1 - p_2}{\gamma}\right)$ is the difference of pressure head at the inlet and the throat. Representing this head by H,

$$\frac{V_2^2}{2g}\left[1-\left(\frac{a_2}{a_1}\right)^2\right] = H - Z$$

or
$$V_2 = \sqrt{\frac{2g(H-Z)}{1-\left(\frac{a_2}{a_1}\right)^2}}$$

Taking the losses in the converging cone into account,

$$V_2 = C\sqrt{\frac{2g(h-Z)}{1-\left(\frac{a_2}{a_1}\right)^2}}$$

where 'h' is the actual measured difference of head and C is a coefficient less than unity.

Now discharge $Q = a_2V_2$

$$Q = Ca_2\sqrt{\frac{2g(h-Z)}{1-\left(\frac{a_2}{a_1}\right)^2}}$$

$$= C\frac{a_1}{\sqrt{\left(\frac{a_1}{a_2}\right)^2 - 1}}\sqrt{2g(h-Z)} \qquad \text{. . . (2.15)}$$

or
$$Q = Ck\sqrt{h-Z} \qquad \text{. . . (2.16)}$$

where k is the constant of the venturimeter, given by

$$k = \frac{a_1\sqrt{2g}}{\sqrt{\left(\frac{a_1}{a_2}\right)^2 - 1}}$$

Downward Flow

Likewise, it can be proved that when the throat is at a level lower than the inlet, i.e. flow is downward,

$$Q = Ck\sqrt{h+Z} \qquad \text{. . . (2.17)}$$

Vertical Venturimeter

The equation derived for the inclined venturimeter may also be used for the vertical venturimeter.

2.14 USE OF DIFFERENTIAL MANOMETERS IN VENTURIMETERS

The difference of pressure between the inlet and throat is usually measured with a differential manometer. The difference of head can be easily calculated if the deflection of the liquid in the manometer is known. We shall consider separately the manometers for the horizontal and the inclined venturimeter.

Horizontal Venturimeter

Figure 2.24(a) shows a differential manometer used for a horizontal venturimeter. In this case, points 1 to 2 are at the same level. From the principle of manometer, the pressures at the same level *AA* in a continuous liquid are equal.

Thus $$p_1 + (y + x)\, s\, \gamma_w = x s_1\, \gamma_w + ys\, \gamma_w + p_2$$

where s is the specific gravity of the fluid in venturimeter, s_1 is the specific gravity of the liquid in manometer and x is the deflection of the liquid in the manometer.

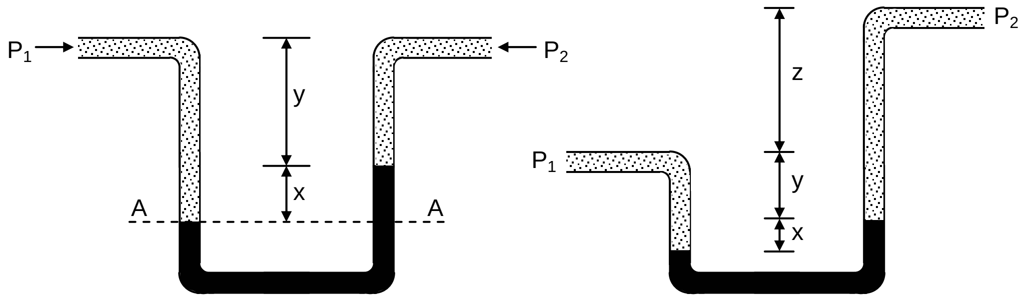


Figure 2.24

Simplifying, $$p_1 - p_2 = x\left[\left(\frac{s_1}{s}\right) - 1\right]\gamma_w\, s$$

The difference of head h between the inlet and the throat can be expressed as

$$h = \frac{p_1 - p_2}{s\gamma_w} = x\left(\frac{s_1}{s} - 1\right)$$

Therefore, $$h = x\,(r - 1) \qquad \text{. . . (2.18)}$$

where r is the ratio of the specific gravity of the liquid in the manometer to that in the venturimeter, i.e. $r = \frac{s_1}{s}$.

Substituting the value of h in Eq. (2.12),

$$Q = Ck\,\sqrt{x\,(r - 1)} \qquad \text{. . . (2.19)}$$

Inclined Venturimeter

Figure 2.24(b) shows the differential manometer which is connected to an inclined venturimeter with the direction of flow upward. Thus, point 2 is at a higher level. Using the principle of manometer,

$$p_1 + (y + x)\, s\gamma_w = xs_1\, \gamma_w + ys\, \gamma_w + zs\, \gamma_w + p_2$$

or $$p_1 + ys\gamma_w + xs\, \gamma_w - xs_1\, \gamma_w - ys\, \gamma_w - zs\, \gamma_w = p_2$$

or $$p_1 - p_2 = xs_1\, \gamma_w - xs\, \gamma_w + zs\, \gamma_w$$

or $$\frac{p_1 - p_2}{s\gamma_w} = x\left(\frac{s_1}{s} - 1\right) + z$$

Thus, the difference of head h between the inlet and the throat is given by,

$$h = x\,(r - 1) + z$$

or $$h - z = x\,(r - 1)$$

Substituting this value of $(h - z)$ in Eq. (2.14)

$$Q = Ck\,\sqrt{x\,(r-1)} \qquad \text{(same as Eq. (2.19))}$$

Hence Eq. (2.19) is a general equation which can be used for both the horizontal and inclined venturimeter.

Example 2.8

A liquid, with a specific gravity 1.25, flows upward through a vertical venturimeter 50 cm × 25 cm. If the mercury manometer (S = 13.6) shows a deflection of 0.1m, find the discharge. Assume C = 0.98.

Solution

$$k = \frac{a_1\sqrt{2g}}{\sqrt{\left(\frac{a_1}{a_2}\right)^2 - 1}}$$

$$k = \frac{(\pi/4) \times 0.50^2 \times 4.43}{\sqrt{16-1}} = 0.224$$

From Eq. (2.18), $$Q = Ck\,\sqrt{x\,(r-1)}$$

$$= 0.98 \times 0.224 \times \sqrt{0.1\,(13.6/1.25 - 1)}$$

$$= 0.218 \text{ m}^3/\text{s}$$

SAQ 3

(a) Gasolene flows through an inclined venturimeter in the upward direction at the rate of 0.22 m^3/s. The venturimeter is inclined at 30º to the horizontal and has the inlet and throat of diameters 30 cm and 15 cm, respectively. If the throat is 0.60 m above the inlet and the pressure gauges at the entrance and throat show pressures of 141.26 kN/m^2 and 75.54 kN/m^2, respectively, calculate the coefficient C of the venturimeter. Take specific gravity of gasoline as 0.82.

(b) A venturimeter having a throat diameter of 100 mm is fitted in a pipe of diameter 250 mm through which oil of specific gravity 0.85 is flowing. The pressure difference between the entry is measured by a U-tuble manometer containing mercury (S = 13.6) and the deflection of the manometer is 0.60 m. Calculate the discharge. Assume coefficient C = 0.97.

2.15 ORIFICE METER

An orifice meter consists of a thin plate with a central hole. The plate is clamped between pipe flanges. The fluid flows through the hole (orifice). The orifice causes the flow to accelerate, as does the throat in a venturimeter. The orifice meter, like a venturimeter, is used to measure discharge in a pipe.

Horizontal Orifice Meter

Figure 2.25 shows an orifice plate inserted in a pipe line. As the fluid passes through the orifice, it contracts in area. The section of the stream where the cross-sectional area is minimum is called the venacontracta. The venacontracta forms at a distance of about $d_1/2$ from the plane of the plate, where d_1 is the diameter of pipe. Pressure connections are made at sections 1 and 2; section 1 is at a distance of 1.5 d_1 from the orifice and section 2 is at the venacontracta. At the venacontracta, the cross-sectional area is minimum and velocity is maximum and hence the pressure is minimum. By measuring the pressure difference between points 1 and 2, discharge may be calculated, as in the case of a venturimeter.

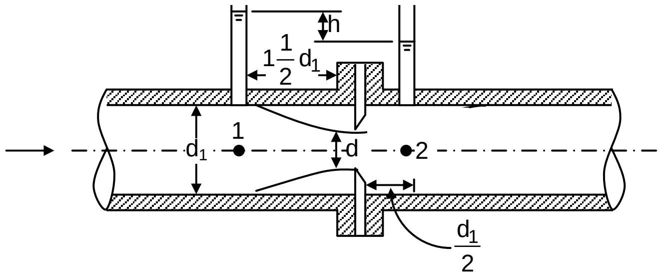


Figure 2.25 : Orifice Meter

Applying the Bernoulli equation to points 1 and 2 assuming that the meter is horizontal.

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g}$$

or $$\frac{V_2^2 - V_1^2}{2g} = \frac{p_1 - p_2}{\gamma}$$

From the continuity equation,

$$a_1 V_1 = a_2 V_2$$

or $$V_1 = \left(\frac{a_2}{a_1}\right) V_2$$

Thus $$\frac{V_2^2}{2g}\left[1 - \left(\frac{a_2}{a_1}\right)^2\right] = H$$

where H is the theoretical difference of head between points 1 and 2 .

Because of loss of head due to friction, the measured difference of head 'h' will be more than H.

Thus $$V_2 = C' \sqrt{\frac{2gh}{1 - \left(\frac{a_2}{a_1}\right)^2}}$$

where C' is a coefficient.

Now $$Q = a_2 V_2$$

$$= C' a_2 \sqrt{\frac{2gh}{1 - \left(\frac{a_2}{a_1}\right)^2}} \qquad \text{. . . (2.20)}$$

It will be noted that Eq. (2.20) is similar to Eq. (2.10) of the venturimeter.

The jet contracts to a minimum area a_2 at the venacontracta. The area a_2 is less than the area of orifice *a*.

$$a_2 = C_2\, a$$

where C_c = coefficient of contraction, less than unity. Because it is more convenient to measure the area of the orifice than the area at the venacontracta, Eq.(2.20) is generally written in a modified form

$$Q = C' \frac{C_c\, a}{\sqrt{1-\left(\dfrac{C_c\, a}{a_1}\right)^2}} \sqrt{2gh} \qquad \text{. . . (2.21a)}$$

or

$$Q = C \frac{a}{\sqrt{1-\left(\dfrac{a}{a_1}\right)^2}} \sqrt{2gh} \qquad \text{. . . (2.21b)}$$

where the coefficient *C* is such that

$$C \frac{a}{\sqrt{1-\left(\dfrac{a}{a_1}\right)^2}} = \frac{C'\, C_c\, a}{\sqrt{1-\left(C_c\,\dfrac{a}{a_1}\right)^2}}$$

Eq. (2.21b) may also be written as

$$Q = C \frac{a_1}{\sqrt{\left(\dfrac{a_1}{a}\right)^2 - 1}} \sqrt{2gh}$$

or

$$Q = Ck\sqrt{h} \qquad \text{. . . (2.22)}$$

where,

$$k = \frac{a_1\sqrt{2g}}{\sqrt{\left(\dfrac{a_1}{a}\right)^2 - 1}}$$

If the manometer deflection *x* is known,

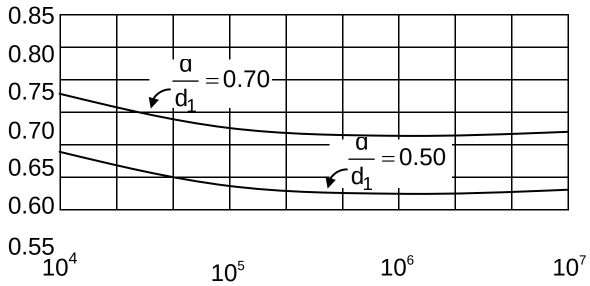
$$Q = Ck\sqrt{x\,(r-1)} \qquad \text{. . . (2.22a)}$$

Eq. (2.22a) is the same as Eq. (2.19) of the venturimeter.

Inclined Orifice Meter

If the orifice meter is inclined, use Eq. (2.16) for the upward flow and Eq. (2.17) for the downward flow. Alternatively, use Eq. (2.22a).

Eq. (2.21b) is similar to Eq. (2.10) of the venturimeter. However, it must be noted that the expression for *k* contains '*a*' and not a_2. The coefficient *C* for the orifice meter has much lower value than that for the venturimeter. Its value ranges from 0.60 to 0.75, depending upon the ratio (d/d_1) and the Reynolds number. The value of *C* also depends upon the shape of the orifice and the location of the pressure connections. Figure 2.26 shows the variation of *C* with the ratio (d/d_1) and the Reynolds number.



$$N_R = \frac{\rho_1 v_1 d_1}{\mu_1}$$

Figure 2.26

Comparison of Orifice Meter with Venturimeter

As stated above, the coefficient of an orifice meter is lower than that of a venturimeter. Another drawback of the orifice meter is that, because of excessive eddies, it is not suitable for measuring high rate of flow in large size pipes.

However, the orifice meter is cheaper than a venturimeter and is very convenient to use. An orifice plate can be inserted between two flanges of the pipe without any difficulty.

The comparison may be summarized as under :

Orifice Meter

Merits

(i) It requires less space.

(ii) It is cheaper.

Demerits

(i) Head loss is large ; C is low.

(ii) Not suitable for large discharges.

Venturimeter

Merits

(i) Head loss is small; The value of coefficient C is high.

(ii) It can be used for large size pipes and for large discharges.

Demerits

(i) It is very long and inconvenient to use.

(ii) It is costly.

Example 2.9

An orifice meter is fixed in a pipe 25 cm diameter conveying oil of specific gravity 0.90. If the diameter of the orifice is 10 cm, calculate the discharge when a mercury differential manometer shows a deflection of 80 cm. $C = 0.65$.

Solution

From Eq. (2.22a),

$$Q = Ck\sqrt{x\,(r-1)}$$

where $$k = \frac{a_1 \sqrt{2g}}{\sqrt{\left(\frac{a_1}{a}\right)^2 - 1}} = \frac{\frac{\pi}{4} \times 0.25^2 \times 4.43}{\sqrt{(2.5)^4 - 1}} = 0.0352$$

Therefore, $$Q = 0.65 \times 0.0352 \sqrt{0.8 \left(\frac{13.60}{0.90} - 1\right)} = 0.077 \text{ m}^3/\text{s}.$$

SAQ 4

Water flows through a 200 mm diameter pipe fitted with a 100 mm diameter horizontal orifice meter at the rate of 0.015 m^3/s. Determine the difference of pressure head between the upstream and the venacontracta. Take coefficient $C = 0.61$.

2.16 SUMMARY

- Different lines of the flow pattern are defined. Generally, streamlines are used in practice. A streamline is an imaginary line such that the tangent at any point on it indicates the velocity at that point.
- Different types of flow are defined. In practice, generally the flow is assumed to be steady and uniform.
- The continuity equation is derived from the principle of conservation of mass.
- The mean velocity of flow (V) and discharge (Q) are explained.
- The Bernoulli equation is derived from the principle of conservation of energy. The assumptions made in the derivation are discussed. The modified Bernoulli equation for real fluids is given.
- The energy gradient line, also called total energy line, and the hydraulic gradient line are plotted from simple flow problems.
- The equation for discharge through a venturimeter is derived using the continuity equation and the energy equation.
- The use of differential manometers for the measurement of the difference of pressure is explained.
- The discharge equation for an orifice meter is derived.
- The relative advantages and disadvantages of venturimeters and orifice meters are discussed. Although a venturimeter is more accurate, it is not as convenient as an orifice meter.

2.17 ANSWERS TO SAQs

SAQ 1

(a) $Q = A \times V$

$$= \frac{\pi}{4} \times (0.20)^2 \times 3 = 0.094 \text{ m}^3/\text{s}$$

(b) Discharge in each pipe $= \frac{0.094}{2}$ m^3/s

$$V = \frac{Q}{A} = \frac{\frac{0.094}{2}}{\frac{\pi}{4} \times (0.1)^2} = 5.98 \text{ m/s}$$

SAQ 2

(a) Pressure head at $A = \frac{68.67}{9.81} = 7.0$ m

Velocity at $A = \frac{0.150}{\frac{\pi}{4} \times (0.15)^2} = 8.49$ m/s

Pressure head at $B = \frac{49.05}{9.81} = 5.0$ m

Velocity at $B = \frac{0.150}{\frac{\pi}{4} \times (0.4)^2} = 1.19$ m/s

Total head at A, assuming datum at A

$$= \frac{p}{\gamma} + \frac{v^2}{2g} + 0$$

$$= 7.0 + \frac{(8.49)^2}{2 \times 9.81} = 7.0 + 3.67$$

$$= 10.67 \text{ m}$$

Total head at $B = 5.0 + \frac{(1.19)^2}{2 \times 9.81} + 4.50$

$$= 9.57 \text{ m}$$

Since the total head A is greater than that at B, the flow is from *A* to *B*.

(b) Loss of head = 10.67 – 9.57 = 1.10 m

SAQ 3

(a) For an inclined venturimeter with upward flow,

$$Q = Ck\sqrt{(h - z)}$$

where $k = \frac{a_1}{\sqrt{\left(\frac{a_1}{a_2}\right)^2 - 1}} \sqrt{2g}$

$$= \frac{\frac{\pi}{4} \times (0.30)^2}{\sqrt{\left(\frac{0.30}{0.15}\right)^4 - 1}} \times \sqrt{2 \times 9.81}$$

or $k = 0.0808$

$$h = \frac{p_1 - p_2}{s\gamma} = \frac{141.26 - 75.54}{0.82 \times 9.81}$$

$= 8.17$ m of gasoline

Since $Z = 0.60$ m,

$$Q = C \times 0.0809 \times \sqrt{8.17 - 0.60}$$

Substituting $Q = 0.220$,

$$0.220 = C \times 0.0808 \times 2.751$$

or $C = 0.989$

(b) $Q = Ck\sqrt{x\,(r-1)}$

where
$$k = \frac{a_1}{\sqrt{\left(\frac{a_1}{a_2}\right)^2 - 1}}\sqrt{2g}$$

$$= \frac{\frac{\pi}{4} \times (0.25)^2}{\sqrt{\left(\frac{0.25}{0.10}\right)^4 - 1}} \times \sqrt{19.62}$$

or $k = 0.0352$

or $r = \frac{s_1}{s} = \frac{13.6}{0.85} = 16.0$

$\therefore$ $Q = 0.97 \times 0.0352 \times \sqrt{0.60\,(16-1)}$

$= 0.1024$ m^3/s.

SAQ 4

$$Q = Ck\sqrt{h}$$

where
$$k = \frac{a_1}{\sqrt{\left(\frac{a_1}{a}\right)^2 - 1}}\sqrt{2g}$$

$$= \frac{\frac{\pi}{4} \times (0.2)^2}{\sqrt{\left(\frac{0.2}{0.1}\right)^4 - 1}} \times \sqrt{2 \times 9.81}$$

or $k = 0.036$

Therefore, $0.015 = 0.61 \times 0.036\sqrt{h}$

or $h = 0.\ 467$ m of water.