UNIT 1 LEARNING AS CONSTRUCTION

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1.1 INTRODUCTION

In this unit, we shall focus on our understanding of learning. Why do we need to do so? As teachers, we plan our teaching strategies on the basis of our understanding of how children learn, and what we want them to learn. That is, each of us has a model of the learning process in our minds. Accordingly, we have our own understanding of the place of memory in learning, whether concepts and processes are learnt by solving many problems on them of the same kind, how important it is to let the student build her own understanding of a concept/process at her own pace, and many other basic issues like these. Here we shall concentrate on the processes that all the stakeholders undergo in a learner-centred mathematics classroom.

To begin with, we shall focus on the kind of understanding of the learning process that we need to have for building such an environment. In particular, we will discuss what the constructivist model means. Then we shall focus on the implications of this understanding for us as mathematics teachers.

Objectives

After studying this unit, you should be able to

- explain the way a constructivist believes that a student learns;
- describe classroom interactions that follow a learner-centred approach;
- explain the outlook required of a teacher for building a classroom environment that truly encourages learning.

1.2 A LEARNER-CENTRED APPROACH

With your experience, you would know that the abilities of your students, are vast. Every child has a desire to explore and learn by discovering. In the process, of course, she makes many mistakes, and learns from them. As you know, this is evidence of an 'active mind'. The approach to learning that regards **the learner as the chief active agent of her learning** is the **constructivist model**. One of the teachers we met, Ms. Jamila, told us that she tries to bring this understanding of learning into classroom. In fact, she gave us a detailed view into one such classroom interactions, which we present below.

Example 1 : Jameela was introducing her Class 10 children to polynomials. She began by chatting with the children, helping them to settle down quickly. Then she told them she would give them some nice puzzles to do, which they could discuss in groups also if they wanted to. The children, as usual, anticipated some fun. She gave each group puzzles like 'think-of-a-number', or 'if the parent is twice the child's age,...', etc. They started discussing noisily amongst themselves. In the meantime, she rotated from group to group, recording in her mind the kind of comments and questions the children were asking each other.

Jameela found, as we all do, that a few children had chosen to do the puzzles on their own, and had solved them. Then there was the other extreme, where some groups were stuck at the first puzzle, trying to do it by using many many specific values, none of which were working. She hinted at the use of a variable, but found no positive response.

Jameela gave the children who had finished a slightly higher level of 'logic' puzzle to do, namely,
There is a critical height (which is a whole number of floors above ground level), such that an egg dropped from that height (or higher) will break, but if dropped from a lower height (no matter how many times), it will not break. You are given two eggs and told that the critical height is between 1 floor and 37 floors (inclusive). What is the minimum number of times you must drop an egg in order to guarantee the successful determination of the critical height?

Soon she found them completely preoccupied with this problem.

Next, she decided to spend time with the children who did not feel comfortable with the idea of 'x'. She started with showing them a closed box with some pens in it, and asking them how many pens there were in it. Some said they didn't know, others said various numbers like 10, 12, 8, etc. One child suddenly said x. She asked this child, Anita, what she meant. With a little bit of help, Anita managed to explain that if one didn't know a quantity then you called it x. After some more discussion, the other children seemed to accept it. But, when Jamila tried to go back to the 'think-of-a-number' puzzle, they still persisted in reverting to particular numbers.

By now, it was time for the class to end, and the 'critical height' puzzle-solving group was still into heavy discussions. She told them to come back after school and work on the problem. They actually did come back in the afternoon, and two of them had the following solution to offer.

"Drop the first egg from the 8th floor, if it survives, drop it from the 15th floor (note 15 = 8+7). If it continues to survive, drop it from the 21st (8+7+6), 26th, 30th, 33rd, 35th, and finally 36th floors. If the first egg breaks when dropped from the 8th floor, drop the second egg from the 1st, 2nd, 3rd floors, etc. If the first egg breaks when dropped from the 15th floor, drop the second from the 9th, 10th floors, etc. In this way you'd know the critical height in at most 8 drops."

They had hit on this ingenious solution after a lot of discussion, hit-and-trial, and looking for patterns. Now another student from the group said, "I have a better solution now. Why not first drop it at 7, if it survives try 14, and so on". To this, the earlier solvers reacted, saying that this could give more drops, "Because suppose the egg broke from the 21st floor. Then you would need to drop it from 15, 16, etc., till you found the floor from which it would break. And, already you would have dropped it from 7 and 14. So the maximum number required could go up to 9 in this situation, and more if the egg breaks from the 35th floor."

This is when the teacher suggested trying the bisection method—drop from $\left[\frac{37}{2}\right]$th floor, see the effect; accordingly drop next from the floor numbered $18+\left[\frac{37-18}{2}\right]$ or the floor $\left[\frac{18}{2}\right]$, and so on. When the children worked it out this way they found it more efficient than the earlier solutions.

After this session, Jameela continued to think about the different levels of readiness of the various children, and disbanded her original plan for teaching polynomials. She now designed a variety of activities that would allow each child to develop her understanding of 'variable' and 'unknown', and take the quicker ones to 'polynomials'.

Though this was strenuous, Jameela managed it, giving the children many different contexts in which they would explore these concepts. She also asked them to think of several other contexts from the world around them. After a few sessions, the children seemed far more confident about the meaning and use of variable, and with linear equations. She found that they were actually looking forward to dealing with more equations of various kinds. In the mean time, the "fast track" children had moved along similar lines, but with monomials, remainder theorem, etc.

Jameela believes, "The learner constructs her own understanding of a concept based on her interaction with it in various contexts. We need to provoke our children to think harder and about many different aspects; to explore the concept on their own, with some support from us of course. We need to provide them with opportunities where they are forced to struggle to find their own methods of solving problems. I just help them now and then, and as and when they need my help."

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What are the features of constructivism that you can gauge from the example above? Firstly, and most importantly, a learner constructs her own understanding of a concept, based on her experiences and exploration related to that concept. She requires several opportunities to solve a wide variety of concrete and abstract problems related to the concept, working out for herself how each problem can be dealt with. Such thinking would be done individually or in groups.

Problems in real-world contexts can function as a source for the learning process. By practising with several different mathematical problems in several different contexts, students learn to use diverse strategies. During the learning process, not only are the interactions between the teacher and the student important, but also those between the students and their peers. Students can learn from each other and each other's strategies. Interaction with peers also promotes conceptual learning. Children in the classroom acquire new knowledge together. Students work together, learn together, and discuss possible solutions to a problem with each other. They develop problem-solving strategies, which they must then explain and justify to each other.

More generally, a constructivist believes **that a child learns by acting on objects**, initially concrete and later abstract. No experience is a waste. Given any task, the child learns **something** while trying to work on it. Given any theorem, the child learns by trying to prove some of its corollaries. She could try to make further conjectures and prove or disprove them along with her peers. A believer in this model expects the child to be able to try to discover patterns and look for generalised rules in empirical data given to her. She helps the child to develop these abilities as well as the **ability to articulate the process followed and the reasons in following that particular process.**

This kind of understanding implies that the child acquires the ability to find answers to questions and apply her mind to create new connections between concepts. Why don't you try some exercises now?

E1) A very experienced and sincere teacher told me that he has no problem regarding teaching of 'variable'. He said, "I tell them that in $x+5 = 15$ or $3x - 5 = 7$, we don't know x. It is some number we have to find." Then he tells them how to find it. "All the children understand it. The ones who are not sincere don't. But about 10% are, and they understand it, so there is no problem."

How do you know they have understood it? I asked. "They can do all the questions I give them on the board after I do one or two for them," said he.

How is this way of looking at learning different from the way in which Jameela looks at it?

E2) List five learning expectations in the constructivist model in the context of learning probability.

E3) Which principles of constructivism that we have discussed above do you find in your classroom interactions? Why are the other principles not present?

So far we have presented what learning means from the viewpoint of a constructivist. You found that **a constructivist focusses on the ability of the child to utilise her abilities, think and attempt to discover answers to new problems and questions.** If we believe that we need to have such an environment, then what does this require of the teacher? We shall consider this in the next section.

1.3 IMPLICATIONS FOR THE TEACHER

If we want the mathematics classroom to be a place in which mathematical thinking is encouraged, we need to ask ourselves several questions like the following ones.

1) How can we give each child a chance **to reflect on** and question her own thought processes? Will this happen if we give the children notes on lots of problems of the same kind to solve as homework?

   We need to give the children time to think, opportunities to try out their own solutions, a chance to reflect on their errors. If we focus on allowing the children different ways of solving a problem, they will be encouraged to think, reflect on and construct their understanding of the concepts involved. This would make them **active and articulate** agents in their own learning.

2) How can we help the child to develop her abilities to **question, investigate and explore** the world of mathematics? Most of us believe that teaching a child a concept means we should give her the formal definitions and related theorems, followed by a few examples and applications, and many problems of the same kind to solve. Do you think this approach would **develop a spirit of enquiry** in the learners. Very unlikely!

   To try and understand what is required, suppose you want to develop an understanding of 'binary operation' in the students' minds. To start with, you would need to use the background that they already have regarding this. This could be their understanding of the four basic arithmetic operations in **N**. Then, rather than **telling** them the definition, you could **encourage them, to themselves try and arrive at the definition**. This could be done, for example, through exposing them to several illustrations of operations which are binary, and those that aren't. You could get them to focus on why addition is a binary operation on **N**, and subtraction isn't, and other such familiar examples. At each stage, you could give them more problems to do. The children would be expected to solve these problems by discussing their own understanding with each other, and with you. This kind of interaction would hold the children's interest and help them build their own understanding.

3) **Shouldn't we teachers reflect** on the effectiveness, or lack of it, of our teaching strategies? Do we think about the implications for our teaching strategies of the way the child is reasoning about a concept/process/skill? This is particularly important for understanding why a child has certain misconceptions and what to do to remedy the situation. (This point is discussed at length in your course 'Teaching-Learning Process'.)

4) Does our interaction with the learners reflect the idea that **the teacher is only a facilitator**, and that **the learner is an important source of knowledge**?

Instruction based on constructivist principles asks students to be proactive participants in the learning process. The task of the teacher is to structure the interaction in such a manner that the students discover new knowledge. This is realised, for instance, by asking questions and posing meaningful problems. The contribution of the students consists of thinking and discussing possible solutions, making further generalisations, assessing the validity of these conjectures, and so on. The teacher can prod the discussion by posing more questions and summarising what the students say. Throughout, the students are responsible for bringing forth the problem-solving strategies to be discussed.

The class would usually start with a review of the pre-requisite knowledge. What the students do and say in this phase is then taken as the starting point for the current lesson. The teacher could clearly state what the topic of the current lesson would be. However, the discussion is often centred on the contributions of the children themselves, which means that the teacher may need to change her plan and discuss other topics.

As a facilitator, the teacher would need to provide much more space for the individual contributions of the students. The main idea is that the teacher presents a problem and the children actively search for a possible solution. The students work together on the solution of the problems, and are given the opportunity to demonstrate their own strategies. The teacher can encourage the discovery of new strategies by offering additional and/or more difficult problems. The teacher **supports the learning process** by asking questions and promoting discussion between the students. Sometimes the children may not discover a particular strategy for solving a problem. In such a situation the teacher could structure the discussions so that the students get a hint of it, and think about this method. She could also help the students classify various strategies, or pose questions about the usefulness of particular strategies.

Before going further, why don't you try the following question?

E4) In what way does your teaching aim at making your students independent confident learners of mathematics? Give examples to explain the effectiveness of your methods.

Let us continue our process of reflection on creating a constructivist learning environment.

5) What is the **purpose of our assessment** tasks? They need to be designed in a manner that brings out the way a student approaches a problem and explains

the reasoning about her approach. We also need to devise tasks that allow the students to reflect on what they have learnt, and assess themselves. The assessment needs to be an integral part of the teacher-learner interaction, not a disjoint activity. Hence, the **aim** of our assessment should be **to guide, focus and take the learning process further**.

6) Do the conventional tests achieve the aim in Point 5 above? In a conventional classroom, we give the child a written test (diagnostic, mastery or achievement test) to do, and collect the responses. We, then, examine the responses and mark them correct or wrong, depending upon whether the children have been able to give the correct answer or not. Then we look at the total number of answers that are correct and give each child a certain mark or a grade, which is a value that is assigned to the performance of the child. In this process, we do not bother to analyse the child's understanding, for instance, why has the child done one problem on continuity erroneously and another one correctly. The entire stress of the assessment through written tests is to show what the children have done incorrectly; but it does not bring out **why** it has been done incorrectly. Through this kind of assessment we don't find out the reason for the error — is it a careless mistake, or a partial understanding of the concepts being tested, or a wrong generalisation, or a misinterpretation of the question, or a total lack of comprehension of the question? Since the assessment does not attempt to discover the logic of the learner, it fails to notice what she **has done** correctly in her solution.

Do the tests you give to your learners show **how** they have produced a correct answer? Or what their understanding is of the underlying mathematics? Or, by what logic they have been able to produce the correct answer?

For instance, a child had answered the question 'If there is a rule r which takes every number x to x + 1, then is r a function?' correctly. Looking at this, some people may say that she 'knows' functions. When I asked her how she reached this answer, she said, "Because r(x) is x + 1, so there is an x on this side (*pointing to the right-hand-side)*, so we get a function." On probing further, she replied that the rule f, defined by f(x) = 0 for every number x, did not give a function since there was no x on the right-hand side!

7) What **type of assessment tests** would be appropriate? We could design worksheets aimed at getting the children to think and actively engage with the concepts. The tasks would, therefore, not be repetitions of what they have done before. They would be new tasks, which the children could even do collectively. These worksheets would need to be structured differently, allowing flexibility so that each student's experience and exploration allows her to give her own responses even with group work. The teacher needs to also participate in the process of the students doing the problems in the worksheet.

The assessment process needs to be a collaboration between the teacher and the students.

As said earlier, **the purpose of the worksheet is not to rank children, but to give them learning opportunities**, and to help the teacher understand how the teaching process should proceed. Therefore, the analysis of the responses need to be carefully done in a manner that brings out patterns of logical and conceptual bottlenecks in the children's understanding. Both the 'correct' and 'wrong' answers need to be examined, but the 'wrong' answers need to be analysed for insights about the way each student is thinking. Further, in order to assess the performance of a child well, the teacher would need to have a dialogue with her and observe her closely while she is trying to solve the problems. This process would also help the child in developing and understanding the concept or operation being learnt.

Children's responses in class, including errors, can be a rich resource for improving the teaching-learning process.

8) How should we react to **students' errors**? It is important that we go beyond merely giving a zero or a cross to students' errors. To really assess the 'errors' of the learner, we need to look for consistency in her answers, observe her while she is solving the problem and have a dialogue with her. If we talk to her about her 'errors', we may find that she has very good reasons for getting those answers. In fact, this is the constructivist's approach to errors.

To bring this point out more clearly, consider the following answers from a notebook of a child.

$$\lim_{x \to 0} (5x^2 + 3) = 3$$

$$\lim_{x \to \pi} \frac{\cos x}{2} = 0.$$

What would your assessment of this child be? Would you just give her marks for doing one problem correctly? Or, would you try and analyse her error to gauge her understanding? And then, had only the first problem been given, would you have assumed that she knows 'limits'? Or, would you have given her problems of the second type and word problems of different kinds to do also? In fact, giving a child several problems related to a concept in a variety of familiar contexts is required to even partially assess her understanding of the concept. Only then can we discover **something** about what she knows and what difficulties she has related to the concept. It is only by talking to her after analysing her answers that we may be able to understand her reasoning. And, only then can we help her to understand the mistake she is making.

There is no separation in a constructivist classroom between assessment and learning.

You may now like to ponder on the issues raised while doing the following questions.

E5) Write down one or more assessment tasks for the children in your mathematics class, and ask them to do the tasks. How did you assess the performance and understanding of the different children? Did you reflect on this assessment? What was the outcome of your reflective thinking based on this assessment?

E6) Where in your courses 'Teaching-Learning Process' and 'Assessment and Evaluation' have you come across the ideas discussed in this unit? Please note down the numbers of the sections or sub-sections of those courses, and the common idea alongside.

We shall end our brief discussion on expectations from the learner as well as teacher if we want a truly learner-centred approach. In the rest of the course, we shall keep referring to these ideas implicitly. So let us summarise the points we have raised in this discussion.

1.4 SUMMARY

In this unit we have tried to provoke you to think about the following points.

1. A child is the chief active agent of her learning.

2. We need to expose our learners to various contexts related to any particular concept/process/skill to allow her to build her understanding of it.

3. We need to give our learners several opportunities to solve a variety of problems around a concept. Also, we must encourage each child to explain the process she has followed and the reasoning behind it.

4. Every error made by a child should be utilised by the teacher to try and understand how her mind works.

5. We teachers must, individually and with our colleagues, reflect on the effectiveness of our teaching strategies.

6. We teachers need to move away from the conventional forms of assessment to a more constructivist understanding of assessment. Accordingly, we need to design tasks to assess individual children's levels and quality of understanding.

7. The teacher needs to be a facilitator, a guide who does not lead, but suggests a way to the learner, who will then be able to discover knowledge on her own.

Before finishing with this unit, you may like to **go back to the objectives listed in Sec. 1.1,** and check if you have achieved them. One way of checking is to do all the exercises and activities we have given you in this unit. You may also find the next section useful, in which we have given remarks on most of the exercises.

1.5 COMMENTS ON EXERCISES

E1) What is the difference in their views about 'understanding'? How do both of them look at the way learning takes place? What are the differences in their ways of assessing how much a child has learnt?
Think of these questions, and other aspects of the teaching-learning process.

E2) For instance, can the learner make connections between data gathered about an event, and the probability of occurrence of the event?
Further, how does the learner link her informal understanding of 'chance' with the formal definition of probability?
How does the learner react to **known situations** requiring her to deal with probability?
How does she react to **unfamiliar situations** requiring her to deal with probability?

Think of at least 3 more expectations from the learner.

E3) We suggest that you list the principles. Then note down which of them are usually present, and which aren't. Explain why the 'present' ones are found and the 'absent' ones aren't.

E4) Is there a difference between your role as a teacher, and the view expressed in Points 1-4? Do you give your students the definitions and some solved problems on the board, and ask them to solve several similar problem? Do you give them unfamiliar problems around the concept to do, and then allow them to discover a solution on their own?

Think about these and several other aspects that would make a learner less or more confident about her understanding.

E6) For instance, in Unit 5 of 'Teaching-Learning Process', you find several points raised here being discussed. Find out other such common focus areas.