UNIT 1 BASIC CONCEPTS

Structure

1.1 INTRODUCTION

Matter exists in two states, *viz*., the solid state and the fluid state. The fluid state is further divided in two states : (i) the liquid state, and (ii) the gaseous state. Fluid Mechanics is the science dealing with the behaviour of fluids at rest and in motion. The fundamental principles employed in fluid mechanics are the same as that used in the mechanics of solids. But fluid mechanics is somewhat more difficult than solid mechanics, because in the mechanics of solids we deal with separate, tangible entities, whereas in fluid mechanics we deal with a continuous mass.

A *fluid is a substance which deforms continuously when subjected to a shear stress.* Even the slightest shear force will cause the fluid to flow. The behaviour of fluid is different from that of solids. A solid always requires a certain amount of shear stress before it yields. The difference in fluids and solids is due to their molecular structure. In solids, the position of molecules is fixed in space. Individual molecules in solids are very close to one another and the inter-molecular forces are rather large. In fluids, the molecules can move and their position is not fixed in space. The spacing between molecules of fluids is very large and the inter-molecular forces are relatively small. A fluid is characterized by the relative ease of the mobility of molecules.

A further difference between solids and fluids is their relative ability to resist external forces. A solid can withstand tensile, compressive and shearing forces. A fluid can sustain only compressive forces and that too when it is confined in a container. In certain cases, a fluid may sustain a small amount of tensile stress, but its tensile strength is usually neglected. When a fluid is subjected to a shearing force, it deforms continuously as long as the force acts. Thus, *the shearing stress exists in a fluid when it is in motion.* When the motion ceases, the shearing stress disappears.

Compressibility is the property, which distinguishes a liquid from a gas. Gases are extremely compressible and expand indefinitely when all external forces are removed. Thus, a gas can remain in equilibrium only when it is completely confined in a container. *Liquids are relatively incompressible*. The cohesion between particles in liquids holds them together due to which they cannot expand indefinitely. Consequently, a liquid may have a free surface, whereas a gas does not have a free surface. A vapour is gaseous in form but its temperature and pressure are such that it is close to the liquid phase in characteristics.

Objectives

After studying this unit, you should be able to

- refresh the basic concepts of mechanics, i.e. dimensions and units, mass density, specific weight, specific gravity, viscosity, bulk modulus, surface tension, pressure, forces, laws of motion etc.,
- describe the core concepts of statics and dynamics, and
- build over the concepts refreshed in this unit during the journey of further units of the course.

1.2 DIMENSIONS AND UNITS

Dimensions

Physical quantities used in fluid mechanics are expressed in five fundamental dimensions, viz, length (L), mass (M), force (F), time (T) and temperature (t). Temperature is important only in compressible flow problems and is not of frequent occurrence in fluid mechanics. Thus, there are four dimensions that are commonly used in fluid problems. However, only three fundamental dimensions are sufficient to describe any fluid phenomenon. Either force (F) or mass (M) can be taken as an independent dimension. These two dimensions are interrelated by Newton's second law of motion :

$$\text{Force } [F] = \text{Mass } [M] \times \text{Acceleration } [L/T^2]$$

If mass [M] is taken as an independent dimension, force [F] is a derived dimension and *vice-versa*. Thus there are two systems of dimensions : (i) M-L-T system, (ii) F-L-T system, depending upon whether M or F is taken as a fundamental dimension. The dimension of any physical quantity may be derived in terms of fundamental dimensions. For example, the dimensions of velocity, acceleration and mass density are respectively

[L/T], [L/T^2], and [M/L^3]. The dimensions are obtained from the definition of the quantity. For example, velocity is define as distance per unit time, therefore its dimension is [L/T]. Likewise, the dimensions of acceleration and mass density are obtained. Dimensions are written in square brackets.

Units

Various physical quantities are measured in different standards called units. There are basically two systems of units, viz. (i) Absolute system and (ii) Gravitational system. In the *absolute system*, the unit of mass is chosen and the unit of force is derived. It is thus based on *M-L-T* system. On the other hand, in the *gravitational system*, the unit of force is chosen, and the unit of mass is derived. It is, therefore, based on *F-L-T* system.

SI Units

System International d' Unites (abbreviated as SI units) has been adopted by most of the countries, including India. This is an absolute system of units. In this system, the unit of mass is chosen as kilogram (kg). The derived unit of force is Newton (N).

One Newton (N) is the force which imparts an acceleration of 1 m/sec^2 to a mass of 1 kg. Thus,

$$1 \text{ N} = 1 \text{ kg} \times 1 \text{ m/sec}^2$$

Sometimes, larger units of force are used.

$$1 \text{ kilo Newton (1 kN)} = 10^3 \text{ N}$$

$$1 \text{ mega Newton (1 mN)} = 10^6 \text{ N}$$

The length unit is metre (m) and the unit for time is second (s).

When it is impractical to use N and m themselves then the use of prefix is convenient. The prefix is representing 10 raised to a power which is a multiple of 3 is recommended in SI units. For example, the distance may be measured in mm (m × 10^{-3}) or km (m × 10^3).

In SI units, the units kg, metre and seconds are used.

The derived unit N or kN are also commonly used in engineering practice.

[**Note :** 1 N is also equal to 10^5 dynes in C.G.S. units.]

1.3 MASS DENSITY, SPECIFIC WEIGHT AND SPECIFIC GRAVITY

Mass Density

Mass Density is defined as the mass per unit volume. It is usually represented by Greek letter ρ (rho). It is measured in kg/m^3.

A molecule has a certain mass regardless of its state. The mass density is proportional to the number of molecules in a unit volume of fluid. Mass density is independent of gravitational pull. It has a fixed value at a constant temperature and pressure. It increases with increase in pressure, but decreases with increase in temperature, which is mainly due to change in the spacing of molecules.

Mass density of water at 4° C is 1000 kg/m^3.

Specific Weight

Specific weight is defined as the weight per unit volume. It is usually represented by Greek letter γ (gamma). The unit of its measurement is N/m^3. [**Note :** In some texts, the specific weight is expressed as *w*].

Specific weight represents the force exerted by the gravity on a unit volume. It depends on both mass density and the acceleration due to gravity. The specific weight changes from one place to another, depending upon the changes in the acceleration due to gravity. As in the case of mass density, the specific weight changes with change in temperature and pressure.

Specific weight and mass density are related by the equation :

$$\gamma = \rho g \qquad \text{. . . (1.1)}$$

where g is the acceleration due to gravity. The value of 'g' will be taken 9.81 m/sec^2, unless otherwise mentioned.

Specific weight of water is generally taken as 9810 N/m^3 (9.81 kN/m^3), unless mentioned otherwise.

Specific Volume

Specific volume is the volume occupied by a unit mass of the fluid. It is represented by V_S. Specific volume is reciprocal of mass density. It is measured in m^3/kg units. Thus

$$V_S = 1/\rho \qquad \text{. . . (1.2)}$$

Specific Gravity

Specific gravity is the ratio of the mass density of the fluid to the mass density of a standard fluid. It is represented by *S.* It is also known as relative density.

For liquids, the standard fluid is water at 4°C. For gases, the standard fluid is taken either air free from carbon dioxide at 0°C or hydrogen at the same temperature. The standard fluid must be mentioned if it is different from water.

Specific gravity is dimensionless and has no units.

Obviously, for any fluid $\rho = \rho_w S$ or $\gamma = \gamma_w S$

where γ_w is the specific weight of water, ρ_w is the mass density of water and S is specific gravity.

Example 1.1

(a) 10 m^3 of kerosene oil weighs 78.48 kN. Calculate its specific weight, mass density and specific gravity.

(b) The specific volume of a certain gas is 0.70 m^3/kg. Determine its specific weight and mass density.

Solution

(a) Specific weight, $\gamma = \dfrac{78.48}{10} = 7.848\ kN/m^3\ (7848\ N/m^3)$

Mass density, $\rho = \frac{\gamma}{g} = \frac{7848}{9.81} = 800 \text{ kg/m}^3$

Specific gravity, $S = \frac{800}{1000} = 0.80$

(b) Mass density, $\rho = \frac{1}{\text{Specific volume}} = \frac{1}{0.70} = 1.43 \text{ kg/m}^3$

Specific weight, $\gamma = \rho g = 14.03 \text{ N/m}^3$

SAQ 1

If 10 m^3 of mercury weighs 1329 kN, calculate its mass density, specific weight and specific gravity.

1.4 VISCOSITY

Viscosity is a property of fluid which determines its resistance to shearing stresses. *An ideal fluid has no viscosity.* In reality there is no fluid which can be classified as a perfectly ideal fluid. However, the fluids with very little viscosity are sometimes considered as ideal fluids. Viscosity of fluids is due to cohesion and interaction between particles.

An expression for viscosity may be obtained by considering two large plates which are placed at a small distance Y apart (Figure 1.1). The space between plates is filled with a fluid. The lower plate is stationary, whereas the upper plate is moving with a velocity V. Particles of the fluid in contact with each plate would adhere to the surface and there would be *no slip*. Thus the velocity of the fluid particles adjacent to the upper plate would be V and that adjacent to the lower plate would be zero. If the distance Y is small, the velocity gradient will be a straight line. Experiments indicate that the force (P) is proportional to the area of the plate (A) and the velocity V, and is inversely proportional to the distance Y, i.e.,

$$P \propto A\,(V/Y)$$

If the constant of proportionality is taken as μ

$$P = \mu\, A\,(V/Y)$$

The shearing stress (τ) may be written as

$$\tau = P/A = \mu\,(V/Y)$$

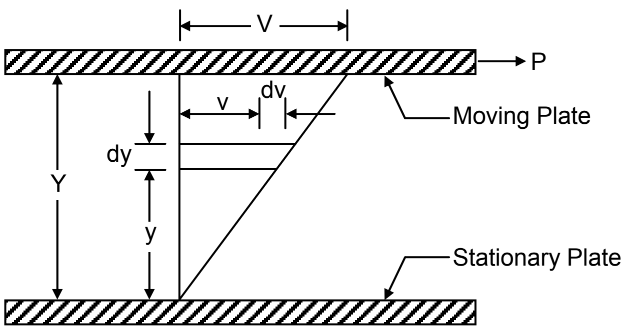


Figure 1.1

The above equation can be expressed in differential form. Let us consider a fluid layer of thickness *dy* at a distance *y*. Let us assume that velocity *v* changes by *dv* in thickness *dy*. Then by similar triangles

$$V/Y = (dv/dy)$$

Therefore, $\tau = \mu\,(dv/dy)$ . . . (1.3)

Eq. (1.3) is known as Newton's equation of viscosity. The ratio *dv*/*dy* is known as the velocity gradient. The constant μ (mu) is called the coefficient of dynamic viscosity or simply the coefficient of viscosity. The coefficient of viscosity may be defined as the amount of tangential stress required to maintain unit relative velocity between two parallel layers of fluid at unit distance. Fluids which obey Eq. (1.3), are known as *Newtonian Fluids*. The fluids which do not obey this law are *Non-Newtonian Fluids*. Figure 1.2 also shows ideal fluids which have zero shear stress. The discussions herein would be confined to Newtonian Fluids only.

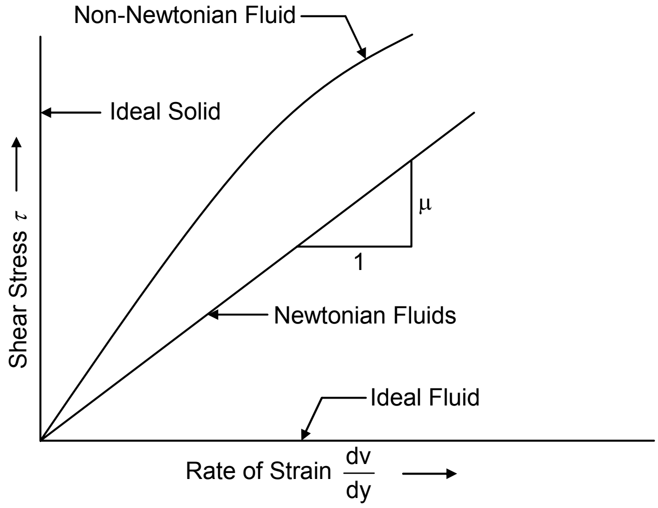


Figure 1.2

Eq. (1.3) may be used to derive the dimensions of μ. Substituting the dimensions of *τ, dv* and *dy.*

$$\left[\frac{F}{L^2}\right] = \mu\left[\frac{L}{T}\times\frac{1}{L}\right] \quad \text{or} \quad \mu = \left[\frac{FT}{L^2}\right]$$

As mentioned before, square brackets indicate that only dimensions are being considered. The dimensions of μ in *M-L-T* can be obtained from Newton's second law of motion, namely,

$$[F] = [M][L/T^2]$$

Therefore, $[\mu] = [M][L/T^2][T/L^2] = [M/(LT)]$

In SI units, the unit of dynamic viscosity is N-s/m^2 (kg m^{-1}s^{-1}). The unit N-s/ m^2 is also called Pascal-second (Pa-s).

In CGS (Centimetre-Gram-Second) units, viscosity is measured in poise which is equal to one dyne-sec/cm^2 or one gm (mass)/cm-sec. The unit is named after Poiseuille, a pioneer in the field. Sometimes, a smaller unit centipoise, which is equal to one-hundredth of poise, is used. The viscosity of water at 20^{o}C is approximately equal to one centipoise.

Obviously, 1 N-s/ m^2 = 10 poise or 1 poise = 0.1 N-s/m^2

In fluid problems, the coefficient of dynamic viscosity μ usually occurs together with mass density ρ in the form μ/ρ. In such problems, it is convenient to use coefficient of kinematic viscosity v (nu). It is the ratio of the coefficient of dynamic viscosity to mass density.

Thus, $$v = \mu / \rho \qquad \text{. . . (1.4)}$$

The dimensions of v can be obtained from Eq. (1.4)

$$[v] = [M/(LT)]\,[L^3/M] = [L^2/T]$$

Thus, v is a kinematic term as it does not involve forces. In CGS units, it is measured in *stoke*, which is equal to one cm^2/sec. The unit is named after Stoke. A smaller unit centistoke, which is one-hundredth of a stoke, is sometimes used. In SI units, the unit is one m^2/sec. Obviously, 1 stoke = 10^{-4} m^2/sec.

Viscosity is practically independent of pressures at ordinary pressures. But at very high pressures, it increases with pressure. The viscosity of liquids decreases with increase in temperature. (The reader might have observed that when a viscous oil is heated, it flows easily). On the contrary, the viscosity of gases increases with an increase in temperature. This behaviour of gases is because of different intermolecular characteristics of gases. In liquids, the viscosity is mainly due to cohesion, which decreases with increase in temperature. In gases, the viscosity is due to molecular activity. As the molecular activity increases with temperature, the viscosity of gases also increases with rise in temperature.

The effect of viscosity on a fluid phenomenon is usually expressed in terms of a non-dimensional parameter called Reynolds Number (N_R) given by

$$N_R = \frac{Vd\rho}{\mu} = \frac{Vd}{v}$$

where V is the velocity and d is any characteristic length. The Reynolds number is also abbreviated as *Re*.

Example 1.2

A fluid has a dynamic viscosity of 0.50 poise. Calculate the velocity gradient and the intensity of shear stress at the boundary if the fluid is filled between two parallel plates 5 cm apart and one plate is moving at a velocity of 1 m/sec, other plate is stationary. Assume distribution of velocity as

$v = 1.0 - k\,(0.05 - y)^2$, where y is in metres.

Solution

The equation of the parabola given is $v = 1.0 - k\,(0.05 - y)^2$

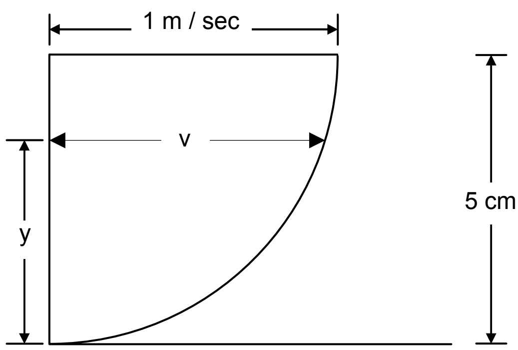


Figure 1.3

Obviously, $v = 0$ when $y = 0$

Therefore, $k = 400$, and the equation becomes $v = 1.0 - 400\,(0.05 - y)^2$

or $$\frac{dv}{dy} = 800(0.05 - y)$$

From Eq. (1.3), since $\mu = 0.5 \times 0.1$ N-s/m^2,

$$\tau = \mu \left(\frac{dv}{dy}\right) = (0.5 \times 0.1)\ 800\ (0.05 - y)$$

The maximum shear stress occurs at the lower plate where $y = 0$.

$$\tau_{max} = (0.5 \times 0.1)\ (800)\ (0.05) = 2\ \text{N/m}^2$$

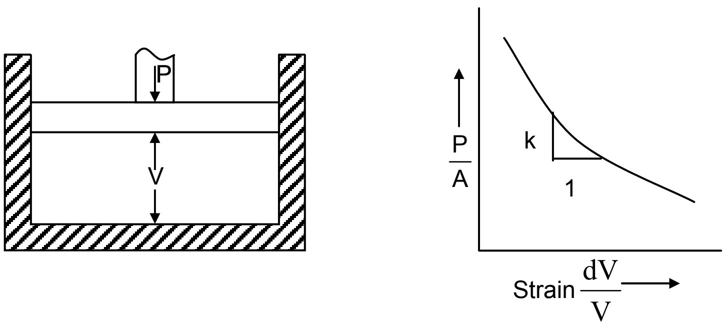
SAQ 2

A cubical block weighing 196.2 kN and having a 200 mm edge is allowed to slide down on an inclined plane surface making an angle of 20º with the horizontal on which there is a thin film of liquid having a viscosity of 2.158 N-s/m^2. What terminal velocity will be attained by the block if the film thickness is 0.025 mm.

1.5 BULK MODULUS

All fluids can be compressed by the application of pressure. However, gases are more compressible than liquids. Liquids are compressed by so little an amount that in most of the cases, they are assumed to be incompressible. Owing to elasticity, strain energy is stored in fluids as they are compressed. Elasticity of fluids is measured in terms of bulk modulus of elasticity (K).

The mechanics of elastic compression of fluids may be demonstrated by means of a cylinder and piston [Figure 1.4(a)]. Let the initial volume of the fluid be V. If a force P is now gradually applied, there will be an increase in pressure. This increase is given by $p=P/A$, where A is the area of cross-section of cylinder. The volumetric strain is given by dV/V, where dV is the change in volume. A plot can be obtained between P/A and dV/V for different value of P as shown in Figure 1.4(b).



(a) (b)

Figure 1.4

The bulk modulus of elasticity is defined as the slope of the stress-strain curve [Figure 1.4(b)] at the point under consideration. Thus,

$$K = \frac{dp}{-dV/V} \qquad \ldots (1.5)$$

The bulk modulus K is always a positive quantity. The negative sign indicates a decrease in volume with an increase in P. Steepening of the curve with increasing pressure shows that as fluids are compressed, it becomes increasingly difficult to compress them further. In other words, the bulk modulus of elasticity increases with an increase in pressure.

Compressibility is inversely proportional to bulk modulus (K).

Bulk modulus of elasticity K is expressed in kN/m^2. Bulk modulus of elasticity of water is approximately 2.1×10^6 kN/m^2 and that of air is 105 kN/m^2. This indicates that air is 20,000 times more compressible than water. However, water is about 80 times more compressible than steel ($K = 170 \times 10^6$ kN/m^2).

Example 1.3

Find the change in volume of 1 m^3 of water when it is subjected to an increase in pressure of 1962 kN/m^2. Take $K = 2.16 \times 10^6$ kN/m^2.

Solution

From Eq. (1.5), $$K = \frac{dp}{-dV/V}$$

or $$2.16 \times 10^6 = \frac{1962}{dV/V}$$

or $$dV = (-9.17 \times 10^{-4}) \times V = -9.17 \times 10^{-4} \text{ m}^3$$

SAQ 3

At a depth of 6.8 km in an ocean, the pressure is 68.67 MN/m^2 above the atmospheric pressure. Assuming that the specific weight of water at the surface is equal to 10 kN/m^3, calculate the specific weight of water at 6.8 km depth. Assume bulk modulus of water as 2.453×10^3 MN/m^2.

1.6 VAPOUR PRESSURE

All liquids have a tendency to vaporize. They tend to change from the liquid to the gaseous state. Molecules of liquids are continuously projected from the free surface of liquids to the atmosphere. These ejected molecules are in gaseous state and exert their own partial vapour pressure on the liquid surface. This pressure is known as the vapour pressure of the liquid (p_v).

As the molecular activity increases with temperature, the vapour pressure also increases with a rise in temperature. Boiling of the liquid occurs when the external pressure imposed on the liquid is equal to or less than the vapour pressure of the liquid at that temperature. Consequently, the boiling point of a liquid depends upon both the temperature and the ambient pressure.

If the space above the liquid is confined, the partial vapour pressure exerted by the molecules increases till the rate at which the molecules re-enter the liquid is equal to the rate at which they leave the surface. When this equilibrium condition is reached, the vapour pressure is called the *saturation vapour pressure*. If the pressure on the liquid surface is lower than or equal to the saturation vapour pressure, boiling occurs.

The saturation vapour pressure is of great practical use in fluid problems. If the pressure at any point in a fluid phenomenon approaches the vapour pressure, the liquid starts vaporizing. Vapour bubbles which are created in the region of low pressure are carried with the liquid to the region of high pressure. These bubbles collapse in the region of high pressure and explosion of bubbles takes place. This explosion causes damage of the walls of the conduit and also creates air pockets in the flow. The phenomenon is known as *cavitation*. Because of the destructive nature of cavitation, its occurrence in flow problems should be avoided. This is possible if the pressure at any point in the fluid phenomenon is not permitted to fall below the saturation vapour pressure.

As the temperature increases, the vapour pressure increases till the boiling point is reached for the ambient pressure. Water boils at 100°C at which temperature the vapour pressure 101.03 kN/m^2 is equal to the atmospheric pressure. Water will boil at 20°C if the ambient pressure is reduced to 2.35 kN/m^2. In problems of flow of water, in order to avoid cavitation, pressure is usually not permitted to fall below 24.5 kN/m^2.

Example 1.4

A vertical cylinder 30 cm diameter is fitted at the top with a tight but frictionless cylinder and is completely filled with water at 20°C. The outside of the piston is exposed to atmospheric pressure of 98.1 kN/m^2. Calculate the minimum force applied to the piston which will cause the water to boil if p_v at 20°C = 2.35 kN/m^2.

Solution

As water cannot expand, a space filled with vapours is obtained as soon as the piston is moved upwards. Boiling will occur when the net downward pressure is equal to vapour pressure. Let P be the force applied in upward direction. Therefore,

$$98.1 \times (\pi/4) \times (0.3)^2 - P = 2.35 \times (\pi/4) \times 0.3^2$$

$$P = (98.1 - 2.35)\,(\pi/4) \times 0.3^2$$

$$= 6.76 \text{ kN.}$$

1.7 SURFACE TENSION

Liquids have properties of cohesion and adhesion. Both these properties are forms of molecular attraction. Cohesion enables a liquid to resist small amount of tensile stresses, whereas adhesion enables it to adhere to another body. Surface tension is caused by the forces of cohesion at the free surface. A liquid molecule in the interior of the liquid mass is surrounded by other molecules all around it and is in equilibrium. At the free surface of the liquid, there are no liquid molecules above the surface to balance the forces of the molecules below it (Figure 1.5). Consequently, there is a net inward force on the molecule. This force is normal to the liquid surface. At the interface between a liquid and a gas (i.e. at free surfaces), a thin layer of molecules is formed because of the difference of forces above and below the layer. It is because of this film that a thin small needle can float on the free surface. The layer acts as a *membrane*.

Surface tension is the force required to maintain the film of the liquid. The surface tension σ (sigma) is defined as the force in the liquid surface normal to a line of unit length drawn on the surface. It has the dimension [F/L] and is measured in N/m. For water at 20°C, the value of σ is approximately 0.0735 N/m. For mercury, its value is 0.51 N/m.

[**Note :** As the surface tension is also a measure of energy per unit area, it can also be expressed as J/m^2 or N-m/m^2.]

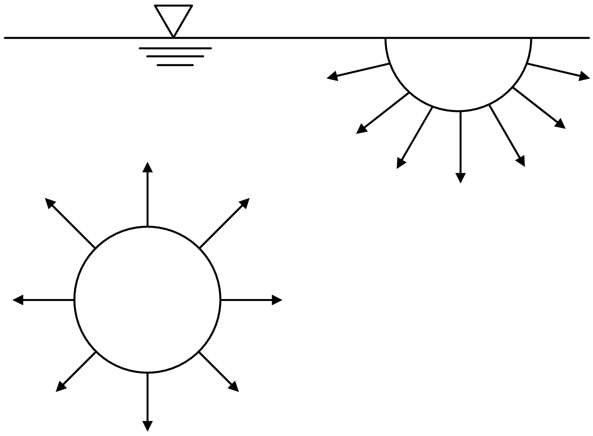


Figure 1.5

Surface tension decreases with an increase in temperature. It depends upon the type of fluid in contact with the liquid surface. It is usually quoted in contact with air.

Expression for Pressure Difference

Let us consider a small element of the liquid. The surface of the element is of double curvature, with radii R_1 and R_2 (Figure 1.6).

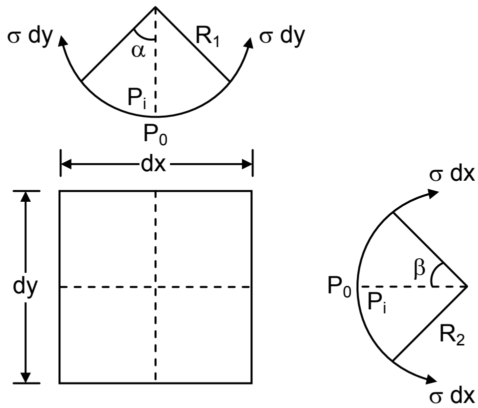


Figure 1.6

The pressure inside and outside the element are p_i and p_0 respectively. The element must be in equilibrium under the pressures and the surface tension forces σ dx and σ dy. Resolving the forces in the direction normal to the element

$$(p_i - p_0)\, dx\, dy = 2\sigma\, dy\, \sin\alpha + 2\sigma\, dx \sin\beta$$

or $$(p_i - p_0)\, dx\, dy = 2\sigma\, dy \left(\frac{dx}{2R_1}\right) + 2\sigma\, dx \left(\frac{dy}{2R_2}\right)$$

or $$(p_i - p_0)\, dx\, dy = \sigma\, dx\, dy \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

or $$(p_i - p_0) = \sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \qquad \text{. . . (1.6)}$$

This is the general expression giving the pressure difference between the inside and outside of the surface.

For a spherical surface, $R_1 = R_2 = R$

Thus, $$(p_i - p_0) = \frac{2\sigma}{R} \qquad \text{. . . (1.7)}$$

Eq. (1.7) can be modified for soap bubbles. Soap bubbles have both inside and outside surfaces on which surface tension acts. Therefore, R. H. S. is doubled.

Thus, $$(p_i - p_0) = \frac{4\sigma}{R} \qquad \text{. . . (1.8)}$$

For a cylindrical surface, $R_1 = R$ (say) and $R_2 = \infty$

Thus $$(p_i - p_0) = \frac{\sigma}{R} \qquad \text{. . . (1.9)}$$

1.8 CAPILLARITY

Capillary action is due to both cohesion and adhesion. If the effect of cohesion is less significant than the effect of adhesion, the liquid will wet the solid surface with which it is in contact and will rise at the point of contact. On the other hand, if the cohesion predominates, the liquid will not wet the surface and the liquid surface will be depressed at the point of contact. That is the reason why water rises in a small glass tube placed in water. If the same tube is placed in mercury, the level of mercury is depressed in the tube.

If a liquid wets the solid boundary, there is a decrease of pressure within the liquid (as $p_i > p_0$). This causes the liquid to rise in a small glass tube [Figure 1.7(a)]. For a cylindrical capillary tube, taking atmospheric pressure as zero,

$$p_i = 0 \text{ and } p_0 = -\gamma h$$

The pressure p_i is zero because the inner surface is exposed to atmosphere. It may be noted that the inner surface is always towards the centre of curvature. From Eq. (1.7), assuming the liquid surface to be spherical,

$$0 + \gamma h = \frac{2\sigma}{R}$$

[**Note :** The pressures with the atmospheric pressure as zero are called gauge pressures. The gauge pressure can be negative. The negative gauge pressure is also called vacuum pressure (see Section 1.10).]

But $\frac{r}{R} = \cos\theta$, where θ is the angle of contact, and r is the radius of tube.

Therefore, $$\gamma h = \frac{(2\sigma\cos\theta)}{r}$$

$$h = \frac{2\sigma\cos\theta}{\gamma r} = \frac{4\sigma\cos\theta}{\gamma d} \qquad \text{. . . (1.10)}$$

where d = diameter of the tube.

Eq. (1.10) gives the capillary rise in a small tube provided both the liquid and the tube are perfectly clean. If they are not clean, the value of h will be less than that given by Eq. (1.10). For water with clean glass surface tube, the angle θ is usually assumed

(a) (b)

Figure 1.7

If the liquid does not wet the solid surface, the liquid level in the tube is depressed [Figure 1.7(b)]. There is an increase in the internal pressure. In this case,
Eq. (1.10) gives the fall of the liquid surface in tube below the liquid surface outside the tube. Mercury behaves in this fashion. The value of θ for mercury in contact with glass is about 140^o.

For tubes of radius 6 mm or more, the capillary action is negligibly small. To minimize the effect of capillary action in pressure measurement by piezometers and manometers, tubes of radius of 6 mm or more should be used.

Example 1.5

Estimate the height to which water column at 20°C will rise in a capillary tube 3 mm diameter. Take σ = 0.0735 N/m.

Solution

From Eq. (1.10), $$h = \frac{2\sigma \cos\theta}{\gamma r}$$

Assuming $$\theta = 0,\ h = \frac{2 \times 0.0735 \times 1}{9810 \times 0.0015} = 0.01 \text{ m} = 10 \text{ mm}.$$

Example 1.6

A small drop of water at 20°C is in contact with air and has a diameter of 0.5 mm. If the pressure within the droplet is 0.59 kN/m^2 greater than the atmospheric pressure, calculate the value of the surface tension (σ).

Solution

From Eq. (1.7), $$p_i - p_0 = \frac{2\sigma}{R}$$

$$0.59 \times 10^3 = \frac{2\sigma}{0.25 \times 10^{-3}}$$

or $\sigma = 0.074$ kN/m

or $\sigma = 74$ N/m.

Table 1.1 gives the properties of some common fluids at 20° and atmospheric pressure.

Table 1.1 : Properties of Common Fluids at 20° and Atmospheric Pressure

Liquid	Mass Density (ρ) kg/m³	Specific Weight (γ) kN/ m³	Bulk Modulus (K) kN/ m² (× 10⁶)	Surface Tension (σ) N/m	Vapour Pressure (p_v) kN/ m²	Dynamic Viscosity (μ)		Kinematic Viscosity (ν) m²/s
						Poise	N-s/m²	
Water	998.00	9.80	2.16	0.073	2.35	1.0×10^{-2}	1.0×10^{-3}	1.00×10^{-6}
Gasoline	680.00	6.67	0.96	0.022	55.00	2.9×10^{-3}	2.9×10^{-4}	4.28×10^{-7}
Mercury	13540.00	132.83	26.24	0.510	1.73×10^{-4}	1.6×10^{-2}	1.6×10^{-3}	1.18×10^{-7}
Glycerine	1268.40	12.44	4.35	0.064	1.37×10^{-5}	8.35	0.835	6.63×10^{-4}
Carbon Tetra Chloride	1594.00	15.64	1.10	0.026	12.75	1.0×10^{-2}	1.0×10^{-3}	6.40×10^{-7}
Kerosene	799.50	7.84	-	0.024	3.20	2.0×10^{-2}	2.0×10^{-3}	2.50×10^{-6}
Benzene	880.94	8.64	1.03	0.026	10.00	7.0×10^{-3}	7.0×10^{-4}	7.95×10^{-7}
Castor oil	959.42	9.41	1.44	0.039	-	9.8	0.98	1.02×10^{-3}
Ethyl Alcohol	788.72	7.74	121	0.022	5.78	1.2×10^{-2}	1.2×10^{-3}	1.53×10^{-6}

SAQ 4

For measuring the surface tension of a mineral oil by the bubble tube method, a tube having an internal diameter of 1.5 mm is immersed to a depth of 12.5 mm in the oil. Air is forced through the tube forming a bubble at the lower end. What magnitude of the surface tension will be indicated by a maximum bubble-pressure intensity of 147.15 N/m²?

Take specific gravity of oil as 0.85.

1.9 PRESSURE

When a fluid is contained in a vessel, it exerts forces on the surface of the vessel. Since the fluid is at rest, there is no relative motion between the layers of the fluid. The velocity gradient is zero, and hence there is no shear stress in the fluid. Consequently, there is no force component acting tangentially to the walls of the container. The force exerted by the fluid on the surface of the vessel is always normal to the surface. Intensity of pressure (or simply pressure) is the normal force per unit area of the surface. In general,

$$p = \frac{dP}{dA}$$

where, p = pressure intensity,

dP = force acting on a small area dA, and

dA = small differential area.

If the total force P acts uniformly over the entire area A,

$$p = \frac{P}{A}$$

The dimension of pressure are $[F/L^2]$. This is measured in N/m^2 or kN/m^2. The unit N/m^2 is also known as Pascal (Pa), and kN/m^2 as kilo Pascal.

The resultant pressure on any plane due to a fluid is always normal to that plane. This fact can be demonstrated as follows : Let us suppose that the resultant pressure P on a plane AB is inclined to the surface (Figure 1.8). This pressure can be resolved into components P_1 and P_2 parallel and perpendicular to AB respectively. The component P_1 can be resisted only by a shearing stress. Since a fluid at rest cannot resist a shearing stress, the pressure must be normal to the plane and there should be no P_1 component. This proves that the pressure must be normal to the plane.

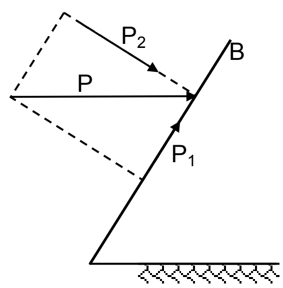


Figure 1.8

1.9.1 Pressure Variation with Depth of Liquid

The intensity of pressure varies with the depth of liquid. The relation between pressure variation and the depth of liquid can be obtained as follows : Let us consider an inclined prism of liquid of length l and cross-sectional area dA (Figure 1.9). The cross-sectional area dA being small, the pressure intensity at the end faces may be assumed to be uniform. If the pressure at the ends A and B are respectively p_1 and p_2, the forces acting on the ends are $p_1\,dA$ and $p_2\,dA$. The forces on the inclined sides of the prism balance themselves. The prism is in equilibrium under the pressure forces at the ends and the self-weight. Resolving these forces along the axis of the prism,

$$p_1\,dA - p_2\,dA - \gamma\,l\,dA\cos\theta = 0$$

Since $\quad l\cos\theta = h, \quad p_1\,dA - p_2\,dA - \gamma\,h\,dA = 0$

or $\quad p_1 - p_2 = \gamma h \quad$ . . . (1.11a)

Alternatively, $p_1 - p_2 = \rho g h$ . . . (1.11b)

Eq. (1.11a) shows that the difference in pressure between two points depends solely upon the difference in elevation between the points and the specific weight of the fluid. If the points lie in the same horizontal plane (i.e. $h = 0$), the pressures are equal. In other words, the pressures at two points at the same level in a continuous fluid are equal.

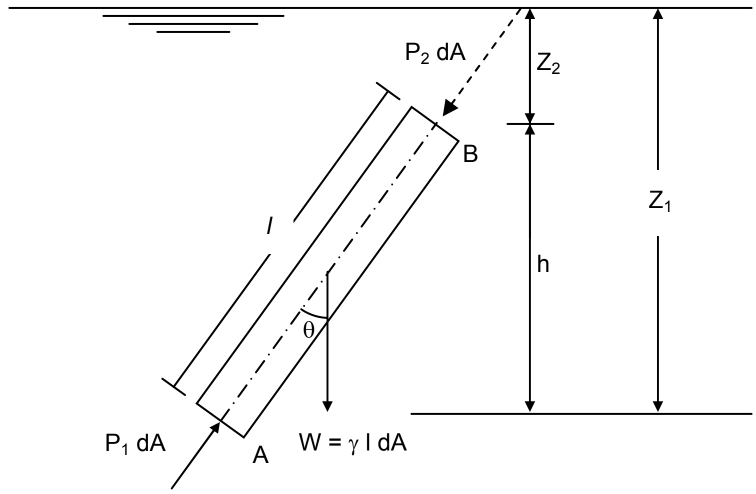


Figure 1.9

The variation in pressure with depth from one point to another point in a body of fluid at rest may also be determined from the free-body concept. Let us consider a cylindrical element of fluid of cross-sectional area *dA* and height *dz* (Figure 1.10). The element is in equilibrium under the pressure forces and the self weight.

Therefore, $$p\,dA - \left(p + \frac{dp}{dz}\,dz\right) dA - \gamma\; dz\; dA = 0$$

$$\frac{dp}{dz}\,dz\;dA = -\,\gamma\;dz\;dA$$

$$dp = -\gamma\;dz \qquad \text{. . . (1.12a)}$$

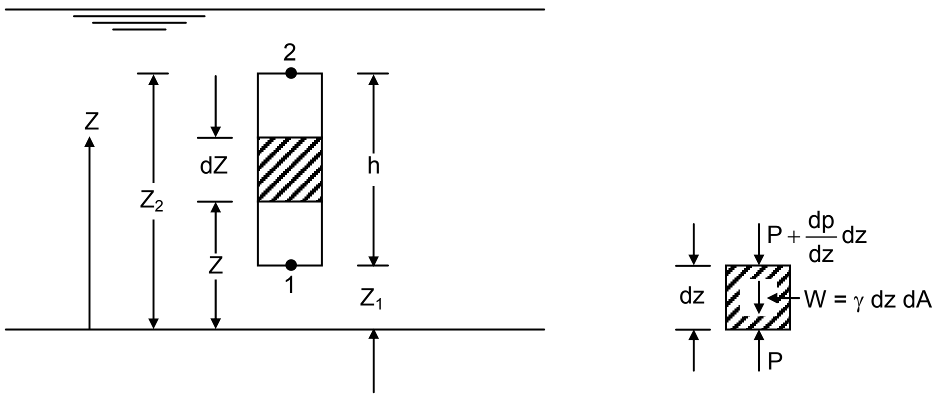


Figure 1.10

Alternatively, $dp = -\rho g\;dz$ . . . (1.12b)

Eq. (1.12a) indicates that the pressure intensity decreases as the height increases. The difference in pressure between two points 1 and 2 may be obtained from Eq. (1.12a) by integration,

$$\int_{p_1}^{p_2} \frac{dp}{\gamma} = - \int_{z_1}^{z_2} dz$$

If γ is assumed to be constant (i.e. the fluid is incompressible),

$$\frac{p_1 - p_2}{\gamma} = -(z_2 - z_1) = z_1 - z_2$$

$$p_1 - p_2 = \gamma (z_2 - z_1) = \gamma h, \text{ same as Eq. (1.11a)}$$

If point 2 is taken at the free surface of the liquid, $p_2 = 0$ (taking the local atmospheric pressure as zero). Therefore,

$$p_1 = \gamma h$$

or, in general, $$p = \gamma h \qquad \ldots (1.13a)$$

The pressure intensity (p) at a depth 'h' below the free surface is equal to the product of the specific weight γ of liquid and the depth 'h'.

Further, p may be expressed in terms of the height h of column of a liquid of specific weight γ as

$$h = \frac{p}{\gamma} \qquad \ldots (1.13b)$$

When the pressure at a point is expressed in units of length, it is commonly called *pressure head.* It is measured in metres or mm of liquid column. For example, the atmospheric pressure of 101.3 kN/m^2 may be expressed as 10.3 m of water or 760 mm of mercury. Atmospheric pressure is also expressed as 1.013 bar, where 1.0 bar = 10^5 N/m^2.

1.9.2 Absolute, Gauge and Vacuum Pressures

If pressure intensity is expressed with respect to complete vacuum, it is called the *absolute pressure.* When it is measured above the local atmospheric pressure, it is called the *gauge pressure.* For example, if the absolute pressure at a point A is 150 kN/m^2 and the atmospheric pressure is 101.0 kN/m^2, the gauge pressure at that point is 49 kN/m^2. If the pressure intensity is greater than the local atmospheric pressure, the difference of these two pressures is called the positive gauge pressure or simply *gauge pressure.* However, if the pressure intensity at a point is less than the local atmospheric pressure, the difference of these two pressures is called the negative gauge pressure or *vacuum pressure.* For example, if the absolute pressure at point B is 53 kN/m^2, the vacuum pressure at that point is 48 kN/m^2. These definitions of absolute, gauge and vacuum pressures are represented diagrammatically in Figure 1.11.

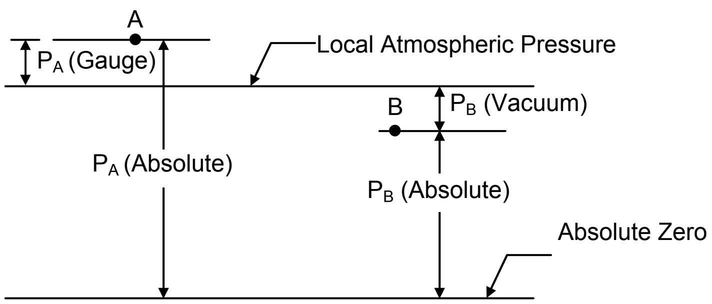


Figure 1.11

It may be noted that high vacuum pressure means a very low absolute pressure. A perfect vacuum means zero absolute pressure.

(**Note :** In all problems the mass density of water is taken as 1000 kg/m^3 and its specific weight as 9.81 kN/m^3 unless mentioned otherwise.)

SAQ 5

At the inlet of a pump, the negative pressure is 7 m of water and at the exit of the pump, there is a gauge pressure of 25 m of water. Calculate the pressure developed by the pump in kN/m^2. Also calculate the absolute pressures at the inlet and exit of the pump if the atmospheric pressure is 10.3 m of water.

1.10 BASIC DEFINITIONS OF MECHANICS

System of Forces

A force is an action which changes or tends to change the state of rest or the state of uniform motion of a body. A force has a magnitude and direction and hence it is a vector. Graphically, a force is represented by a straight line to scale where length of the line indicates the magnitude and the arrow head indicates the direction. A force is fully specified if its magnitude, direction and the point of application are known.

The forces may be classified as :

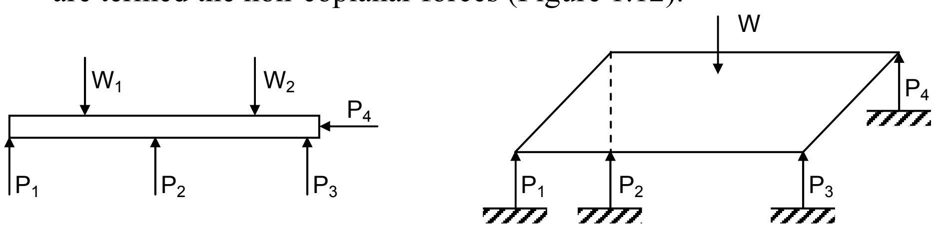
(a) Coplanar or non-coplanar.

(b) Concurrent or non-concurrent.

(c) Parallel or non-parallel.

Coplanar or Non-coplanar Forces

If all the forces lie in the same plane, they are called coplanar forces; on the other hand, if all the forces do not lie in the same plane, they are termed the non-coplanar forces (Figure 1.12).

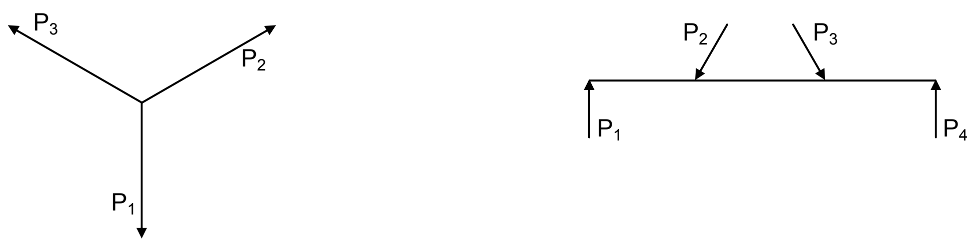


(a) Coplanar Forces **(b) Non-coplanar Forces**

Figure 1.12

Concurrent or Non-concurrent Forces

If all the forces pass through a common point, they are known as concurrent forces. In contrast, if all the forces do not pass through a common point, they are called non-concurrent forces (Figure 1.13).

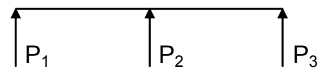


(a) Concurrent Forces **(b) Non-concurrent Forces**

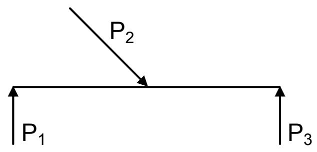
Figure 1.13

Parallel or Non-parallel Forces

If the lines of action of all the forces are parallel to one another, they are called parallel forces. However, if the lines of action of all the forces are not parallel to one another, they are known as non-parallel forces (Figure 1.14).



(a) Parallel Forces



(b) Non-parallel Forces

Figure 1.14

Various combinations based on above three classifications are possible. The following 4 combined systems are quite common.

Coplanar Concurrent Forces

In this system, all the forces lie in one plane and they also pass through a common point (Figure 1.15a).

Coplanar Non-concurrent Forces

In this system, all the forces lie in one plane but they do not pass through a common point (Figure 1.15b). The non-concurrent forces may be parallel or non-parallel.

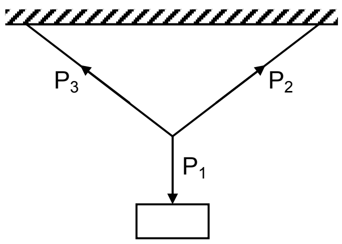
Non Coplanar-concurrent Forces

In this system, all the forces do not lie in one plane but they pass through a common point (Figure 1.15c).

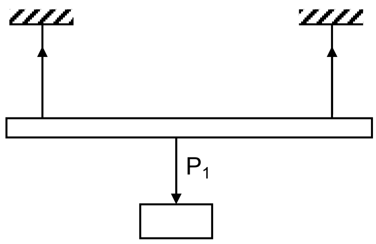
Non-coplanar Non-concurrent Forces

In this system, all the forces do not lie in one plane. Moreover, they do not pass through a common point (Figure 1.15d). The non-concurrent forces may be parallel or non-parallel.

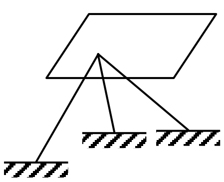
(**Note :** In the following discussions, non-coplanar forces are not discussed further.)



(a)



(b)



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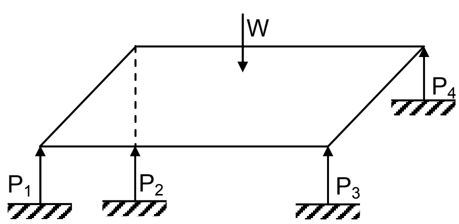


(d)

Figure 1.15

Composition of Forces

If a system of forces is acting on a body, their total effect can be expressed in terms of their resultant force. Thus, a resultant force is a single force which can replace two or more forces acting on a body, and has the same effect on the body as all the forces. Since a force is a vector quantity, the resultant of a number of forces can be found out by using vector algebra. The method of determination of the resultant of a system of forces acting simultaneously on a body is known as the *composition of forces*.

Resolution of Forces

A single force can be replaced by two parts acting in different directions which will produce the same effect on the body as the given force has. This process of splitting a force into two parts is called the *resolution of forces*. The force which is broken into parts is called the resolved force and the parts are called resolved parts, component forces or the resolutes.

A force can be resolved either into two mutually perpendicular parts or into two mutually non-perpendicular parts. Generally, the force is resolved into two mutually perpendicular parts. Figure 1.16 shows a force F inclined at an angle θ with x-axis. The force is represented by an inclined OA. The component F_x of the force in x-direction is equal to OB. Thus,

$$F_x = F \cos \theta \qquad \text{. . . (a)}$$

Likewise, the component F_y of the force in y-direction is equal to OC.

Thus, $$F_y = F \sin \theta \qquad \text{. . . (b)}$$

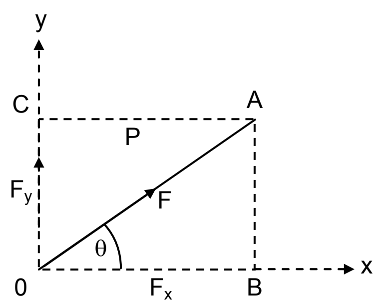


Figure 1.16

Moment of a Force

The moment of a force on a body is defined as the tendency of the force to produce rotation about a point. The point need not be the actual pivot point. It may be any other point on the body or even may be elsewhere. The moment of a force about a point is equal to the product of the magnitude of the force and the perpendicular distance between the line of the force and the axis of rotation through the point. For example, the moment of force F about point O is given by

$$M = F\,x$$

where x is the perpendicular distance OC of point O from AB as shown in Figure 1.17.

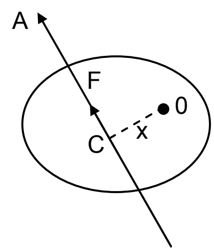


Figure 1.17

Point O is known as the moment centre (or fulcrum) and the distance x is called the moment arm.

Equilibrium of Forces

If a body is subjected to a number of forces and it does not move or accelerate, it is said to be in equilibrium or static equilibrium. All external forces acting on a body in equilibrium must have a resultant equal to zero. Moreover, the algebraic sum of the moment caused by these forces about any point must also be equal to zero.

$$R = 0 \qquad \sum M_0 = 0$$

where R is the resultant force and $\sum M_0$ is the algebraic sum of the moment of all the forces about any convenient point O.

1.10.1 Coplanar Concurrent Forces

Laws of Forces

The following laws of forces are used for the determination of the resultant of forces acting on a body.

Parallelogram Laws of Forces

According to this law, if two forces on a body are represented in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram which passes through their point of intersection. In Figure 1.18, the two forces F_1 and F_2 acting on a body are represented by sides AB and AD, respectively, of a parallelogram $ABCD$. According to the parallelogram law of forces, their resultant R is represented by the diagonal AC.

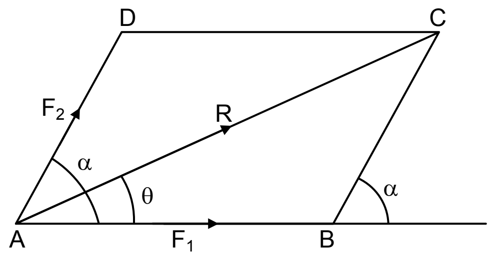


Figure 1.18

The value of R can be determined graphically by drawing the parallelogram to the scale and measuring the diagonal AC and angle θ. The value of R can also be determined analytically from the trigonometric relation.

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos \alpha} \qquad \ldots (1.14)$$

where α is the angle between F_1 and F_2.

The angle θ which the resultant R makes with the force F_1, is computed from the relation

$$\tan\theta = \frac{F_2 \sin\alpha}{F_1 + F_2 \cos\alpha} \qquad \text{. . . (1.15)}$$

Triangle Law of Forces

According to this law, if two forces acting an a body are represented in magnitude and direction by the two sides of a triangle, taken in order, then their resultant is represented in magnitude and direction by the third side of the triangle, taken in opposite direction.

Figure 1.19 shows two forces F_1 and F_2 represented by two sides AB and BC respectively of a triangle ABC. Their resultant R is represented by the side AC, with the direction from A to C.

The resultant can be determined graphically by drawing the triangle to the scale and measuring the angle θ and the length AC.

The resultant can also be determined analytically from the trigonometric relation.

$$\frac{R}{\sin\alpha} = \frac{F_1}{\sin(\alpha - \theta)} = \frac{F_2}{\sin\theta} \qquad \text{. . . (1.16)}$$

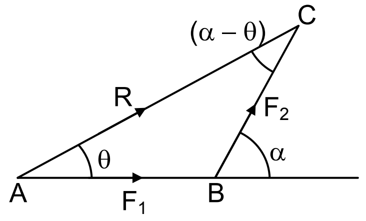


Figure 1.19

Polygon Law of Forces

According to this law, if a number of coplanar concurrent forces acting on a body are represented in magnitude and direction by the sides of an open polygon, taken in order, then their resultant is represented in magnitude and direction by the closing side of the polygon, taken in the opposite order.

In Figure 1.20, the forces F_1, F_2, F_3 and F_4 are represented by the sides *AB, BC, CD* and *DE* of an open polygon *ABCDE*. Their resultant R is represented by side AE, which is in opposite order (i.e. not in the same sense as the forces F_1, F_2, F_3 and F_4.

The polygon law of forces is an extension of the triangle law of forces. According to which R_1 is the resultant of forces F_1 and F_2 and R_2 is the resultant of R_1 and F_3, and finally R is the resultant of R_2 and F_4. In other words, R is the resultant of all forces.

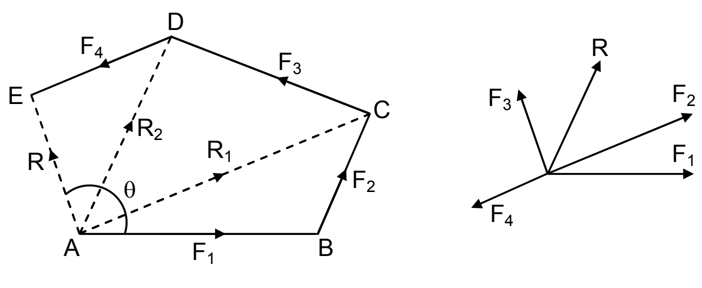


Figure 1.20

The resultant R can be determined by drawing the polygon $ABCDE$ to scale and measuring the side AE and angle θ. However, for the determination of resultant R the analytical method by using the resolved parts of forces is more convenient. The resolved parts of the resultant R in x and y directions are obviously equal to the algebraic sum of the resolved parts of all the forces in x and y directions. Thus,

$$R \cos\theta = F_1 \cos\theta_1 + F_2 \cos\theta_2 + F_3 \cos\theta_3 + F_4 \cos\theta_4$$

or $$R \cos\theta = \sum H \qquad \text{. . . (1.17a)}$$

and $$R \sin\theta = F_1 \sin\theta_1 + F_2 \sin\theta_2 + F_3 \sin\theta_3 + F_4 \sin\theta_4$$

$$R \sin\theta = \sum V \qquad \text{. . . (1.17b)}$$

where θ_1, θ_2, θ_3, and θ_4 are the angles which F_1, F_2, F_3 and F_4 make with the x-axis, respectively.

From Eqs. (1.17a) and (1.17b)

$$R = \sqrt{(\sum H)^2 + (\sum V)^2} \qquad \text{. . . (1.17c)}$$

and $$\tan\theta = \frac{\sum V}{\sum H} \qquad \text{. . . (1.18)}$$

While computing the resolved parts, proper signs must be considered. The resolved parts in the upward direction (+ y) and that in the left to right direction (+ x) are taken positive.

Equilibrium Conditions for Coplanar Concurrent Forces

A body subjected to coplanar concurrent forces will be in equilibrium if the resultant of all the forces is zero. The body will, therefore, be in equilibrium if the force polygon of forces closes, i.e. when the last force vector ends at the starting point.

Equilibrium conditions for coplanar concurrent forces may also be written as

$$\sum H = 0 \qquad \text{. . . (1.19)}$$

where $\sum H$ indicates the algebraic sum of the components of all the forces in the horizontal direction (or along x- axis).

Also $$\sum V = 0 \qquad \text{. . . (1.20)}$$

where $\sum V$ indicates the algebraic sum of the components of the forces in the vertical direction (or along y-axis).

Obviously, when the above two conditions are satisfied, the resultant will be zero,

$$R = \sqrt{(\sum H)^2 + (\sum V)^2} = 0 \qquad \text{. . . (1.21)}$$

Example 1.7

A force of 20 kN acts in x-direction and another force of 16 kN acts at an angle of 60° with the x-axis. Determine the resultant of the two forces.

Solution

From Eq. (1.14), $R = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos\alpha}$

$$= \sqrt{(20)^2 + (16)^2 + 2 \times 20 \times 16 \cos 60^\circ} = 31.24 \text{ kN}$$

From Eq. (1.15), $\theta = \tan^{-1} \dfrac{F_2 \sin\alpha}{F_1 + F_2 \cos\alpha} = \tan^{-1}\left(\dfrac{16 \sin 60^\circ}{20 + 16 \cos 60^\circ}\right)$

$$\theta = 26.33^\circ$$

The resultant force is 31.24 kN acting at an angle of 26.33° with the x-axis.

Example 1.8

Determine the resultant of the following four coplanar concurrent forces.

(a) Force of 15 N acting along x-axis.

(b) Force of 20 N acting at an angle of 30° with x-axis.

(c) Force of 10 N acting at an angle of 60° with x-axis.

(d) Force of 25 N acting in the y-direction.

Solution

From Eq. (1.17a)

$$\Sigma H = 15 \cos 0^\circ + 20 \cos 30^\circ + 10 \cos 60^\circ + 25 \cos 90^\circ$$

$$= 15 + 17.32 + 5 + 0 = 37.32 \text{ N}$$

From Eq. (1.17b)

$$\Sigma V = 15 \sin 0^\circ + 20 \sin 30^\circ + 10 \sin 60^\circ + 25 \sin 90^\circ$$

$$= 0 + 10 + 8.66 + 25 = 43.66 \text{ N}$$

From Eq. (1.17c)

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(37.32)^2 + (43.66)^2}$$

$$= \sqrt{1392.78 + 1906.20} = 57.44 \text{ N}$$

From Eq. (1.18), $\theta = \tan^{-1} \dfrac{\Sigma V}{\Sigma H}$

$$= \tan^{-1}\left(\frac{43.66}{37.32}\right) = 49.48^\circ$$

The resultant force is 57.44 N acting at an angle of 49.48° with x-axis.

Example 1.9

A weight of 10 kN is suspended by two cords AC and BC as shown in Figure 1.21. Determine the tension in the cords if the system is in equilibrium.

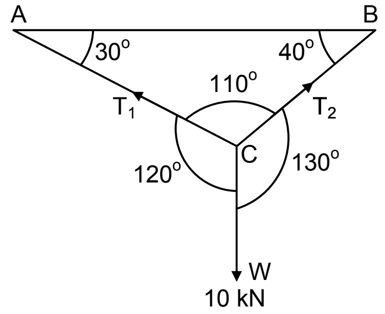


Figure 1.21

Solution

At point C, three forces T_1, T_2 and W are acting where T_1 is the tension in the cord AC, T_2 is the tension in the cord BC and W is the weight.

From Eq. (1.19), for equilibrium

$$\sum H = 0$$

or $$T_2 \cos 40^\circ - T_1 \cos 30^\circ + W \cos 90^\circ = 0$$

or $$0.766\ T_2 - 0.866\ T_1 + 0 = 0 \quad \ldots (1)$$

From Eq. (1.20)

$$T_2 \sin 40^\circ + T_1 \sin 30^\circ - W = 0$$

or $$0.643\ T_2 + 0.5\ T_1 - 10 = 0 \quad \ldots (2)$$

Solving simultaneous Eqs. (1) and (2),

$$T_1 = 8.15 \text{ kN} \qquad T_2 = 9.21 \text{ kN}$$

SAQ 6

(a) Two forces F_1 and F_2 act at a point O. The force F_1 has a magnitude of 10 kN and is inclined at 45° to positive x-axis, whereas the force F_2 has a magnitude of 20 kN and is inclined at 120° to positive x-axis. Determine the resolved parts of the two forces in x and y-directions and hence determine the resultant.

(b) Four coplanar forces F_1, F_2, F_3 and F_4 acting at a point are in equilibrium. If the magnitudes and inclinations of the forces F_1, F_2, and F_3 are as given in the following table, determine the magnitude and inclination of the force F_4.

Force	**Magnitude**	**Inclination with + x-axis**
F_1	5 kN	30°
F_2	10 kN	90°
F_3	20 kN	150°

1.10.2 Coplanar Non-Concurrent Forces

Resultant of Coplanar Non-concurrent Forces

In this case, in addition to the magnitude and direction of the resultant force, the third unknown; namely, the point of application of the resultant force is required.

The magnitude of the resultant can be found by resolving all the forces horizontally and vertically. Thus

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

where ΣH = algebraic sum of the components of all the given forces in the horizontal direction.

ΣV = algebraic sum of the components of all the given forces in the vertical direction.

The direction of the resultant is determined using Eq (1.18)

$$\tan\theta = \frac{\Sigma V}{\Sigma H}$$

where θ is the angle which the resultant makes in the horizontal axis.

The position of the resultant is determined by taking moments of the forces (or their components) about any point in their plane and equating the algebraic sum of moments of all the given forces to that of the resultant by using the following relation.

Moment of resultant R about that point

= algebraic sum of all the given forces about the same point

= algebraic sum of the rectangular components of all the forces about the same point.

or $$R \times d = \Sigma M_0 \qquad \ldots (1.22)$$

where R is the resultant force, d is the perpendicular distance of R from any point O, and ΣM_0 is the algebraic sum of the moments of all the given forces about O.

Note :

(i) While computing the algebraic sum of the moments, a suitable sign convention is taken. Generally, clockwise moments are taken as positive.

(ii) If all the given forces are parallel, their resultant will also be parallel to the forces. Hence, angle θ is not required for the determination of the direction of the resultant.

Equilibrium Conditions for Coplanar, Non-concurrent Forces

If a body is subjected to a number of coplanar, non-concurrent forces, the body will be in equilibrium if their resultant is equal to zero and the algebraic sum of moments of all the forces about any point in their plane is also equal to zero.

For equilibrium, the following three conditions must be satisfied:

The algebraic sum of the components of all the forces in the horizontal direction is equal to zero, i.e.

$$\Sigma H = 0$$

The algebraic sum of the components of all the forces in the vertical direction is equal to zero, i.e.

$$\Sigma V = 0$$

The algebraic sum of moments of all the forces about any point O in their plane must be equal to zero, i.e.

$$\Sigma M_0 = 0 \qquad \text{. . . (1.23)}$$

Example 1.10

Four forces equal to 5 kN, 10 kN, 20 kN and 40 kN respectively acting along the four sides of a square ABCD of side 2 m each, taken in order as shown in Figure 1.22. Find the magnitude, direction and the position of the resultant.

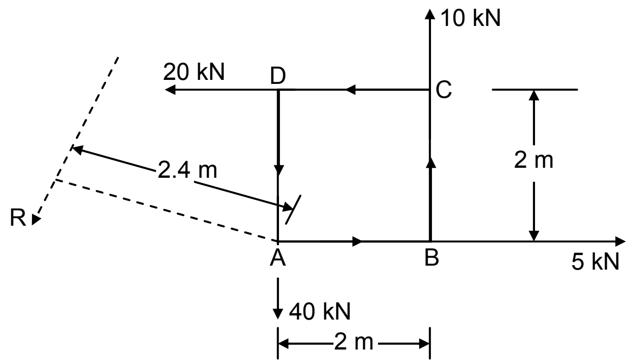


Figure 1.22

Solution

From Eq. (1.17a),

$$\Sigma H = 5 - 20 = -15 \text{ kN}$$

From Eq. (1.17b),

$$\Sigma V = 10 - 40 = -30 \text{ kN}$$

From Eq. (1.17c),

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$R = \sqrt{(-15)^2 + (-30)^2}$$

$$= 33.54 \text{ kN}$$

From Eq. (1.18),

$$\tan \theta = \frac{\Sigma V}{\Sigma H}$$

$$= \frac{-30^{\circ}}{-15}$$

$$\theta = 63.43° \text{ or } 243.43°$$

Since both ΣH and ΣV are negative, angle θ must be 243.43°.

From Eq. (1.22),

$$R \times d = \sum M_0$$

Taking moments of all forces about point *A*,

$$33.54 \times d = -10 \times 2 - 20 \times 2$$

or $$d = -1.79 \text{ m}$$

The resultant acts at a perpendicular distance of 1.79 m from *A*. The resultant is shown by the dotted line in Figure 1.22.

Example 1.11

Determine the reaction components R_1, R_2 and R_3 for the beam shown in Figure 1.23.

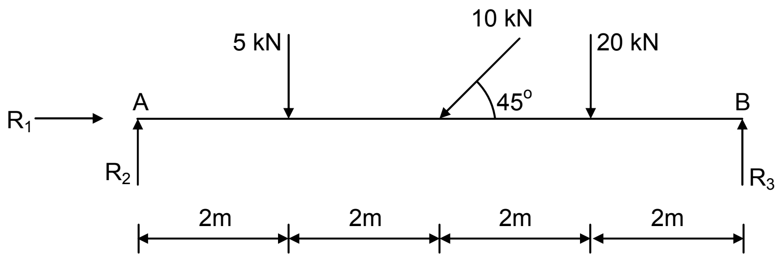


Figure 1.23

Solution

$$\sum H = 0$$

or $$R_1 - 10\cos 45° = 0$$

or $$R_1 = 7.07 \text{ kN} \text{ (towards right)}$$

$$\sum V = 0$$

or $$R_2 - 5 - 10\sin 45° - 20 + R_3 = 0$$

or $$R_2 + R_3 = 32.07 \quad \ldots \text{(i)}$$

From Eq. (1.23),

$$\sum M_0 = 0$$

Taking moments of all forces about point *A*, with clockwise moments as positive,

$$5 \times 2 + (10 \sin 45°) \times 4 + 20 \times 6 - 8R_3 = 0$$

or $$10 + 28.28 + 120 - 8R_3 = 0$$

or $$8\,R_3 = 158.28$$

or $$R_3 = 19.79 \text{ kN}$$

Substituting the value of R_3 in Eq. (i),

$$R_2 + 19.79 = 32.07$$

$$R_2 = 12.28 \text{ kN}$$

SAQ 7

(a) Determine the support reactions R_1, R_2 and R_3 of the beam shown in Figure 1.24.

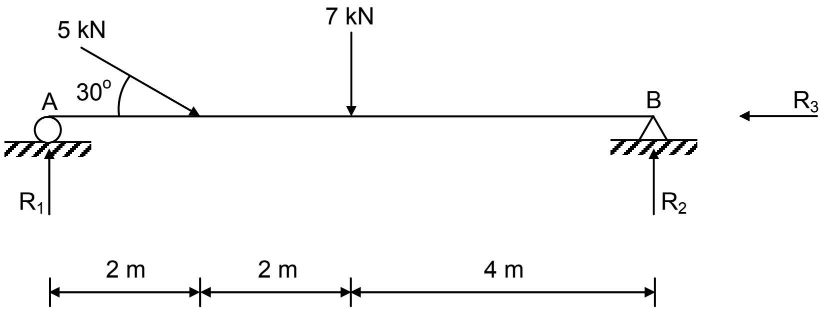


Figure 1.24

(b) Determine the resultant of the four coplanar non-concurrent forces shown in Figure 1.25.

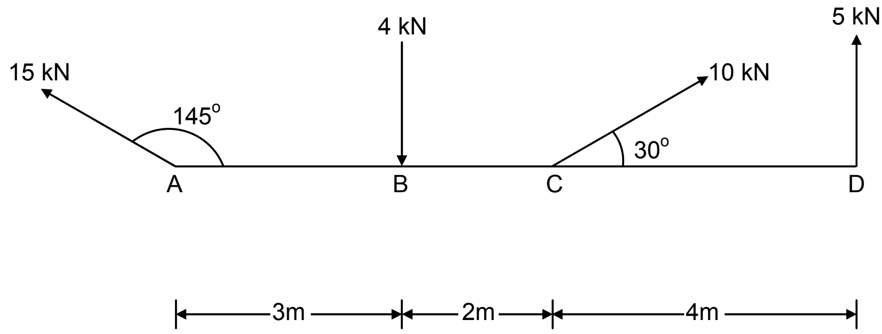


Figure 1.25

1.11 LAWS OF MOTION

The following three laws were enunciated by Sir Issac Newton in 1686 and are known as Newton's laws of motion :

First Law of Motion

It states that a body continues in the state of rest or in the state of uniform motion in a straight line unless it is acted upon by some external force to change its state.

We know from our experience that if a body is at rest, it will remain in that condition unless some external force is applied to move it. Similarly, if a body is moving at a uniform velocity in a straight line, it will continue its motion unless some external force is applied to it.

Second Law of Motion

It states that the rate of change of momentum of a body is directly proportional to the impressed force and takes place in the direction of the straight line in which the force acts.

Momentum of a body is equal to the product of its mass and velocity. Because velocity is a vector, momentum is also a vector. If the mass is expressed in kg and velocity in m/s, the momentum is in kg m/s.

Thus, $\text{Momentum} = \text{Mass} \times \text{velocity} = m\,v$

where m is mass and v is the velocity.

If the velocity changes from v_1 to v_2 in time t,

$$\text{Rate of change of momentum} = \frac{mv_2 - mv_1}{t}$$

$$= \frac{m(v_2 - v_1)}{t}$$

Rate of change of momentum = $m\,a$

where a is acceleration (i.e. rate of change of velocity).

Now according to Newton's second law of motion,

Impressed force $\propto$ rate of change of momentum

or $$F \propto ma \qquad \text{. . . (i)}$$

where F is the impressed force.

or $$F = kma$$

where k is a constant of proportionality.

If we take m = 1 kg, a = 1 m/s^2, and F = 1 Newton, k = 1. Thus

$$F = ma \qquad \text{. . . (ii)}$$

Thus, a force of one Newton when acting upon a mass of 1 kg produces an acceleration of 1 m/s^2.

It is useful to express Newton's second law of motion as given in Eq. (ii).

Third Law of Motion

According to this law, to every action there is an equal and opposite reaction.

We know from our experience that when we push a body, the body pushes us back with an equal force. When we pull a block by a rope, the block pulls us back with an equal force. Similarly, if a block of weight W is lying on the ground, a reaction R acts upward on the block and $R = W$.

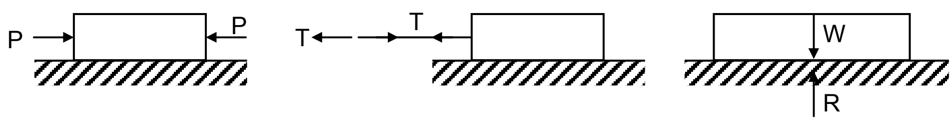


Figure 1.26

1.12 IMPULSE-MOMENTUM EQUATION

Impulse of a force F applied over a time t is equal to the product $F \times t$. If the force is expressed in Newtons and t in seconds, the unit of impulse are N-s. Impulse is a vector.

According to Newton's second law of motion,

$$\text{Force} = \text{Mass} \times \text{Acceleration}$$

or $$F = m \times \frac{\text{Change in velocity}}{\text{Time}}$$

If the velocity changes from v_1 and v_2 in time t,

$$F = \frac{m(v_2 - v_1)}{t}$$

or $$F \times t = m(v_2 - v_1)$$

$$= mv_2 - mv_1$$

$$\text{Impulse} = \text{Final momentum} - \text{Initial momentum}$$

In words, the impulse of a force in any direction is equal to change in momentum in that direction. This is known as Impulse-momentum equation and is very useful in the study of fluid mechanics.

If the force varies over the time t, the impulse of the force is obtained by integration.

$$\int F dt = \text{Final momentum} - \text{Initial momentum}$$

Example 1.12

If a body has a mass of 100 kg, what is the force required to produce an acceleration of 2 m/sec^2.

Solution

From Newton's second law,

$$F = ma$$

or $$F = 100 \times 2 = 200 \text{ N}$$

Example 1.13

A person weighs 981 N on the earth. What will be his weight on the moon where acceleration is 1.4 m/sec^2. Assume acceleration due to gravity on the earth as 9.81 m/sec^2.

Solution

The mass of the person is found from Newton's second law of motion.

$$F = ma$$

$$981 = m \times 9.81$$

or $$m = 100 \text{ kg}$$

The weight on the moon is given by

$$F = 100 \times 1.4 = 140 \text{ N}$$

Example 1.14

The velocity of a body of mass 100 kg changes from 2 m/s to 4 m/s in the same direction in a time of 25 seconds. Determine the applied force and the impulse.

Solution

According to Newton's second law,

$$\text{Force} = \text{Rate of change of momentum}$$

$$= \frac{\text{Final momentum} - \text{Initial momentum}}{\text{Time}}$$

$$= \frac{100 \times 4 - 100 \times 2}{25} = \frac{200}{25} = 8 \text{ N}$$

Now Impulse = Change of momentum

$$= 100 \times 4 - 100 \times 2 = 200 \text{ N-s}$$

Example 1.15

A mass of 10 kg is subjected to a varying force expressed as $F = 2\,t + 4$. Determine the velocity of mass after 4 seconds if its initial velocity is 10 m/s

Solution

In this case,

$$\int F\,dt = \text{Final momentum} - \text{Initial momentum}$$

or $$\int_0^4 (2t + 4)\,dt = mv_2 - mv_1$$

or $$\left[\frac{2}{2}t^2 + 4t\right]_0^4 = 10(v_2 - 10)$$

or $$[16 + 4 \times 4] = 10(v_2 - 10)$$

or $$3.2 = v_2 - 10$$

or $$v_2 = 13.2 \text{ m/s}$$

SAQ 8

The velocity of a body of mass 10 kg changes from 4 m/s to 12 m/s in 10 seconds. Determine the applied force in that direction and the impulse.

1.13 PRINCIPLE OF CONSERVATION OF ENERGY

The energy of a body is its capacity of doing work. The units of energy are the same as that of work.

The work done by a force is equal to the product of the force and the displacement in the direction of force. Work is said to be done by a force in any direction when the point of application of the force moves in that direction. For example, if a force F is applied on the body and the force is inclined at an angle of θ with
x-axis, then work done by force is given by

$$W = F \cos \theta \times s$$

where s is the distance moved in x-direction (Figure 1.27).

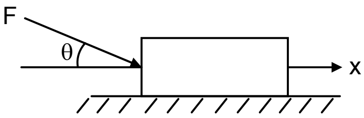


Figure 1.27

The SI unit of work as well as energy is Joule (J), where

$$1 \text{ Joule} = 1 \text{ Newton} \times 1 \text{ metre}$$

or $$1 \text{ J} = 1 \text{ N-m}$$

Power is the rate of doing work. In SI system of units, the unit of power is Watt (W), where

$$1 \text{ Watt} = 1 \text{ Joule per second}$$

or $$1 \text{ W} = 1 \text{ J/sec}$$

The power is usually expressed in kW.

Different Forms of Energy

The energy may exist in various forms such as :

(a) Mechanical energy
(b) Electrical energy
(c) Light energy
(d) Heat energy
(e) Sound energy
(f) Chemical energy
(g) Magnetic energy
(h) Nuclear energy

Principle of Conservation of Energy

According to the principle of conservation of energy, the energy can neither be created nor it can be destroyed. It can, however, change from one form to another form.

For example, electrical energy is transformed into heat energy in an electric oven and into light energy in an electric bulb. However, the total energy contained in a given system of bodies remains constant.

It may be noted that according to the principle of conservation of energy if some work is done on a body, the energy content of the body is increased by the amount of work done. On the other hand, if some work is done by a body, the energy content of the body is reduced by the amount of work done. Thus,

Final energy = Initial energy ± Work done

This equation is also known as **Work energy equation**.

In mechanics, we are generally concerned with only mechanical energy. In fluid mechanics, we also consider pressure energy. In thermodynamics, we are also concerned with heat energy. The following discussion is limited to mechanical energy only.

Mechanical Energy

The mechanical energy of a body consists of potential energy and kinetic energy.

Potential Energy (*PE*)

It is the energy possessed by a body by virtue of its position or configuration.

If a body of weight W is located at a height of h above an arbitrary datum, the body has a potential energy equal to $W\,h$ stored in it. If the body comes to the datum, it will do a work equal to $W\,h$. Thus,

$$\text{Potential Energy} = \text{Weight of the body} \times h$$
$$= m\,g\,h$$

where m is the mass of the body, g is the acceleration due to gravity (9.81 m/sec^2), h is the height above the datum.

Kinetic Energy (*KE*)

It is the energy possessed by a body by virtue of its motion.

If a body of weight W has a velocity of v, it has a kinetic energy equal to $\frac{1}{2g}Wv^2$. The kinetic energy can also be expressed as $\frac{1}{2}mv^2$, where m is the mass of the body. The amount of work required to be done on the body to bring it to rest is equal to the kinetic energy of the body.

Total Energy (*TE*)

The total energy in a body of weight W and having velocity v is equal to the sum of the potential energy and the kinetic energy. Thus,

$$\text{Total Energy} = \text{Potential Energy} + \text{Kinetic Energy}$$
$$= Wh + \frac{1}{2g}Wv^2$$
$$= mgh + \frac{1}{2}mv^2$$

According to the law of conservation of energy, the total energy of a body is a constant. If a body of weight W falls from rest position at a height of h above the datum (say, ground level), its kinetic energy will increase but potential energy will decrease and the sum of its potential energy and kinetic energy remains constant.

Thus, Potential Energy + Kinetic Energy = Constant

Note : In actual practice, because of friction and other losses of energy, the total energy may change. However, in the following discussions, it is assumed that there is no loss of energy from one position to the other.

Example 1.16

If a body of mass of 10 kg falls from a height of 5 m, what will be the velocity of the body when it strikes the ground.

Solution

Potential energy of the body when at a height of 5 m is equal to $(10 \times g)\ 5$.

According to the principle of conservation of energy, the total energy remains constant.

Therefore, kinetic energy when the body strikes the ground is also equal to $(10 \times g)\ 5$.

Thus $\frac{1}{2} m v^2 = 10 \times g \times 5$

or $\frac{1}{2} \times 10 \times v^2 = 10 \times 9.81 \times 5$

or $v^2 = 98.1$ or $v = 9.90$ m/s

Example 1.17

One thousand litres of water is lifted to a height of 25 m and delivered with a velocity of 4 m/s to consumers. If the operation takes 15 seconds, determine the amount of work done on water and the power used.

Solution

We know that the work done on water is equal to the change in the potential energy and kinetic energy of water when it is lifted and delivered.

One litre of water has a mass of 1 kg (i.e. a weight 9.81 N)

Potential Energy = mgh

$= 10^3 \times 9.81 \times 25$

$= 245.25 \times 10^3$ Nm

Kinetic Energy = $\frac{1}{2} m v^2$

$= = \frac{1}{2} \times 10^3 \times 4^2 = 8000$ Nm

Work done = $245.25 \times 10^3 + 8000 = 253.25 \times 10^3$ Nm

Power used = Work done per second

$= \frac{253.25 \times 10^3}{10} = 25.325 \times 10^3$ W = 25.325 kW .

1.14 HYDROSTATIC FORCE ON A SUBMERGED PLANE SURFACE

Let a plane surface of area A with its centroid at C be placed at an angle 'α' with respect to the free surface of a liquid.

Let us consider an elementary shaded strip at a depth z as in Figure 1.28(a).

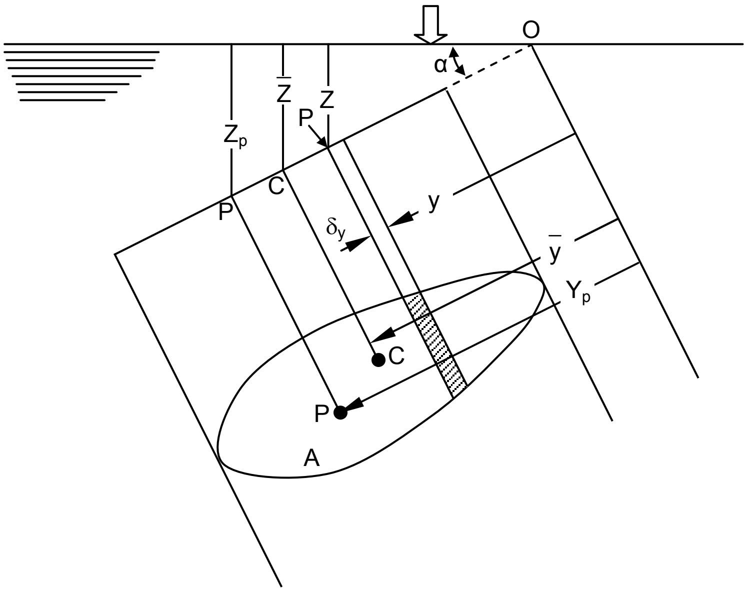


Figure 1.28(a) : A Submerged Plane Surface

We know that the pressure p on the strip = $\rho\, g\, z$

Force on the strip, $\delta F = p\, \delta A = \rho\, g\, z\, \delta A = \rho\, g\, y \sin \alpha\, \delta A$

Force on the entire surface, $F = \int dF = \int p\, dA$

$$= \int \rho\, g\, y \sin \alpha\ dA = \rho\, g \sin \alpha \int y\, dA$$

$$= \rho\, g \sin \alpha\ \bar{y}\, A = \rho\, g\, \bar{z}\, A$$

or $$F = \rho\, g\, \bar{z}\ .\ A \qquad \ldots \text{(i)}$$

This equation may be interpreted by stating that the hydrostatic force on a plane surface equals the pressure $\rho\, g\, \bar{z}$ at the centroid of the area times the submerged area A of the surface and acts normal to it.

It is interesting to arrive at the same result by looking at the pressure distribution over the surface. The pressure at any point on the surface must act normal to the surface and the intensity of pressure is given by

$$p = \rho\, g\, z$$

where z is the depth of the liquid at that point. The pressure distribution is consequently linear, i.e. trapezoidal as shown in a typical end view in Figure 1.28(b). Volume under the pressure plot should represent the hydrostatic force acting on the surface. Utilizing the fact that the volume under the pressure plot must be given by

$$V = \int \rho\, g\, z\ .\ dA = \rho\, g \int z\, dA = \rho\, g\, \bar{z}\ A$$

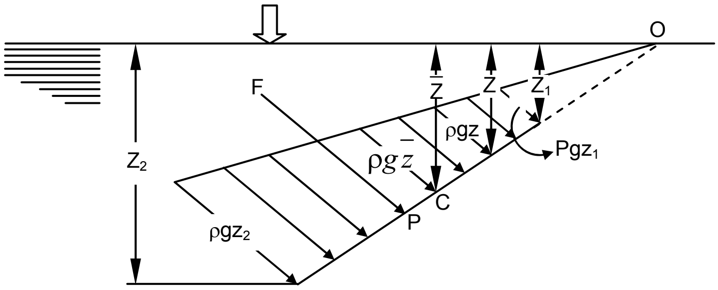


Figure 1.28(b) : Pressure Distribution and Force on an Inclined Plane Surface Submerged in a Liquid

The force F on the surface is given by

$$F = \rho g \bar{z} A \qquad \text{. . . (ii)}$$

Example 1.18

A tank contains water of density 1000 kg/m^3 up to a height of 3 m above the base. An immiscible liquid of specific gravity 0.8 is filled on top of that over 2 m depth. Calculate the pressures at a point 1.5 m below the free surface, at the interface and at another point 2.5 m below the free surface. Calculate also the force on a vertical wall, 6 m wide.

Solution

The pressure at a point A, 1.5 m below the free surface must be given by

$$p_A = (0.8 \times 1000) \times 9.81 \times 1.5 = 11770 \text{ N/m}^2$$

$$= \mathbf{11.77 \text{ kN/m}^2}$$

Similarly, the pressure at a point B at the interface of the two liquid is

$$p_B = (0.8 \times 1000) \times 9.81 \times 2 = 15700 \text{ N/m}^2$$

$$= \mathbf{15.70 \text{ kN/m}^2}$$

The pressure at a point C, 2.5 m below the free surface may be determined by either of the following considerations :

By considering that a liquid of specific gravity 0.8 acts over the 2.5 m depth and, in addition, liquid of differential specific gravity (1.0 – 0.8), i.e. 0.2 over the 0.5 m below the interface;

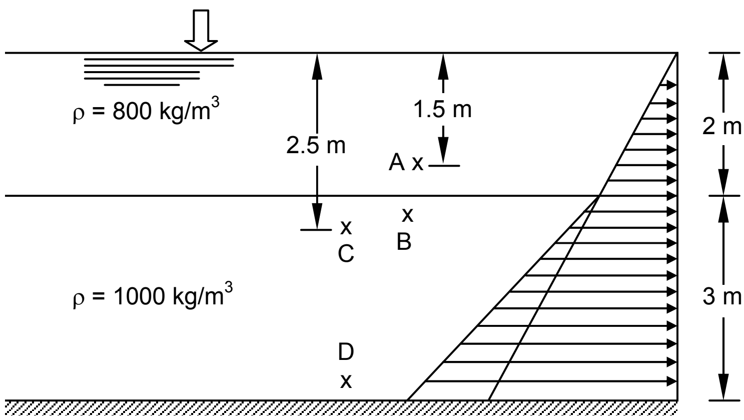


Figure 1.29

1.14.1 Forces on Immersed Plane Surfaces

Sometimes, it is essential to find out the location of the line of application of the hydrostatic force and the point of action on a plane submerged surface.

The point of action of the total hydrostatic force on the surface is called the centre of pressure, *P*, and it is not necessary that it should coincide with the centroid. We now find out the location of *P* by equating the moment of the resultant acting at *P* to the summation of the moments due to the elementary forces on the small strips.

$$F \cdot y_p = \int \delta F \cdot y$$

$$= \int \rho g\, y^2 \sin\alpha\, \delta A$$

or $$y_p\, \rho g \sin\alpha \int y\, dA = \rho g \sin\alpha \int y^2\, dA$$

or $$y_p = \frac{\int y^2\, dA}{\int y\, dA}$$

We can also write

$$y_p = \frac{\text{Second moment of area about O}}{\text{First moment of area about O}}$$

$$= \frac{I_0}{\bar{y}\, A}$$

According to parallel axis theorem,

$$I_0 = I_C + A\, \bar{y}^2$$

Hence, $$y_p = \frac{I_C + A\, \bar{y}^2}{\bar{y}\, A}$$

This relation locates the centre of pressure *P* in relation to the centroid of the submerged surface. It may be observed that each of the quantities I_C, $\sin^2 \alpha$, A and z may either by positive or zero. This means that z_p must be more than or at most equal to z. In other words, the centre of pressure must be below *C* or at most coincide with *C*. This fact can be appreciated by observing the pressure distribution on a plane surface as well. Since the intensity of pressure increase as depth increases, the point of application of the total force due to the pressure must lie below the centroid of the area on which it acts.

Let us examine the action of hydrostatic force and the location of centre of pressure for the special cases of horizontal and vertical surfaces :

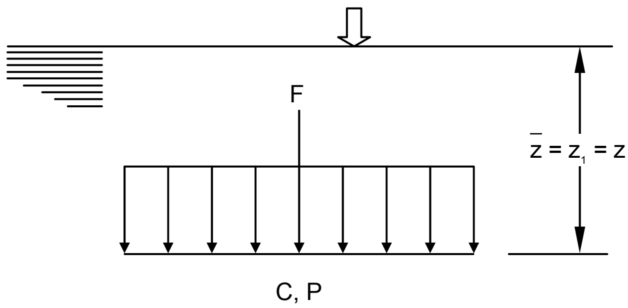
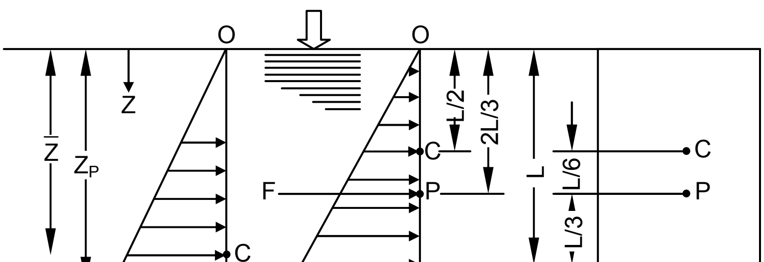


Figure 1.30(a) : Pressure Distribution and Force on a Horizontal Submerged Plane Surface



(b) Pressure Distribution and Force on a Horizontal Submerged Plane Surface

Figure 1.30

Submerged Horizontal Surface, $\alpha = 0$.

The pressure distribution on a submerged horizontal surface must be uniform over the entire area because every point on the surface is at a depth z below the free level of the liquid as shown in Figure 1.30(a).

The hydrostatic force F on it must be

$$F = \rho g \bar{z} A$$

where $\bar{z} = z$, the depth of the centroid of the area A of the surface.

The centre of pressure P must be located on the surface at the centroid

$$z_p = z = \bar{z}$$

as is also seen by examining Eq. (2.5), by substituting $\sin \alpha = 0$ for $\alpha = 0$.

Submerged Vertical Surface, $\alpha = \frac{\pi}{2} = 90^o$

The pressure distribution on a submerged vertical surface must be such that, in the end view as shown in Figure 1.30(b), pressure increases linearly with the depth z, according to the relation

$$P = \rho g z$$

The hydrostatic force F on it must be

$$F = \rho g \bar{z} A$$

where $\bar{z} =$ depth of the centroid of the area A of the surface.

Since the pressure increases with the depth, the centre of pressure P must lie-below the centroid of the area of the surface. From Eq. (2.5), for $\alpha = 90^o$ $\sin \alpha = 1$,

$$z_p = z + \frac{I_c}{A\bar{z}}$$

In particular, if the surface is rectangular, with dimensions $l \times b$, and its one edge is placed immediately beneath the free level of the liquid as shown in Figure 1.30(b) the centroid is located at $l/2$ below the free level of the liquid,

$$\bar{z} = \frac{l}{2}$$

Also, $I_c = \frac{1}{12} b l^2$ and $A = bl$

which yield $z_p = \frac{l}{2} + \frac{1}{12} \frac{bl^2}{b.l.\frac{l}{2}} = \frac{2}{3} l$

and $$z_p - z = \frac{l}{6}$$

In other words, the centre of pressure of a rectangular surface lies at a depth equal to two-third the length of the surface below the free level of the liquid; the distance between the centroid C and the centre of pressure P is one sixth the length of the surface. It may be noted that this criterion does not hold good if one edge of the rectangular surface does not coincide with the free level. In that case,

$$\bar{z} < z_p < \frac{2}{3}\, l$$

Example 1.19

A box of rectangular base 3 m × 4 m contains gasoline (specific gravity 0.8) up to a height of 5 m. Calculate the force on the base and on each of the vertical faces and locate their lines of action.

Solution

The base is a horizontal surface at $\bar{z} = 5$ m. The force acting on it

$$= \rho\, g\, \bar{z}\, A = 1000 \times 0.8 \times 9.81 \times 5 \times (3 \times 4) = 470.88 \times 10^3 \text{ N} = 470.88 \text{ kN}.$$

Since the pressure is uniform over the base, the resultant must pass through the centroid of the base and is directed vertically downwards.

Each vertical face has the centroid at depth

$$\bar{z} = \frac{5}{2} = 2.5 \text{ m}$$

Force on each 3 m × 5 m face is given by

$$F = 1000 \times 0.8 \times 9.81 \times 2.5 \times (3 \times 5) = 294.3 \times 10^3 \text{ N} = 294.3 \text{ kN}$$

and on a 4 m × 5 m face

$$F = 1000 \times 0.8 \times 9.81 \times 2.5 \times (4 \times 5) = 392.4 \times 10^3 \text{ N} = 392.4 \text{ kN}$$

1.14.2 Total Force on Immersed Curved Surfaces

Evaluation of total force R on a curved immersed surface can be made by determining the horizontal component R_x and the vertical component R_y of this force.

As regards the curved surface ABC, principles of statics yield the following relations :

$$\sum F_x = F_x - R_x = 0 \qquad \text{. . . (i)}$$

$$\sum F_y = F_y + W - R_y = 0 \qquad \text{. . . (ii)}$$

Σ moments of all forces about axis passing through $O = 0$. . . . (iii)

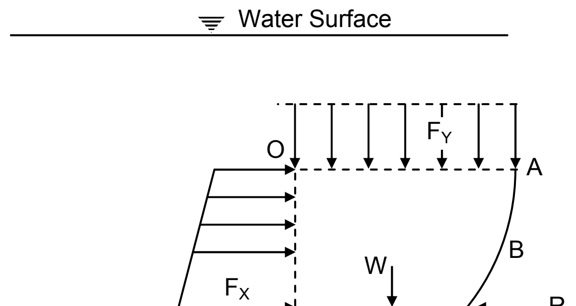


Figure 1.31 : Forces on a Curved Immersed Surface

Solution of these equations will give R_x, R_y and point of application of resultant force R. It can be seen from Eq. (ii) that R_y, the vertical component of the force on the curved surface is equal to weight of water above it. One can also consider a free body of fluid *AOC* and consider external force on it and write the condition of equilibrium. In that case R_x and R_y will be the components of the force exerted by the surface *ABC* on the free body. Such analysis leads to the same equations viz., Eqs. (i), (ii) and (iii).

1.14.3 Floating Bodies

Basic principles of buoyancy and flotation were first established by Archimedes (287-212 BC). These principles can be stated as follows. A body completely immersed in a fluid is acted upon by an upward buoyant force equal to the weight of fluid displaced by the body and it acts through the centre of gravity of displaced fluid. When the body is floating in a liquid, the weight of the body is equal to the buoyant force on the immersed part of the body. Therefore,

$$\text{Buoyant force } F_y = \gamma \forall$$

where $\forall$ is the volume of immersed part of the body and γ is the specific weight of the liquid.

1.15 SUMMARY

- A fluid is a substance which deforms continuously when subjected to a shear stress. Thus the shearing stress exists in a fluid only when it is in motion.
- A brief introduction to SI units is given. The basic properties of a fluid such as mass density, specific weight, specific volume and specific gravity are defined.
- Newton's law of viscosity is explained.
- The importance of vapour pressure and cavitation is discussed.
- The effects of surface tension and capillarity are outlined.
- The basic equation of the hydrostatic pressure is derived.
- The difference between the absolute pressure and gauge pressure is explained.
- Basic definitions in mechanics are given.
- The composition and resolution of coplanar concurrent and non-concurrent forces are discussed.

- The parallelogram law of forces, triangle law of forces and polygon law of forces are explained.
- The equilibrium condition for coplanar forces are discussed.
- A brief introduction to Newton's law of motion and the impulse-momentum equation is given.
- The principle of conservation of energy is introduced.
- Computation of hydrostatic force on submerged plane surfaces as given.

1.16 ANSWERS TO SAQs

SAQ 1

Specific weight, $\gamma = \frac{1329}{10}$

$$= 132.9 \text{ kN/m}^3 \ (= 132.9 \times 10^3 \text{ N/m}^3)$$

Mass density, $\rho = \frac{\gamma}{g}$

$$= \frac{132.9 \times 10^3}{9.81} = 13.547 \times 10^3 \text{ kg/m}^3$$

Specific gravity, $S = \frac{\rho}{\rho_w}$

$$= \frac{13.547 \times 10^3}{10^3} = 13.547$$

SAQ 2

Component of the weight in the direction of plane

$$= 196.2 \cos 70^{\circ} = 67.104 \text{ kN}$$

Shear stress $= \frac{\text{Sliding Force}}{\text{Area}}$

$$= \frac{67.104}{0.2 \times 0.2} = 1677.61 \text{ kN/m}^2$$

Now $\quad T = \mu \frac{dv}{dy}$

$$1677.61 = 2.158 \times 10^{-3} \times \frac{dv}{dy}$$

or $\quad \frac{dv}{dy} = 777.39 \times 10^3 \approx \frac{v}{y}$

Therefore, $\quad v = 777.39 \times 10^3 \times 0.025 \times 10^{-3}$

$$= 19.435 \text{ m/s.}$$

SAQ 3

Mass density at the surface $= \frac{10 \times 10^3}{9.81} = 1020 \text{ kg/m}^3$

Specific volume at the surface, $V_s = \frac{1}{\rho}$

$$= \frac{1}{1020} = 0.98 \times 10^{-3} \text{ m}^3/\text{kg}$$

We know $\quad K = -\dfrac{dp}{\frac{dV}{V}}$

or $\quad 2.453 \times 10^3 = -\dfrac{68.67}{\frac{dV}{V}}$

or $\quad \dfrac{dV}{V} = -0.028$

Change in specific volume $= -0.028 \times 0.98 \times 10^{-3} = -0.027 \times 10^{-3}$

Specific volume at 6.8 km depth $= (0.98 - 0.027) \times 10^{-3}$

$$= 0.953 \times 10^{-3} \text{ kg/m}^3$$

Mass density $= \dfrac{1}{0.953 \times 10^{-3}} = 1.049 \times 10^3 \text{ kg/m}^3$

Specific weight $= 1.049 \times 10^3 \times 9.81 \times 10^{-3} = 10.291 \text{ kN/m}^3$

SAQ 4

Pressure at a depth of 12.5 mm in oil

$$p_0 = \gamma h$$

$$= 0.85 \times 9.81 \times 12.5 \times 10^{-3}$$

$$= 0.10423 \text{ kN/m}^2 = 104.23 \text{ N/m}^2$$

Bubble pressure, $p_i = 147.15 \text{ N/m}^2$

Now $\quad p_i - p_0 = \dfrac{2\sigma}{R}$

or $\quad 147.15 - 104.25 = \dfrac{2 \times \sigma}{0.5 \times 1.5 \times 10^{-3}}$

or $\quad \sigma = 16.09 \times 10^{-3} \text{ N/m}.$

SAQ 5

Pressure developed by the pump

= Exit Pressure – Inlet Pressure

= 25 – (– 7) = 32 m of water

Now $\quad p = \gamma h$

$$= 9.81 \times 32 = 313.92 \text{ kN/m}^2$$

Absolute pressure at inlet = 10.3 – 7 = 3.3 m of water

Absolute pressure at exit = 10.3 + 25 = 35.3 m of water.

SAQ 6

(a) Resolved parts of F_1,

$$F_{x_1} = 10 \cos 45^o = +7.07 \text{ kN}$$

$$F_{y_1} = 10 \sin 45^o = +7.07 \text{ kN}$$

Resolved parts of F_2,

$$F_{x_2} = 20 \cos 120^o = -10.0 \text{ kN}$$

$$F_{y_2} = 20 \sin 120^o = +17.32 \text{ kN}$$

$$\sum H = +7.07 - 10.0 = -2.93 \text{ kN}$$

$$\sum V = +7.07 + 17.32 = +24.39 \text{ kN}$$

Resultant $R = \sqrt{(\sum H)^2 + (\sum V)^2} = \sqrt{(-2.93)^2 + (24.39)^2}$

$$= \sqrt{8.58 + 594.84} = 24.57 \text{ kN}$$

$$\tan \theta = \frac{\sum V}{\sum H}$$

$$= \frac{24.39}{-2.93} = -8.32$$

$$\theta = 96.85^o$$

(b) Resolved parts of F_1, F_2 and F_3, are given in table

Force	Magnitude	θ	F_x	F_y
F_1	5	30°	4.33	2.50
F_2	10	90°	0	10
F_3	20	150°	– 17.32	10
			Σ = – 12.99	Σ = 22.50

If F_{x_4} and F_{y_4} are the resolved parts of F_4, then $\sum H = 0$

or $F_{x_4} - 12.99 = 0$ or $F_{x_4} = 12.99$ kN

and $\sum V = 0$

or $F_{y_4} + 22.50 = 0$ or $F_{y_4} = -22.5$ kN

Therefore, $F_4 = \sqrt{(F_{x_4})^2 + (F_{y_4})^2} = \sqrt{(12.99)^2 + (-22.5)^2}$

$$= \sqrt{168.74 + 506.25} = 25.98 \text{ kN}$$

$$\tan \theta = \frac{F_{y_4}}{F_{x_4}}$$

$$= \frac{-22.5}{12.99} = -1.732$$

$$\theta = 120^o$$

SAQ 7

(a) For equilibrium, $\sum H = 0$

or $5 \cos 30^o - R_3 = 0$

or $R_3 = 4.33$ kN

Also $\sum V = 0$

or $-5 \sin 30^o - 7 + R_1 + R_2 = 0$

or $R_1 + R_2 = 9.5$ . . . (a)

Taking moments of all the forces about A,

$$(5 \sin 30^o) \times 2 + 7 \times 4 - 8 R_2 = 0$$

or $R_2 = 4.125$ . . . (b)

From Eq. (a),

$$R_1 = 9.5 - 4.125 = 5.375 \text{ kN}$$

(b) $\sum H = 15 \cos 145^o + 10 \cos 30^o$

$= -12.29 + 8.66 = -3.63$ kN

$\sum V = 15 \sin 145^o - 4 + 10 \sin 30^o + 5$

$= 8.60 - 4 + 5.0 + 5 = 14.6$ kN

$$R = \sqrt{(\sum H)^2 + (\sum V)^2} = \sqrt{(3.63)^2 + (14.6)^2}$$

$$= \sqrt{13.18 + 213.16} = 15.04 \text{ kN}$$

$$\tan \theta = \frac{14.6}{-3.63} = -4.022$$

$$\theta = -103.96^o$$

SAQ 8

Force = Rate of Change of Momentum

$$= \frac{10\,(12 - 4)}{10} = 8 \text{ N}$$

Impulse = Force $\times$ Time

$$= 8 \times 10 = 80 \text{ N-s}$$