UNIT 4 FLOW THROUGH MOUTHPIECES AND MINOR LOSSES

Structure

4.1 INTRODUCTION

An orifice with prolonged sides is called a *mouthpiece* or *tube*. In other words, a mouthpiece is an attachment in the form of a small tube fixed to an orifice. When the tube is fixed externally, it is known as an external mouthpiece. If it is fixed internally (i.e. if it projects inside the vessel), it is called an internal mouthpiece. Depending on the shape, a mouthpiece can be cylindrical, convergent, divergent, convergent-divergent, or of any other form.

Objectives

After studying this unit, you should be able to

- identify the different types of mouthpiece, e.g. external and internal mouthpiece, and
- describe and calculate the loss of head due to shock, e.g. sudden enlargement, sudden contraction, entrance and exit, obstruction and bends, etc.

4.2 EXTERNAL MOUTHPIECE

An external mouthpiece consists of a small tube attached to the vessel such that it projects outside. Figure 4.1 shows a tank to which an external mouthpiece of internal diameter '*d* ' is attached. Let the head over the axis of the mouthpiece be '*H* '. If the length of the mouthpiece is more than 2.5*d* to 3*d*, the jet after passing through the venacontracta expands and fills the tube. When the jet leaves the tube, it occupies the complete cross-section of the tube and the tube runs full.

Taking datum through the axis of the mouthpiece, and applying Bernoulli's equation to points 1 and 3,

$$\frac{p_a}{\gamma} + H = \frac{p_a}{\gamma} + \frac{V_3^2}{2g} + H_L \qquad \text{. . . (a)}$$

where p_a is the atmospheric pressure. (For mouthpieces, it is convenient to work in absolute pressures and not in the gauge pressures. The absolute pressure is measured with respect to complete vacuum.)

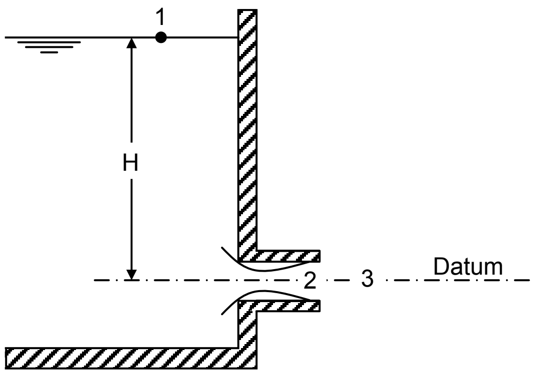


Figure 4.1 : Cylindrical Mouthpiece

H_L = loss of head due to sudden enlargement (see Section 4.8)

$$H_L = \frac{(V_2 - V_3)^2}{2g}$$

There is practically no loss of head between points 1 and 2, as the streamlines are converging and the flow is accelerating.

If 'a' is the cross-sectional area of the mouthpiece, $a_2 = C_c\, a$

From the continuity equation,

$$V_2\, a_2 = V_3\, a_3 = V_3\, a$$

or $\quad V_2\, C_c\, a = V_3\, a \quad$ or $\quad V_2 = \frac{V_3}{C_c}$

Thus, $$H_L = \left(\frac{V_3}{C_c} - V_3\right)^2 \times \frac{1}{2g} = \frac{V_3^2}{2g}\left(\frac{1}{C_c} - 1\right)^2$$

If $\quad C_c = 0.62,$

$$H_L = \frac{V_3^2}{2g}\left(\frac{1}{0.62} - 1\right)^2 = 0.375\, \frac{V_3^2}{2g}$$

Substituting this value of H_L in Eq. (a),

$$H = \frac{V_3^2}{2g} + 0.375\,\frac{V_3^2}{2g} = 1.375\,\frac{V_3^2}{2g}$$

or $$V_3 = \frac{1}{\sqrt{1.375}}\sqrt{2gH}$$

or $$V_3 = 0.853\sqrt{2gH} \qquad \text{. . . (b)}$$

Eq. (b) indicates that the coefficient of velocity is 0.853. Since the tube runs full, the coefficient of contraction is unity. Hence the coefficient of discharge C_d is given by

$$C_d = C_v = 0.853$$

Thus, $$Q = 0.853\,a\sqrt{2gH} \qquad \text{. . . (4.1)}$$

It may be noted that the coefficient of discharge of a mouthpiece is greater than that of a similar sharp-edged orifice.

The pressure at the venacontracta may be obtained by applying Bernoulli's equation to points 2 and 3,

$$\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + 0 = \frac{p_a}{\gamma} + \frac{V_3^2}{2g} + 0.375\,\frac{V_3^2}{2g}$$

or $$\frac{p_2}{\gamma} = \frac{p_a}{\gamma} + 1.375\,\frac{V_3^2}{2g} - \frac{V_2^2}{2g}$$

or $$\frac{p_2}{\gamma} = \frac{p_a}{\gamma} + 1.375\,\frac{V_3^2}{2g} - \left(\frac{V_3}{C_c}\right)^2 \times \frac{1}{2g}$$

Taking $C_c = 0.62$,

$$\frac{p_2}{\gamma} = \frac{p_a}{\gamma} + 1.375\,\frac{V_3^2}{2g} - \left(\frac{V_3}{0.62}\right)^2 \times \frac{1}{2g}$$

or $$\frac{p_2}{\gamma} = \frac{p_a}{\gamma} - 1.226\,\frac{V_3^2}{2g}$$

Substituting the value of $\frac{V_3^2}{2g}$ from Eq. (b),

$$\frac{p_2}{\gamma} = \frac{p_a}{\gamma} - 1.226 \times \frac{H}{1.375}$$

or $$\frac{p_2}{\gamma} = \frac{p_a}{\gamma} - 0.891\,H \qquad \text{. . . (4.2a)}$$

Eq. (4.2a) indicates that the pressure at the venacontracta is less than the atmospheric pressure. In fact, a mouthpiece decreases the pressure at the venacontracta and thus increases the effective head causing flow, and hence it increases the discharge.

Eqs. (4.1) and (4.2a) have been derived on the *assumption that the coefficient of contraction is 0.62*. If its value is not 0.62, these equations will be modified accordingly. The reader should use the equations carefully.

Eq. (4.1) gives the theoretical value of C_d as 0.853. It has been found from experiments that because of the frictional resistance at the walls of the mouthpiece, the actual value of C_d is about 0.81. If this value of C_d is taken, Eq. (4.2a) becomes

$$\frac{p_2}{\gamma} = \frac{p_a}{\gamma} - 0.82\ H \qquad \text{. . . (4.2b)}$$

If the length of the mouthpiece is less than 2.5*d* to 3*d*, the jet after passing through the venacontracta does not occupy the tube fully and the mouthpiece acts as an orifice. On the other hand, if the length of the mouthpiece is much greater than 3*d*, the mouthpiece will act as a pipe, and owing to friction, the value of the coefficient of discharge will decrease.

It may be mentioned that although a mouthpiece gives more discharge than a free orifice, it is not commonly used in practice. A mouthpiece creates the negative pressure at the venacontracta, given by Eqs. (4.2a and b). As soon as the absolute pressure at the venacontracta approaches the vapour pressure, cavitation occurs. Thus there is a limit on the head under which the mouthpiece can work properly. This drawback, along with practical difficulties in its construction, makes the mouthpiece unsuitable for the measurement of discharge, especially under large heads.

Example 4.1

Water is discharged through an external mouthpiece of 25 cm^2 area, under a head of 3 m. Find the discharge through the mouthpiece and the pressure at the venacontracta. Take $C_c = 0.62$ and atmospheric pressure = 10.3 m of water.

Solution

As $C_c = 0.62$, Eqs. (4.1) and (4.2a) are applicable.

From Eq. (4.1),

$$Q = 0.853\ a\ \sqrt{2gH} = 0.853 \times (25 \times 10^{-4}) \times \sqrt{2 \times 9.81 \times 3}$$

$$Q = 0.853 \times (25 \times 10^{-4}) \times 7.67 = 0.0164 \text{ cumecs}$$

From Eq. (4.2a),

$$\frac{p_2}{\gamma} = \frac{p_a}{\gamma} - 0.89\ H$$

$$= 10.3 - 0.89 \times 3 = 10.3 - 2.67 = 7.63 \text{ m of water (absolute).}$$

SAQ 1

A 10 cm diameter external cylindrical mouthpiece discharges under a head of 4 m. Determine the discharge and the pressure at the venacontracta. Take $C_c = 0.60$ and atmospheric pressure as 10.3 m of water.

4.3 INTERNAL MOUTHPIECE

An internal mouthpiece consists of a tube projecting inside the tank. Internal mouthpieces are of two types :

(i) Mouthpiece running free, also called *Borda's mouthpiece*, and

(ii) Mouthpiece running full.

In a mouthpiece running free, the jet after contraction does not touch the walls of the tube. In a mouthpiece running full, after contraction the jet occupies the full cross-sectional area of the tube. An internal mouthpiece is also called a '*re-entrant tube*'.

4.4 INTERNAL MOUTHPIECE RUNNING FREE

Figure 4.2 shows an internal mouthpiece running free. Let V be the velocity of flow through the mouthpiece and p be the pressure intensity at Section 1 – 1. By the impulse momentum equation, the force acting on the mouthpiece must be equal to the change in momentum per second. Thus,

$$pa = \left(\frac{\gamma a_c V}{g}\right) V$$

where a = area of the mouthpiece,

a_c = area of the jet at the venacontracta.

In Borda's mouthpiece, the velocity along the walls of the vessel is almost zero, and the pressure distribution is hydrostatic.

Thus $$p = \gamma H.$$

Therefore, $$\gamma H a = \frac{\gamma a_c V^2}{g} \qquad \text{. . . (a)}$$

Assuming the coefficient of velocity as unity,

$$\frac{V^2}{2g} = H$$

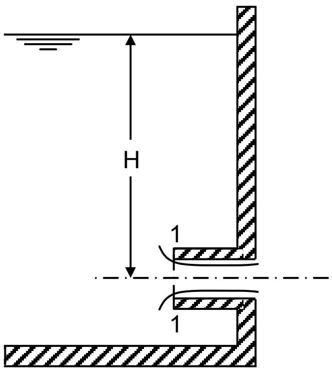


Figure 4.2 : Internal Mouthpiece Running Free

Substituting this value of V in Eq. (a),

$$\gamma Ha = \gamma a_c (2H)$$

or $$a_c = \frac{a}{2}$$

The coefficient of contraction of an internal mouthpiece running free is, therefore, 0.50.

Discharge $$Q = a_c V$$

or $$Q = 0.50\, a \sqrt{2gH} \qquad \text{. . . (4.3)}$$

Eq. (4.3) has been derived on the assumption that the coefficient of velocity is unity. It has been found from experiments that the practical value of this coefficient is 0.98 or so. Eq. (4.3) needs modification in the light of this change in the value of C_v. The value of the coefficient of contraction becomes 0.52, as derived below.

If $\qquad C_v = 0.98, \qquad V = 0.98\sqrt{2gH}$

or $$\frac{V^2}{g} = 1.92\ H$$

Therefore, $\qquad \gamma\ Ha = \gamma\ a_c\ (1.92\ H)$ or $a_c = 0.52$

The mouthpiece runs free when its length is less than the diameter (d). When the length of the mouthpiece is more than 2.5d, it runs full.

4.5 INTERNAL MOUTHPIECE RUNNING FULL

Figure 4.3 shows an internal mouthpiece running full. The jet first contracts at the venacontracta and then fills the tube. The loss of head occurs between points 2 and 3 owing to sudden enlargement. From the continuity equation, $V_2\ a_c = V_3\ a$

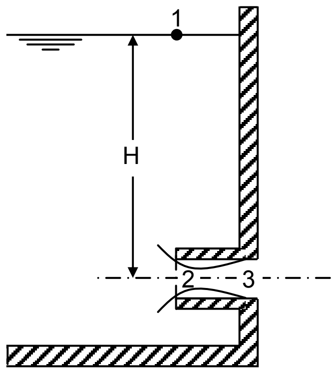


Figure 4.3 : Internal Mouthpiece Running Full

Taking the coefficient of contraction as 0.50,

$$a_c = \frac{a}{2} \text{ and } V_2 = 2V_3 \qquad \text{. . . (a)}$$

Therefore, loss of head due to enlargement

$$H_L = \frac{(V_2 - V_3)^2}{2g} = \frac{V_3^2}{2g}$$

Taking the datum level at the axis of the mouthpiece (Figure 4.3) and applying Bernoulli's equation to points 1 and 3,

$$\frac{p_a}{\gamma} + H = \frac{p_a}{\gamma} + \frac{V_3^2}{2g} + H_L$$

$$= \frac{p_a}{\gamma} + \frac{V_3^2}{2g} + \frac{V_3^2}{2g}$$

or $$H = \frac{V_3^2}{g} \qquad \text{. . . (b)}$$

or $$V_3 = \sqrt{gH}$$

or $$V_3 = \frac{1}{\sqrt{2}}\sqrt{2gH} = 0.707\sqrt{2gH}$$

The coefficient of velocity is, therefore, 0.707. Since the coefficient of contraction is unity, the coefficient of discharge is also 0.707. Thus

$$Q = 0.707\, a\sqrt{2gH} \qquad \ldots (4.4)$$

where a is area of the mouthpiece.

The pressure at the venacontracta may be obtained by applying Bernoulli's equation to points 2 and 3.

$$\frac{p_2}{\gamma} + \frac{V_2^2}{2g} = \frac{p_a}{\gamma} + \frac{V_3^2}{2g} + H_L$$

$$\frac{p_2}{\gamma} + \frac{V_2^2}{2g} = \frac{p_a}{\gamma} + \frac{V_3^2}{2g} + \frac{V_3^2}{2g}$$

From Eq. (a), $V_2 = 2V_3$

$\therefore$ $$\frac{p_2}{\gamma} + \frac{4V_3^2}{2g} = \frac{p_a}{\gamma} + \frac{V_3^2}{2g} + \frac{V_3^2}{2g}$$

or $$\frac{p_2}{\gamma} = \frac{p_a}{\gamma} - \frac{V_3^2}{g}$$

From Eq. (b), $$\frac{V_3^2}{g} = H$$

Therefore, $$\frac{p_2}{\gamma} = \frac{p_a}{\gamma} - H \qquad \ldots (4.5)$$

As in the case of an external mouthpiece, a negative pressure is created at the venacontracta. This increases the discharge. To avoid cavitation, it is essential that the absolute pressure should not fall below about 2.5 m for water.

Example 4.2

An internal mouthpiece has a diameter of 4 cm. If the head above the mouthpiece is 1.5 m and the coefficient of velocity is 0.95, determine

(a) The coefficients of contraction and discharge when the mouthpiece is running free, and

(b) The discharge when the mouthpiece is running full.

Solution

(a) In this case, Eq. (4.3) cannot be directly used.

From the impulse-momentum principle,

Force = Rate of change of momentum

or $$pa = \frac{\gamma\, a_c V}{g} V \qquad \ldots (a)$$

where p = pressure on the mouthpiece, a = cross-sectional area of the mouthpiece, a_c = cross-sectional area of the venacontracta, and V = velocity.

Substituting $p = \gamma H$ in Eq. (a),

$$\gamma Ha = \left(\frac{\gamma a_c V^2}{g}\right) \qquad \text{. . . (b)}$$

Also $\quad V = C_v \sqrt{2gH}$

or $\quad \frac{V^2}{2g} = C_v^2 H = (0.95)^2 H = 0.90 H$

Substituting this value of $\frac{V^2}{2g}$ in Eq. (b),

$$\gamma Ha = \gamma a_c \times 1.80 H$$

or $\quad a_c = \frac{a}{1.80} = 0.555 a$

Therefore, coefficient of contraction, $C_c = 0.555$

And coefficient of discharge, $C_d = C_c \times C_v = 0.555 \times 0.95 = 0.528$

(b) From the continuity equation (refer Figure 4.3),

$$a_c V_2 = a V_3$$

or $\quad V_2 = \left(\frac{a}{a_c}\right) V_3$

Taking $\quad C_c = 0.555$, $V_2 = 1.80 V_3$

Loss of head due to sudden enlargement = $\frac{(V_2 - V_3)^2}{2g}$

or $\quad H_L = \frac{(1.80-1)^2 V_3^2}{2g} = 0.64 \frac{V_3^2}{2g}$

Applying Bernoulli's equation to points 1 and 3,

$$\frac{p_a}{\gamma} + H = \frac{V_3^2}{2g} + H_L + \frac{p_a}{\gamma}$$

or $\quad H = \frac{V_3^2}{2g} + 0.64 \frac{V_3^2}{2g} = 1.64 \frac{V_3^2}{2g}$

or $\quad V_3 = \sqrt{\frac{2gH}{1.64}}$

or $\quad V_3 = \sqrt{\frac{2 \times 9.81 \times 1.50}{1.64}} = 4.24 \text{ m/sec}$

Discharge $\quad Q = V_3 a$

$$= 4.24 \times \left(\frac{\pi}{4}\right)(0.04)^2 = 5.33 \times 10^{-3} \text{ cumecs}.$$

SAQ 2

An internal mouthpiece of 15 cm diameter discharges water under a head of 3 m. Calculate the discharge and the diameter of the jet at the venacontracta. The mouthpiece is 12 cm long. Assume coefficient of velocity as 0.97.

4.6 CONVERGENT MOUTHPIECE

A convergent mouthpiece is a frustum of cone with the larger end attached to the tank wall (Figure 4.4). The jet first contracts and then expands in the mouthpiece. The coefficient of discharge varies with the angle of convergence θ. The coefficient of contraction (C_c) of the convergent mouthpiece, which is based on the area of the mouthpiece at the exit, decreases as the angle of convergence θ increases. On the other hand, the coefficient of velocity (C_v) increases as the angle of convergence increases.

For a convergent mouthpiece, the maximum value of coefficient of discharge (C_d) is 0.946. This occurs when the angle θ is about 13.5˚. Thus

$$Q = 0.946\ a \sqrt{2gH} \qquad \ldots (4.6)$$

where 'a' is the area of the mouthpiece at the exit.

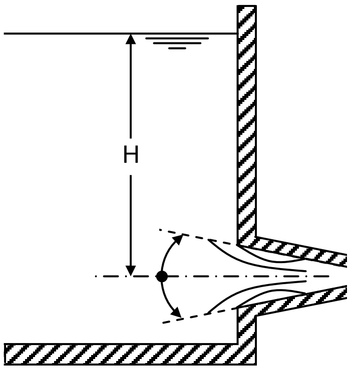


Figure 4.4 : Convergent Mouthpiece

Table 4.1 gives the typical values of C_c, C_v and C_d for different orifices and mouthpieces.

Table 4.1 : Coefficients of Various Types of Mouthpiece

Sl. No.	Type	C_c	C_v	C_d
1	External cylindrical mouthpiece	1.00	0.81	0.81
2	Internal mouthpiece running free	0.52	0.98	0.51
3	Internal mouthpiece running full	1.00	0.71	0.71
4	Convergent mouthpiece, θ = 13.5˚	1.00	0.946	0.946

4.7 MINOR LOSSES

Minor losses are the secondary losses due to shock in a pipe. These losses occur whenever there is a sudden change in the area of flow and/or the direction of flow.

The following minor losses usually occur in a pipe. (Pipe flow is discussed in Unit 5.)

(a) Loss of head due to sudden enlargement

(b) Loss of head due to sudden contraction

(c) Loss of head at the entry

(d) Loss of head at the exit

(e) Loss of head due to obstruction

(f) Loss of head at bends

(g) Loss of head due to fittings.

4.8 LOSS OF HEAD DUE TO SUDDEN ENLARGEMENT IN A PIPE

Whenever there is a sudden change in the cross-section of a conduit carrying a fluid, eddies are formed and there is a loss of head. The impulse-momentum equation may be used to find the loss of head due to sudden enlargement in the cross-section of a pipe.

Let us consider a pipe of the cross-sectional area A_1 which suddenly enlarges to the cross-sectional area A_2 (Figure 4.5). The velocity and pressure at section 1 are V_1 and p_1 and those at section 2 are V_2 and p_2 respectively.

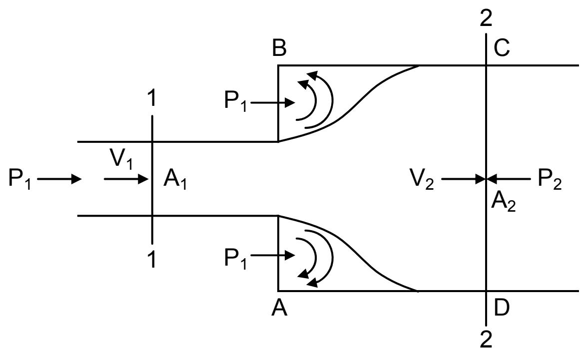


Figure 4.5

Backwash of eddies is formed in the corners of the pipe. These eddies are responsible for the loss of energy. It has been experimentally found that the pressure in the annular ring at section *AB* of cross-sectional area $(A_1 - A_2)$ is equal to the pressure p_1. Let us consider the control volume *ABCD*. The discharge through the control volume is given by

$$Q = A_1 V_1 = A_2 V_2 \qquad \ldots \text{(a)}$$

Applying the impulse-momentum equation to the fluid in the control volume, *ABCD*,

$$p_1 A_1 + p_1 (A_2 - A_1) - p_2 A_2 = \frac{\gamma Q}{g} [V_2 - V_1]$$

Substituting the values of Q from Eq. (a),

$$A_2 (p_1 - p_2) = \frac{\gamma A_2 V_2}{g} [V_2 - V_1]$$

or $$\frac{p_1 - p_2}{\gamma} = \frac{V_2 [V_2 - V_1]}{g} \quad \text{. . . (b)}$$

Applying the Bernoulli equation to sections 1 and 2

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + h_L$$

where h_L is the loss of head due to sudden enlargement.

Substituting the value of $\frac{p_1 - p_2}{\gamma}$ from Eq. (b),

$$\frac{V_2 (V_2 - V_1)}{g} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} = h_L$$

or $$h_L = \frac{V_1^2}{2g} + \frac{V_2^2}{2g} - \frac{V_1 V_2}{g} = \frac{V_1^2 + V_2^2 - 2V_1 V_2}{2g}$$

or $$h_L = \frac{[V_1 - V_2]^2}{2g} \quad \text{. . . (4.7)}$$

Eq. (4.7) is the well-known *Borda-Carnot equation* for the loss of head due to sudden enlargement.

Example 4.3

A pipe carrying water suddenly enlarges from a diameter of 40 cm to 60 cm. If the discharge is 0.615 cumecs, calculate the loss of head due to sudden enlargement.

Solution

From the continuity equation,

$$Q = A_1 V_1 = A_2 V_2$$

or $$V_1 = \frac{Q}{A_1} = \frac{0.615}{\frac{\pi}{4} \times 0.40^2} = 4.89 \text{ m/sec}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.615}{\frac{\pi}{4} \times 0.60^2} = 2.18 \text{ m/sec}$$

From Eq. (4.7),

$$h_L = \frac{[V_1 - V_2]^2}{2g} = \frac{(4.89 - 2.18)^2}{2 \times 9.81}$$

$$= 0.374 \text{ m of water.}$$

SAQ 3

At a sudden enlargement of a pipeline from 240 to 480 mm diameter, the hydraulic gradient line rises by 10 mm. Determine the discharge.

4.9 LOSS OF HEAD DUE TO SUDDEN CONTRACTION

Figure 4.6 represents a pipeline in which sudden contraction occurs. The streamlines are converging from sections *A*-*A* to section 1-1, and there is very little loss of head between these sections. At section 1-1, the diameter of the jet is minimum. This section forms the venacontracta. The loss of head occurs mainly between sections 1-1 and 2-2. Thus the loss of head is not actually due to contraction, but it is due to enlargement which takes place from section 1-1 to 2-2.

Let the cross-sectional area of pipe at section *A*-*A* and 2-2 be a_1 and a_2 respectively. The cross-sectional area of the jet at the venacontracta is given by, $a_c = C_c\, a_2$, where C_c is the coefficient of contraction.

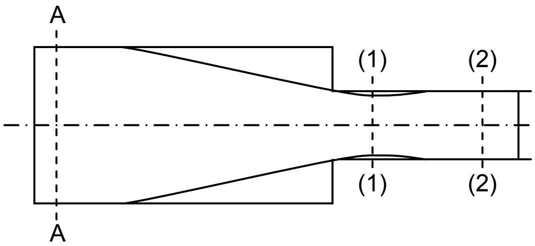


Figure 4.6

The loss of head from sections 1-1 to 2-2 is due to sudden enlargement (Eq. (4.7)),

$$H_L = \frac{(V_c - V_2)^2}{2g} \qquad \text{. . . (a)}$$

From the continuity equation,

$$a_2 V_2 = a_c V_c = a_2 C_c V_c \;\text{ or }\; V_c = \frac{V_2}{C_c}$$

Substituting the value of V_c in Eq. (a),

$$H_L = \frac{\left(\dfrac{V_2}{C_c} - V_2\right)^2}{2g} = \frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1\right)^2 \qquad \text{. . . (4.9)}$$

Assuming $C_c = 0.62$,

$$H_L = \frac{V_2^2}{2g}\left(\frac{1}{0.62} - 1\right)^2 = 0.375\,\frac{V_2^2}{2g}$$

In general, $$H_L = \frac{kV_2^2}{2g}$$

The constant 'k' depends upon the ratio a_2 / a_1. In practice, the loss of head due to sudden contraction is found to be $\frac{0.5\ V_2^2}{2g}$. Unless otherwise mentioned, the value of k will be taken as 0.50.

Example 4.4

A pipe carrying 0.05 cumecs of water suddenly contracts from 20 cm to 15 cm diameter. Calculate the coefficient of contraction if the loss of head is 0.5 m.

Solution

From Eq. (4.9) $$H_L = \frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1\right)^2$$

From the continuity equation,

$$V_2 = \frac{Q}{A_2} = \frac{0.05}{\frac{\pi}{4} \times 0.15 \times 0.15} = 2.83 \text{ m/sec}$$

$$H_L = \frac{2.83^2}{2 \times 9.81}\left(\frac{1}{C_c} - 1\right)^2$$

or $$0.5 = 0.408\left(\frac{1}{C_c} - 1\right)^2$$

or $$\left(\frac{1}{C_c} - 1\right)^2 = \sqrt{1.226} = 1.11$$

$$C_c = 0.475$$

SAQ 4

A 150 mm diameter pipe reduces in diameter abruptly to 100 mm diameter. If the pipe carries water at 0.03 m^3/s, calculate the loss of head across the contraction. Take the coefficient of contraction as 0.60.

4.10 LOSS OF HEAD AT ENTRANCE TO PIPE

The loss of head at the entrance where the fluid enters the pipe from the reservoir is similar to that in sudden contraction. As the fluid enters, it first contracts and then expands in the pipe. The loss of head is caused mainly by the turbulence created by the sudden enlargement of the jet after it has passed through the venacontracta.

The loss of head depends on the condition at the inlet. If the entrance to the pipe is well-rounded, with a bell-mouth, the loss of head is very small and is about $0.04\,\frac{V^2}{2g}$, where V is the velocity in the pipe.

If the entrance is conical, with included angle between 30° and 60°, the loss of head is about $0.18\,\dfrac{V^2}{2g}$.

For a flush entrance, as in the case of an external cylindrical mouthpiece, the loss of head is given by,

$$H_L = k\,\frac{V^2}{2g}$$

in which 'k' varies from 0.485 to 0.56.

In the re-entrant (internal) mouthpiece, the value of 'k' varies from 0.62 to 0.93.

Usually, an average value of $k = 0.5$ is taken.

Thus,
$$H_L = 0.5\,\frac{V^2}{2g} \qquad \text{. . . (4.10)}$$

4.11 LOSS OF HEAD AT EXIT

When a pipe carrying a fluid discharges into a reservoir, the entire velocity is dissipated. If V is the velocity in the pipe, the loss of head at the exit is given by

$$H_L = \frac{V^2}{2g}$$

The loss of head may also be obtained from the Borda-Carnot equation. In this case as A_2 is very large, the velocity V_2 may be neglected. Hence

$$H = \frac{(V_1 - V_2)^2}{2g} = \frac{(V - 0)^2}{2g} = \frac{V^2}{2g}$$

If a pipe discharges into atmosphere, the velocity V is dissipated in air and the loss of head is $\dfrac{V^2}{2g}$.

4.12 LOSS OF HEAD DUE TO OBSTRUCTION

The loss of head due to an obstruction placed in a pipe may be looked upon as the loss due to sudden enlargement beyond the obstruction. Figure 4.7 represents the conditions when an obstruction, such as a diaphragm, is inserted.

Let the cross-sectional area of the pipe be A. If the cross-sectional area of the opening in the diaphragm is 'a', the area at the venacontracta $a_c = C_c\, a$. The loss of head due to enlargement from sections 1-1 to section 2-2 is given by

$$H_L = \frac{(V_1 - V_2)^2}{2g} \qquad \text{. . . (a)}$$

From the continuity equation,

$$V_1\, C_c\, a = V_2\, A \quad \text{or} \quad V_1 = \frac{AV_2}{C_c\, a}$$

Substituting this value of V_1 in Eq. (a),

$$H_L = \left[\frac{A}{C_c\, a} - 1\right]^2 \frac{V_2^2}{2g} \qquad \text{. . . (4.11)}$$

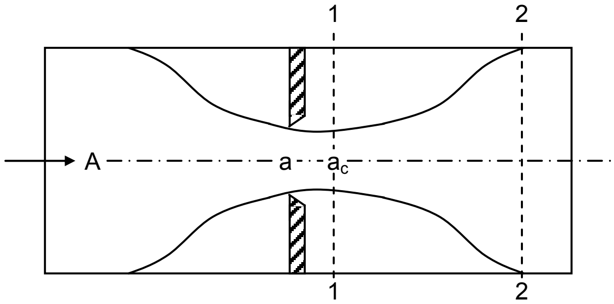


Figure 4.7

The loss of head due to obstruction may be obtained from Eq. (4.11) if the coefficient of contraction (C_c) is known. In general,

$$H_L = \frac{kV^2}{2g} \qquad \text{. . . (4.11a)}$$

The value of k depends upon the type and shape of obstruction.

Example 4.5

A diaphragm with a central hole 8 cm in diameter is placed in a 15 cm diameter pipe. If the velocity of water in the pipe is 0.3 m/sec, find the loss of head. Take $C_c = 0.60$

Solution

From Eq. (4.11), $$H_L = \left[\frac{A}{C_c\, a} - 1\right]^2 \frac{V_2^2}{2g}$$

or $$H_L = \left[\frac{\frac{\pi}{4} \times 0.15 \times 0.15}{0.60 \times \frac{\pi}{4} \times 0.08 \times 0.08} - 1\right]^2 \frac{(0.3)^2}{19.62}$$

or $$H_L = 0.108 \text{ m.}$$

4.13 LOSS OF HEAD AT BENDS

When a fluid flows around a bend or an elbow, it has to overcome additional resistance. The loss of head depends upon the ratio of the radius of curvature (R') of the bend to the diameter of the pipe (D). The loss of head is expressed as

$$H_L = k\,\frac{V^2}{2g} \qquad \text{. . . (4.12)}$$

where k is the coefficient depending upon the ratio R'/D. The value of k ranges from 0.19 to 0.42, the larger values being for low R'/D ratio. For a circular right-angled bend the value of k varies from 0.1 to 1.20, and an average value of 0.50 is usually taken.

4.14 LOSS OF HEAD IN PIPE FITTINGS

Pipe fittings when inserted in a pipe cause obstruction to flow and the loss of head occurs. The loss of head may be expressed as

$$H_L = \frac{kV^2}{2g} \qquad \ldots (4.13)$$

in which the coefficient k depends upon the size, shape and type of fitting. The value of k for various types of valves ranges from 0.2 to 24. A reference may be made to the tables provided by the manufacturer to get an accurate value of k. Its value may also be determined experimentally by actual measurement of loss of head H_L.

It may be mentioned that for very long pipes the minor losses are relatively insignificant as compared to the loss of head due to friction and are usually neglected.

Table 4.2 gives the typical values of losses for different types of fitting.

Table 4.2 : Approximate Values of the Minor Losses Coefficient *k* for Different Types of Fitting

Valve/Fitting	*K*
Globe valve, fully open	10.00
Angle valve, fully open	10.00
Gate valve, fully open	0.20
Gate valve, Half open	5.60
Pump foot valve	1.50
Standard 90° elbow	0.90
Standard 45° elbow	0.40
Standard 90° bend	0.10
Return bend	2.20
Standard *T*, line flow	0.90
Standard *T*, branch flow	1.80

SAQ 5

Water is discharged from a tank through a pipe of 300 mm diameter at the rate of 0.18 m^3/s. Calculate the loss of head at

(a) the entry of the pipe if it is short-edged,

(b) the exit of the pipe, and

(c) the gate valve.

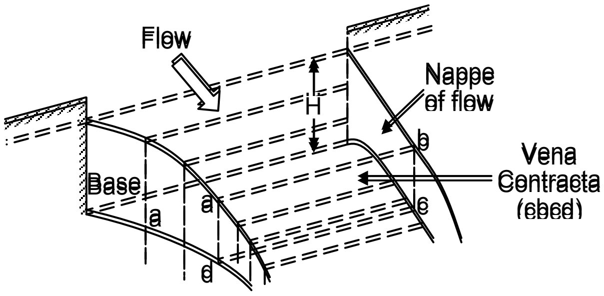
Assume $k = 0.20$.

4.15 MEASUREMENT OF DISCHARGE THROUGH AN OPEN CHANNEL BY A WEIR, A NOTCH OR A VENTURI FLUME

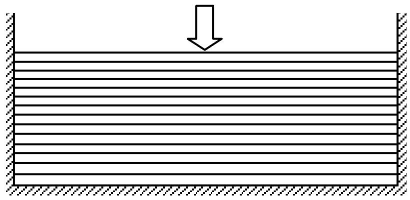
Flow through an open channel occurs under atmospheric pressure. The motive force to create the flow is derived from the slope of the bed and the kinetic energy possessed by the liquid. Mechanics of flow through open channels will be presented at a later stage; the purpose here is to study the common methods of discharge measurement.

A weir is an obstruction placed in an open channel over which the flow occurs. The weir is generally in the form of a vertical wall with a sharp edge at the top, running all the way across the open channel. When the liquid flows over the weir, the height of the liquid above the tip of the sharp edge bears a relationship with the discharge across it.

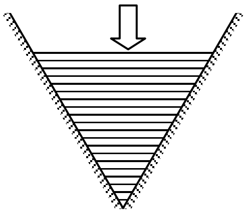
A notch is a sharp-edged device which permits the liquid to go through it, the liquid being exposed to the atmospheric pressure. Notches may be rectangular, triangular, circular or trapezoidal in shape. A triangular notch, for example, has a sharp-edged base and sharp-edged vertical-end walls. A triangular notch, also called V-notch consists of a V-cut sharp-edged passage through which the liquid passes. The only difference between a weir and a rectangular notch is that a weir runs all the way across the channel whereas a rectangular notch may be as wide as the channel.



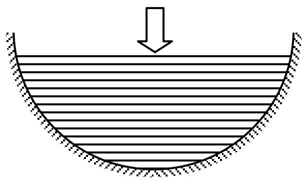
(a) Flow Through a Rectangular Notch



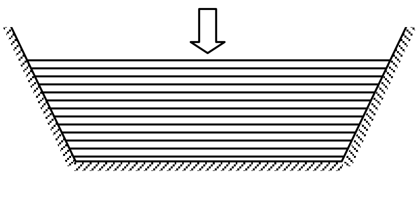
(b) End View of the Idealised Flow Through a Rectangular Notch



(c) V or Triangular Notch



(d) Circular Notch



(e) Trapezoidal Notch or Weir

Figure 4.8 : Various Shapes of Notches

Consider the flow through a notch of a given shape and dimensions. The flow in the open channel far upstream approaches the notch with a velocity U_1 and while crossing over the notch, the total height H and the velocity distribution must be in accordance with the continuity principle and the Bernoulli equation. Consider a horizontal strip of depth dh located at a depth h below the liquid level. If the width of the strip is b, the area is $b\ dh$. The velocity of the liquid at a point on the strip must be given by

$$U = \sqrt{2\,g\,h}$$

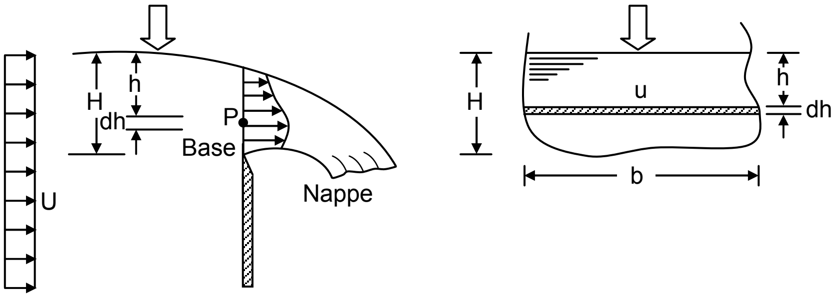


Figure 4.9 : Flow Through a Notch

The flow rate across the strip is

$$dQ = u\,dA = \sqrt{2\,g\,h}\;\; b\; dh$$

The total discharge across the notch must be given by

$$Q = \int dQ = \int_0^H \sqrt{2\,g\,h}\; b\,dh$$

The expression for the discharge is evaluated with the knowledge of the shape and dimensions of the notch.

4.15.1 Flow Through a Rectangular Notch

The expression for the discharge through a notch;

$$Q = \int_0^H \sqrt{2\,g\,h}\; b\,dh$$

may be evaluated for flow through a rectangular notch by taking

$$b = \text{constant}$$

$$Q = \sqrt{2\,g}\,b \int_0^H h^{1/2}\,dh$$

or $$Q = \frac{2}{3}\sqrt{2\,g}\,b\;H^{3/2} \qquad \text{. . . (4.14a)}$$

Taking note of the fact that the minimum area of cross-section, i.e. the vena contracta is less than the area of flow across the notch and these are some real-flow effects such as side-wall boundaries and eddy formation and the three-dimensional nature of flow, a factor called the coefficient of discharge C_d is introduced. The Actual discharge is, therefore, expressed as,

$$Q = \frac{2}{3}\,C_d\;\sqrt{2\,g}\,b\;H^{3/2} \qquad \text{. . . (4.14b)}$$

This expression takes no note of the fact that the liquid approaches the notch with an initial velocity U_1 and that the liquid possesses kinetic energy per unit weight given by $\frac{U_1^2}{2g}$. In order to take the velocity of approach into account, the limits of integration in the expression for Q should be

$$\frac{U_1^2}{2g} \text{ to } \left(\frac{U_1^2}{2g} + H\right)$$

instead of 0 to H. with this correction, the improved expression for the discharge through a rectangular notch becomes,

$$Q = \frac{2}{3} C_d \sqrt{2g}\, b \left[\left(H + \frac{U_1^2}{2g} \right)^{3/2} - \left(\frac{U_1^2}{2g} \right)^{3/2} \right] \qquad \ldots (4.14c)$$

When the flow occurs over a rectangular notch, the nappe of flow is contracted at the ends and an area of minimum cross-section called vena contracta occurs at a short distance from the notch as shown in Figures 4.9(a) and 4.10. The effect of the end contraction is that the effective width of flow is reduced. According to Francis, each end contraction is of the order of 10 per cent of head over the notch, i.e. 0.1 H. For a rectangular notch with two end contractions,

Effective width = $b - 0.2\, H$

and the expression for the distance becomes

$$Q = \frac{2}{3} C_d \sqrt{2g}\ (b - 0.2H)\ H^{3/2} \qquad \ldots (4.14d)$$

which implies that the end contractions reduce the discharge by

$$q = \frac{2}{3} C_d \sqrt{2g}\ (0.2H)\ H^{3/2}$$

$$= \frac{2}{15} C_d \sqrt{2g}\ H^{5/2}$$

Comparison of this expression with the expression for discharge for a triangular notch (Figure 4.11) shows that, for an equivalent triangular weir,

$$\frac{2}{15} C_d \sqrt{2g}\ H^{5/2} = \frac{8}{15} C_d \sqrt{2g}\ \tan\frac{\theta}{2}\ H^{5/2};\ \tan\frac{\theta}{2} = \frac{1}{4} \qquad \ldots (4.15)$$

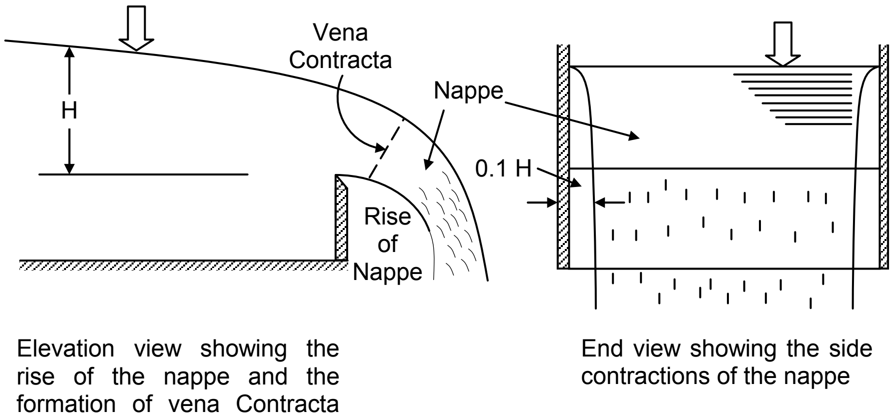


Figure 4.11 : Flow Over a Rectangular Weir

If the rectangular weir is enlarged to become a trapezoidal weir with the side slopes ‘1 horizontal to 4 vertical’ it would give the discharge that an uncontracted rectangular weir would have given. This was first discovered by Cippoleti and the compensated weir is called Cippoleti weir.

It may also be noted that the lower surface of the nappe rises by 10% of the head over the notch as also shown in Figure 4.10. This however does not change the width of flow. Effect of the rise of the nappe is included in the coefficient of discharge C_d for the notch.

4.15.2 Flow Through a Triangular Notch

Consider a triangular or V-notch with an included angle θ and semi-angle $\frac{\theta}{2}$. Let the liquid flow through it with the level H above the base point. Consider the flow through an elementary strip of depth dh located at a depth h below the level of the liquid. The discharge through it must be

$$dQ = \sqrt{2\,g\,h}\,b\,dh$$

and the discharge through the entire notch should be

$$Q = \int dQ = \int_0^H \sqrt{2\,g\,h}\;b\,dh$$

For a triangular notch,

$$b = 2\,(H - h)\,\tan\frac{\theta}{2} \qquad \text{. . . (4.16)}$$

Substituting this relation in the expression for Q,

$$Q = 2\tan\frac{\theta}{2}\sqrt{2g}\int_0^H (H - h)\,h^{1/2}\,dh$$

or

$$Q = 2\tan\frac{\theta}{2}\,\sqrt{2g}\left[\frac{2}{3}\,H\,h^{3/2} - \frac{2}{3}\,h^{5/2}\right]_0^H$$

$$= \frac{8}{15}\,\tan\frac{\theta}{2}\,\sqrt{2g}\;H^{5/2} \qquad \text{. . . (4.17a)}$$

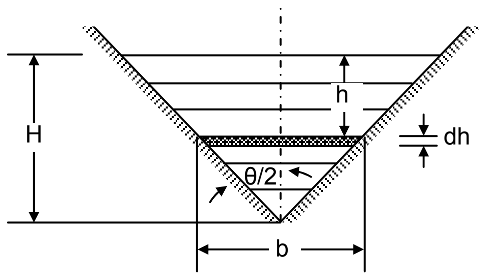


Figure 4.12 : Flow Through a Triangular Notch

Introducing the efficient of discharge C_d in order to provide an expression for the actual flow through V notch,

$$Q = \frac{8}{15}\,C_d\,\tan\frac{\theta}{2}\,\sqrt{2g}\;H^{5/2} \qquad \text{. . . (4.17b)}$$

In particular, for a right-angled V notch,

$$\theta = 90^{\circ},\ \frac{\theta}{2} = 45^{\circ};\ \tan\frac{\theta}{2} = 1$$

the expression for the actual discharge becomes

$$Q = \frac{8}{15}\,C_d\,\sqrt{2g}\;H^{5/2} \qquad \text{. . . (4.17c)}$$

A word of comparison between the accuracy of measurement by a rectangular and for a V notch is worthwhile. For a rectangular notch,

$$Q = K\,H^{3/2}$$

and for V notch $\quad Q = C\,H^{5/2}$

where K and C are appropriate constants.

The accuracy of measurement of the discharge depends upon the accuracy of measurement of the head H. A small error dH in H would correspond to an error dQ in Q.

For a rectangular notch,

$$dQ = \frac{3}{2}\,K\,H^{1/2}\,dH$$

and
$$\frac{dQ}{Q} = \frac{3}{2}\,\frac{dH}{H} = 1.5\,\frac{dH}{H} \qquad \ldots (4.18a)$$

Similarly, for a V notch

$$dQ = \frac{5}{2}\,C\,H^{3/2}\,dH$$

and
$$\frac{dQ}{Q} = \frac{5}{2}\,\frac{dH}{H} \quad 2.5\,\frac{dH}{H} \qquad \ldots (4.18b)$$

It is seen that a 1% error in the measurement of the head result in 1.5% error in discharge measured by a rectangular notch and 2.5% error in discharge measured by a V notch.

There is one more point to it. For low discharge, the reading 'H' for a V notch is greater than the reading 'H' for a rectangular notch for the same discharge. This is obvious because the width of a rectangular notch is b everywhere whereas the width at the base of a V notch is zero. A notch is, therefore, preferred for low discharge. For high discharge, a rectangular notch is preferred because the incremental head is more for an increment in discharge than that for a V notch.

If it is required to have a single notch suitable for a large range of discharge, a combination of 'V notch and rectangular notch' may be used.

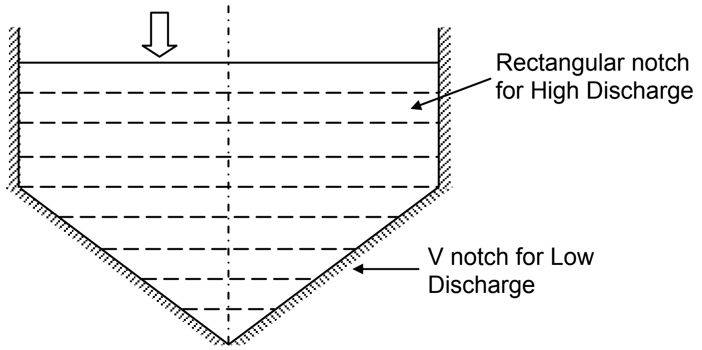


Figure 4.13 : A Combination Notch

4.15.3 Venturi Flume

A device called venturi flume is also useful for estimating the flow rate through an open channel. The width of the channel b_1 is reduced to b_2 to create a throat section. Higher velocity at the throat results in a drop of the depth of the liquid as shown in Figure 4.14.

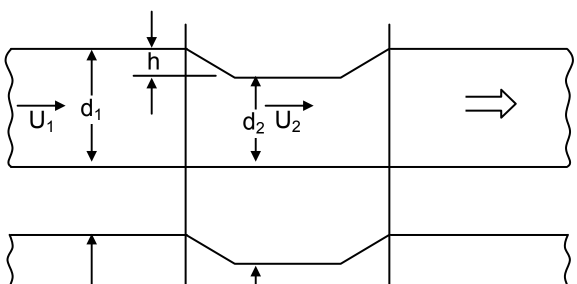


Figure 4.14 : Venturi Flume

By continuity, $A_1 U_1 = A_2 U_2$

By the energy equation, $d_1 + \frac{U_1^2}{2} = d_2 + \frac{U_2^2}{2}$

Combining and simplifying these, using $h = d_1 - d_2$

$$Q = \frac{C_d \; A_1 \, A_2}{\sqrt{(A_1^2 - A_2^2)}} \sqrt{2 g \, h}$$

which bears a similarity with the expression for flow through a venturimeter.

Example 4.6

A reservoir having a surface area of 800 m^2 is emptied by a 0.5 m wide rectangular weir. How long should it take to empty the reservoir from a height 0.3 m to 0.2 m above the sill? Take $C_d = 0.65$.

Solution

With reference to a rectangular weir,

$$A \; dh = - \, Q \; dt$$

The negative sign appears because h decreases as t increases.

For a rectangular weir,

$$Q = \frac{2}{3} \sqrt{2g} \; h^{3/2} \, C_d$$

Hence $$dt = \frac{- \, A \; h^{3/2} \; dh}{\frac{2}{3} \sqrt{2g} \; b \; C_d}$$

After integrating, we get

$$T = \int_0^T dt = \frac{3A}{\sqrt{2g} \; b \; C_d} \left(\frac{1}{\sqrt{H_2}} - \frac{1}{\sqrt{H_1}} \right)$$

$$= 3 \times \frac{800}{(4.43 \times 0.5 \times 0.65)} \times \left(\frac{1}{\sqrt{0.2}} - \frac{1}{\sqrt{0.3}} \right)$$

= 684 s = 11 minutes and 24 seconds.

4.16 SUMMARY

- The difference between an orifice and a mouthpiece is explained. The monthpieces can be of different shapes. They may be fitted either externally or internally to vessel.
- The discharge equation for an external mouthpiece is derived. The importance of the negative pressure at the venacontracta is emphasized.
- The discharge equations for internal mouthpiece running free and running full are developed. The mouthpiece runs free if it is of small length.
- In general, the coefficient of discharge of a mouthpiece is greater than that for a similar orifice, but it is not commonly used as it is not convenient to construct and there is a possibility of cavitation under large heads.
- A brief introduction is given to convergent mouthpieces.
- The typical coefficients of various types of mouthpieces are given.
- The difference between minor losses and the major loss due to friction in pipes is explained.
- The minor losses are important for pipes of small lengths. For long pipes, these losses are usually negligible as compared to the loss due to friction.
- The loss of head due to sudden enlargement, sudden contraction, at the entry, at the exit, at the bends and at the obstruction can be determined from the equations given.
- The loss of head at the pipe fittings can be determined using the values of the coefficient supplied by the manufacturers or by actual measurement. The typical values for different types of fitting are given.
- The measurement of discharge through an open channel by a view, a notch or a venturi flume is explained.

4.17 ANSWERS TO SAQs

SAQ 1

Loss of head,

$$H_L = \frac{V_3^2}{2g}\left(\frac{1}{0.60} - 1\right)^2 = 0.444\,\frac{V_3^2}{2g}$$

Therefore, $$\frac{p_a}{\gamma} + H = \frac{p_a}{\gamma} + \frac{V_3^2}{2g} + 0.444\,\frac{V_3^2}{2g}$$

or $$V_3 = 0.832\,\sqrt{2gH}$$

$$Q = 0.832 \times \frac{\pi}{4}\,(0.10)^2 \times \sqrt{2 \times 9.81 \times 4} = 0.0579\ \text{m}^3/\text{s}$$

Pressure at the venacontracta,

$$\frac{p_2}{\gamma} = \frac{p_a}{\gamma} + 1.444\,\frac{V_3^2}{2g} - \left(\frac{V_3}{0.60}\right)^2 \times \frac{1}{2g}$$

$$= 10.3 - 1.334\, \frac{V_3^2}{2g}$$

$$= 10.3 - 1.334 \times \left(\frac{H}{1.444} \right)$$

$$= 10.3 - \frac{1.334 \times 4}{1.444} = 6.605 \text{ m} \ \text{(absolute)}$$

SAQ 2

Since the length of mouthpiece is less than its diameter, it runs free.

Now $$V = 0.97 \sqrt{2 \times g \times h}$$

or $$\frac{V^2}{g} = 1.88\, H$$

From the impulse-momentum equation,

$$\gamma\, ha = \gamma\, a_c\, \frac{V^2}{g}$$

$$= \gamma\, a_c\ (1.88\, H)$$

or $$a_c = 0.53\, a$$

or $$C_c = 0.53$$

$$\left(\frac{d_c}{d} \right)^2 = 0.53$$

or $$d_c = \sqrt{0.53}\ d$$

$$= \sqrt{0.53} \times 15 = 10.92 \text{ cm}$$

$$C_d = C_c \times C_v = 0.53 \times 0.97 = 0.51$$

$$Q = C_d\, a\, \sqrt{2gH}$$

$$= \sqrt{0.51} \times \frac{\pi}{4} \times (0.15)^2\ \sqrt{2 \times 9.81 \times 3}$$

$$= 0.0718 \text{ m}^3\text{/s}$$

SAQ 3

The rise in pressure is given by

$$\frac{p_2 - p_1}{\gamma} = \frac{-V_2\,(V_2 - V_1)}{g}$$

$$0.01 = \frac{(V_1 - V_2)\, V_2}{g}$$

But $$\frac{\pi}{4} \times (0.24)^2\ V_1 = \frac{\pi}{4} \times (0.48)^2 \times V_2$$

$$V_1 = 4V_2$$

Therefore, $$0.01 = \frac{(4V_2 - V_2)\,V_2}{9.81}$$

or $$V_2 = 0.181 \text{ m/s}$$

$$Q = \frac{\pi}{4} \times (0.48)^2 \times 0.181 = 0.0327 \text{ m}^3/\text{s}$$

SAQ 4

Loss of head at sudden contraction,

$$H_L = \frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1\right)^2$$

or $$H_L = \frac{V_2^2}{2g}\left(\frac{1}{0.60} - 1\right)^2 = 0.444\,\frac{V_2^2}{2g}$$

Now $$\frac{\pi}{4} \times (0.1)^2 \times V_2 = 0.03$$

or $$V_2 = 3.82 \text{ m/s}$$

$$H_L = 0.444 \times \frac{(3.82)^2}{2 \times 9.81} = 0.33 \text{ m}$$

SAQ 5

$$V = \frac{0.18}{\frac{\pi}{4} \times (0.3)^2} = 2.55 \text{ m/s}$$

(a) Loss of head at entry $= 0.5\,\dfrac{V^2}{2g} = 0.5 \times \dfrac{(2.55)^2}{2 \times 9.81} = 0.166 \text{ m}$

(b) Loss of head at exit $= \dfrac{V^2}{2g} = \dfrac{(2.55)^2}{2 \times 9.81} = 0.331 \text{ m}$

(c) Loss of head at valve $= \dfrac{kV^2}{2g} = 0.20 \times \dfrac{(2.55)^2}{2 \times 9.81} = 0.066 \text{ m}$