UNIT 1 INTRODUCTION TO SETS

Structure

1.1 INTRODUCTION

Sometimes, we deal with some types of collections e.g.

i) Collection of books in a library of a university.

ii) Collection of natural numbers which are factors of say, 80 or any other natural number.

Set is also a collection of objects but it is a well defined collection (we will learn more about this in Sec 1.2). Consider the collection of states in India. We know that presently there are 28 states in India and this figure is exactly 28 (neither one less nor one more). Also if any number of persons (having a general knowledge of states in India) are asked to write the names of the states then final list of every body will contain the same 28 names (order in which they write the names of the states does not matter). Such type of a well defined collection is known as set.

By **well defined** collection, we mean that given any object we must be able to know as to whether it belongs to the collection or does not belong to the collection.

In this unit, we will introduce the notations and terminology used for sets. The unit defines set, its various types, discusses hierarchy of sets, Venn diagrams, various operations on sets and finally the unit is closed by giving an idea of some important and commonly used laws. Concept related to sets is very elementary and it is used directly or indirectly in the rest of our courses. So you must understand the concepts discussed in this unit before you proceed further in the course.

Objectives

After completing this unit, you should be able to:

- define a set;
- write a set in different forms;
- explain the types of sets;
- draw Venn diagrams.
- apply the operations on sets;
- get the idea of super sets and subsets; and
- get an idea of some important laws related to sets like associative, De-Morgan's laws, etc.

1.2 SETS

A well defined collection of distinct objects is called a set. A set is generally denoted by capital letters such as A, B, C, X, Y, Z, etc. and the objects which belong to the set are known as elements or members of the set and are generally denoted by small letters a, b, c, x, y, z, etc.

If 'a' is an element of a set A then we write $a \in A$ (read it as 'a' belongs to A). If 'a' is not an element of A then we write $a \notin A$ (read as 'a' does not belong to A).

> A set remains the same if some or all of its elements are repeated or rearranged.
>
> For example, if a set contains the elements 0, 1, – 1 and another set contains the elements – 1, – 1, 1, 0, 0, 0, 1, 0, 1, then these two sets are nothing but represent the same set having three elements 0, 1, – 1.

Following example illustrates the term "well defined collection" or "set".

Example 1: Consider the following collections and state reasons whether they form set or not.

(i) Collection of good cricketers in India.

(ii) Collection of honest students in a particular university in India.

(iii) Collection of natural numbers which are less than 5.

(iv) Collection of rich persons in India.

(v) Collection of letters of the word "ASSIGNMENT"

Solution:

(i) This collection does not form a set, because a given player may be good according to some person but the same player may not be good according to some other person.

(ii) This collection does not form a set, because a student may be honest according to some person but the same student may not be honest according to some other person.

(iii) Yes, this collection forms a set and elements of this set are 1, 2, 3, 4.

(iv) Richness is not a well defined property, because according to someone, a person may be rich while he/she may not be rich in view of some other person. So this collection does not form a set.

(v) It is a set and elements of this set are A, S, I, G, N, M, E, T.

Now, you can try the following exercise:

E1) Give reasons whether the following collections are sets or not.

(i) Collection of intelligent students in a particular school.

(ii) Collection of good hockey players in India.

(iii) Collection of good actors in India.

(iv) Collection of vowels in the word "INDIA".

Methods of Representing a Set

A set is generally represented by two methods as given below.

1. Roster Method

Dictionary meaning of 'Roster' is 'a list showing persons who perform their duties in turn'**.** As its meaning suggests, in this method each and every element is listed and put, separating by commas, in curly brackets. This method is also known as **Tabular Form** or **Listing Method.**

For example,

(i) If A is the set of vowels of English alphabets, then A = {a, e, i, o, u}

(ii) If N is the set of natural numbers, then N = {1, 2, 3, 4, 5, …}

(iii) If W is the set of whole numbers, then W = {0, 1, 2, 3, 4, 5, …}

(iv) If Z is the set of integers, then Z = {… , – 3, – 2, – 1, 0, 1, 2, 3,…}

(v) If E is the set of even natural numbers, then E = {2, 4, 6, 8, 10, 12, …}

(vi) If O is the set of odd natural numbers, then O = {1, 3, 5, …}

(vii) If P is the set of prime numbers, then
P = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, …}

Remark 1: Throughout the course, we use N, W and Z for the sets of natural numbers, whole numbers, and integers, respectively.

2. Set-Builder Method

In this method, we consider one or more properties that are exclusive to the elements of a set so that no other elements can be the member of the set. This method is also known as **Property Method** or **Rule Method.**

For example,

(i) Let A = {x : x is a vowel of English alphabet}, then elements of A are a, e, i, o, u and having exclusive property of being a vowel no other alphabet can be considered as an element of set A.

(ii) Let A = {x : x is a natural number and x is a multiple of 3}, then elements of A are 3, 6, 9, 12, … which have the exclusive property of being multiple of 3 and no other element can be consider as an element of A.

(iii) Let A = {x : x is a factor of 10 and x > 0}, then elements of A are 1, 2, 5, 10 and having exclusive property of being a factor of 10 and no other element can be consider as an element of A.

(iv) If Q is the set of rational numbers then Q = $\left\{x : x = \frac{p}{q}, p, q \in Z, q \neq 0\right\}$

Remark 2:

(i) In terminology of set the symbol ":" used in each part above is read as "such that".

(ii) Throughout the course the sets of rational numbers, irrational numbers and real numbers will be denoted by Q, I and R, respectively.

Note: Advantage of the second method, i.e. Set-Builder method lies in the fact that sometimes (or in some situations) we cannot list the elements of the set or even if we can list them, it may not be practical or feasible to do so.

For example, consider the set {x: x is a person who born in 2010 in India}. Obviously you will be more comfortable with property method in this example.

Consider another set {x : x is a real number and 1 < x < 4}. This set cannot be described by listing method, because number of elements in this set is uncountable.

Thus, above two examples show that in some situations either it is too difficult to describe the set by listing method or it is impossible to describe it.

No doubt, there are some examples in which listing method has its advantage For example, consider the set {28, Bihar, India}.

Three dots '…' are read as "and so on" which means all the elements following this pattern are also included in the set.

Prime number: A natural number (> 1**)** is called a prime number if and only if it has only two divisors 1 and itself. For example, 2 is a prime number as its only divisors are 1 and the number itself.

But 15 is not a prime number because it has 3 and 5 as its factors other than 1 and the number itself.

Rational number: A number which can be expressed in the form $\frac{p}{q}$ such that $p, q \in Z, q \neq 0$ is known as rational number. For example, $\frac{2}{3}, \frac{-4}{13}$, etc.

Irrational number: A number which cannot be expressed in the form $\frac{p}{q}$ where p, q are integers and q $\neq$ 0, is called an irrational number. For example, $\sqrt{2}, \sqrt{5}, \sqrt[3]{10}$, etc.

Real number: A number which is either rational or irrational is called real number.

It is a set having three elements 28, Bihar and India.

Let us now consider some examples to make the ideas of two methods discussed above more clear.

Example 2: Write the following sets by roster method:

(i) A = {x : x is a letter of the word "FUNCTION"}

(ii) B = $\{x : 2x + 5 < 17, x \in N\}$

(iii) C = $\{x : x^2 - x - 12 = 0, x \in N\}$

(iv) D = $\{x : x^2 - 4x - 21 = 0, x^2 - 49 = 0, x \in N\}$

Solution:

(i) A = {F, U, N, C, T, I, O} [Repeated elements are written once only]

(ii) B = $\{x : 2x < 17 - 5, x \in N\} = \{x : 2x < 12, x \in N\}$
$= \{x : x < 6, x \in N\} = \{1, 2, 3, 4, 5\}$

(iii) C = $\{x : x^2 - x - 12 = 0, x \in N\} = \{x : x^2 - 4x + 3x - 12 = 0, x \in N\}$
$= \{x : x(x-4) + 3(x-4) = 0, x \in N\} = \{x : (x-4)(x+3) = 0, x \in N\}$
$= \{x : x = 4, -3, x \in N\} = \{4\}$ $[\because -3 \notin N]$

(iv) D = $\{x : x^2 - 4x - 21 = 0, x^2 - 49 = 0, x \in N\}$
$= \{x : x^2 - 7x + 3x - 21 = 0, x^2 - 7^2 = 0, x \in N\}$
$= \{x : x(x-7) + 3(x-7) = 0, (x-7)(x+7) = 0, x \in N\}$
$= \{x : (x-7)(x+3) = 0, x = 7, -7, x \in N\}$
$= \{x : x = 7, -3, x = 7, -7, x \in N\}$

$= \{7\}$ $\left[\begin{array}{l}\because \text{here are two properties one says } x = 7, -3 \text{ and second says} \\ x = 7, -7 \text{ but in case we have more than one properties, we} \\ \text{take common element(s) between them and in our case it is } 7.\end{array}\right]$

Example 3: Express the following sets in the set-builder form:

(i) A = {3, 6, 9, 12, 15, 18, …}

(ii) B = {1, 2, 3, 5, 6, 10, 15, 30}

(iii) C = {5, 25, 125, 625, …}

(iv) D = $\{1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \frac{1}{243}\}$

(v) E = {1, 4, 9, 16, 25, 36, 49, 64, 81, 100}

Solution:

(i) Here we see that elements in this set are multiple of 3 so in set-builder form it can be written as A = $\{x: x \text{ is a multiple of } 3, x \in N\}$.

Similarly, by observing the pattern obeyed by the elements of other parts we can write them as given below.

(ii) B = $\{x: x \text{ is a factor of } 30, x \in N\}$

(iii) C = $\{x: x = 5^n, n \in N\}$

(iv) D = $\{x: x = \frac{1}{3^n}, 0 \le n \le 5, n \in W\}$

(v) E = $\{x : x = n^2, 1 \le n \le 10, n \in N\}$

Here are some exercises for you.

E 2) Describe the following sets by roster method:

(i) $A = \{x: x = 2n + 3, n \in W\}$ (ii) $B = \{x : x = 7^n, 0 \le n \le 3, n \in N\}$

(iii) $C = \{x : x \in N \text{ and } x \in W\}$ (iv) $D = \{x : x \in W \text{ and } x \in Q\}$

E 3) Express the following sets in set-builder form:

(i) $\{5, 10, 15, 20, \ldots\}$ (ii) $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\right\}$ (iii) $\{2, 4, 6, 8, 10, \ldots\}$

1.3 TYPES OF SETS

We have seen that set is a well defined collection of distinct objects. Also repetition of elements in a set is not allowed. So once a set is defined, automatically number of elements contained by it has also become fixed. In this section, we shall discuss the different names given to a set on the bases of the number of elements contained by the set. Equivalent and equal sets are also defined in this section.

Null Set

Consider a collection of those sons having their ages more than their respective fathers. Of course we will find no such son in this world. This type of collection is nothing but simply known as null set or **empty set** or **void set** in the terminology of sets.

Let us now formally define null set.

A set is said to be null (or empty or void) if it has no element in it. Null set is denoted by ϕ or { }.

For example, $A = \{x : x \text{ is a natural number}, 1 < x < 2\}$ is a null set as there is no natural number between 1 and 2.

Singleton Set

Consider the collection of mothers of a baby. Obviously a baby has only one mother. This type of collection having a single element is known as singleton set in the terminology of sets. Thus, a singleton set is defined as follow.

A set is said to be singleton set if it contains only one element.

For example, $A = \{x : x \text{ is an even prime number}\}$ is a singleton set as there is only one even prime number, i.e. 2 .

Finite Set

A set is said to be finite set if either it is an empty set or it has a finite number of elements.

For example,

(i) $A = \{2, 5, 7, 15\}$ is a finite set because it contains 4 elements, i.e. finite number of elements.

(ii) $B = \{1, 2, 3, 4, 5, \ldots\}$ is not a finite set as the number of elements in it are infinitely many.

(iii) $C = \{x : x + 1 = 0, x \in N\} = \{x : x = -1, x \in N\} = \{\ \}$ is an empty set. So, it is a finite set.

Cardinal Number of a Finite Set

The number of elements in a finite set say A is called its cardinality and is denoted by n(A).

For example,

(i) If A = {x, y, z}, then n(A) = 3, i.e. cardinality of A is 3.

(ii) If B = {14, 2, 3, 9, 15}, then n(B) = 5, i.e. cardinality of B is 5.

Infinite Set

A set is said to be infinite if it is not finite.

For example, A = {1, 4, 9, 16, 25, 36, …} is an infinite set.

Remark 3: Infinite sets are either countable or uncountable. We shall discuss it in Sec. 2.6 of Unit 2 of this block.

Equivalent Sets

Two finite sets A and B (say) are said to be equivalent if number of elements in both the sets are equal in numbers, i.e. n(A) = n(B) and we denote it by A ~ B (read as A is equivalent to B).

For example, if A = {a, b, c, d} and B= {2, 3, 5, 7}, then
A ~ B [$\because$ n(A) = n(B) = 4]

Equal Sets

Two sets A and B are said to be equal if every element of A is in B and every element of B is in A and is written as A = B.

For example, if A = {a, b, c, d} and B = {c, b, d, a}, then A = B as order of elements does not matter.

If two sets A and B are not equal then we write $A \neq B$.

Example 4: Give reasons whether the following statements are true or false:

(i) If A = {x : x is a vowel of English alphabet} and B = {x : x is a natural number, 7 < x < 13}, then A = B

(ii) If $A = \{x : x^2 = 9, x \in z)$ and B = {3, – 3}, then A = B

(iii) If A = {x, y, z, w} and B = {d, e, 7, 9}, then A ~ B

(iv) If A = {x, x, y, z} and B = {x, y, z, w}, then A ~ B

Solution:

(i) Here, A = {a, e, i, o, u} and B = {8, 9, 10, 11, 12}.
Clearly, $A \neq B$ (as there elements are not same). Hence, the given statement is false.

(ii) Here, A = {3, – 3} and $\left[\begin{array}{l}\because \text{we know that if } x^2 = a \text{ then } x = \pm\sqrt{a} \\ \therefore x^2 = 9 \Rightarrow x = \pm\sqrt{9} = \pm 3\end{array}\right]$

B = {3, – 3}, Clearly, A = B $\left[\begin{array}{l}\because \text{all the elements of A are in B and} \\ \text{all the element of B are in A.}\end{array}\right]$

$\therefore$ the statement is true.

(iii) Here, n(A) = 4, n(B) = 4 $\Rightarrow$ A ~ B, therefore, it is a true statement.

(iv) Here, A = {x, x, y, z} = {x, y, z} [As repetition in a set is not allowed] and B = {x, y, z, w}, so n(A) = 3, n(B) = 4.
$\Rightarrow$ A ~ B, is false because $n(A) \neq n(B)$.

Here is an exercise for you.

E 4) Give reasons whether the following statements are true or false:

(i) If A = {2, 9, 7, 7, 5}, B = {5, 2, 2, 9, 7}, then A = B

(ii) If A = { α, β, γ }, B = { λ, μ, ν }, then A ~ B

(iii) If A = {4, –4, 5, 5}and
B = {x : either $x^2 = 16$ or $x^2 + x - 20 = 0, x \in Z$}, then A = B

(iv) If A = {a, a, b, b, b, c} and B = {d, e, e, f, g, h}, then A ~ B

1.4 HIERARCHY OF SETS

For given any two real numbers a and b, you know that either a = b or a < b or a > b. This section will focus on how this type of association is setup in case of sets. Actually, here, we consider the sets contained in some other sets and define them with their appropriate designations.

Subset

Suppose A be the set of all working ladies and B be the set of all ladies then obviously all working ladies are ladies first. That is all the members of the set A are members of the set B. If it is so, then in the terminology of the sets A is known as subset of B.

Now, let us formally define the term subset.

Let A and B be two sets. Then A is said to be subset of B (or B is **super set** of A) if every element of A belongs to B and is denoted by $A \subseteq B$.
"$A \subseteq B$" read as A is contained in B or A is a subset of B.
If we write it as "$B \supseteq A$" then we read it as B contains A and we call B is a super set of A.

Remark 4: From above definition of subset, we see that a set A(say) will not be subset of another set B(say) if there is at least one element in A which is not in B. And it is denoted by $A \not\subset B$ (read as A is not a subset of B or A is not contained in B).

For example,

(i) If N = {1, 2, 3, 4, …}, W = {0, 1, 2, 3, 4, 5, …},

$$Z = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\},\ Q = \left\{\frac{p}{q} : p, q \in Z, q \neq 0\right\},$$

then $N \subseteq W$, $W \subseteq Z$, $Z \subseteq Q$, i.e. $N \subseteq W \subseteq Z \subseteq Q$.

(ii) If A = {1, 2, 4}, B = {1, 2, 4, 7, 9}, then $A \subseteq B$

(iii) If $A = \{a, b, c\}$, $B = \{b, c, d, e\}$, then $A \not\subset B$ $[\because a \in A \text{ but } a \notin B]$

Proper Subset

Let A and B be two sets. Then A is said to be proper subset of B if all the elements of A are in B and B has at least one element other than elements of A and is denoted by $A \subset B$.

For example, if A = {1, 2, 3} and B = {1, 2, 3, 4, 5},

then $A \subset B$. $\left[\begin{array}{l}\because \text{all the elments of A are in B and B} \\ \text{has two extra elements, i.e. 4 and 5.}\end{array}\right]$

Remark 5:

(i) Empty set is subset of every set, i.e. $\phi \subseteq A$ for any set A.

(ii) Every set is a subset of itself, i.e. $A \subseteq A$ for every set A.

Power Set

Let us first consider some examples:

(i) If A = { }, then ϕ is only subset of A.
That is, there is only 1 $(= 2^0)$ subset of A.

(ii) If A = {a}, then possible subsets of A are ϕ, {a}.
That is, there are only 2 $(= 2^1)$ subsets of A.

(iii) If A = {a, b}, then possible subsets of A are ϕ, {a}, {b}, {a, b}.
That is, there are only 4 $(= 2^2)$ subsets of A.

(iv) If A = {a, b, c}, then possible subsets of A are ϕ, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}.
That is, there are only 8 $(= 2^3)$ subsets of A.

(v) Similarly, if A has n elements then total number of subsets of A are 2^n.

Now, let us define what we mean by power set of a set.

Let A be any set. Then set of all subsets of A is known as power set of A and is denoted by P (A).

For example, in above discussed cases (i) to (iv) power set of A is given by

(i) P (A) = {ϕ}

(ii) P (A) = {ϕ, {a}}

(iii) P (A) = {ϕ, {a}, {b}, {a, b}}

(iv) P (A) = {ϕ, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}}

Here is an exercise for you.

E 5) If A = {a, b, c, d}, then write P(A).

Universal Set

In IGNOU there are 21 schools such as school of humanities (SOH), school of social sciences (SOSS), school of sciences (SOS), etc. (source IGNOU dairy 2011). If U is the set of all faculties of IGNOU and $A_1, A_2, A_3, ..., A_{21}$ are sets representing the faculties of 21 schools. Then, of course, faculties of all these 21 schools are faculties of IGNOU. That is, all the members of these 21 schools are present in the set U. Here U plays the role of universal set for the sets $A_1, A_2, A_3, ..., A_{21}$.

Now, let us formally define the universal set.

A set U is said to be **universal set** if all the sets under study are subsets of U.

For example,

(i) If A = {1, 2, 3, 5, 7, 9}, B = {2, 4, 6, 8, 10}, C = {3, 5, 7}, D = {8, 9, 10}, then U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} can play the role of universal set.

(ii) If in a study, only integers are involved as the elements of the sets, then Z, the set all integers, is the universal set.

You can now solve the following exercise.

E 6) If A = {1, 3, 5, 7, 9}, B = {2, 4, 6, 8}, C = {2, 3, 5, 7, 11, 13, 17, 19, 23}, and D = {5, 10, 15, 20, 25}, then what will be the smallest universal set?

1.5 VENN DIAGRAMS

As we know that examples play an important role to understand the concepts of theory/definitions. Similarly a diagram speaks more than the words that we may use and they also make the ideas simple and easy to understand even for a fresh reader. Euler (1707-1783), a Swiss mathematician was the first who took the step to represent the sets diagrammatically. Then John Venn (1834-1923), a British mathematician, moving a step ahead simplifying the ideas and made it more user friendly. That is why diagrammatical representation of sets is also known as Venn-Euler diagram. But usually they are known as Venn-diagrams. In Venn diagrams, sets are represented by enclosed areas in a plane as described below:

Notations Used in Venn Diagrams

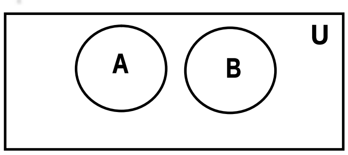
1. **Universal Set**

   Universal set U is represented by the interior of a rectangle as shown in Fig. 1.1

Fig. 1.1

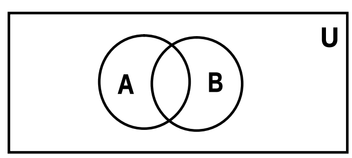
2. **Subsets**

   Subsets of U are described by the interior of closed curves (known as circular discs) within the rectangle, representing the universal set U. Fig. 1.2 shows the case when A and B have no common element, while Fig.1.3 shows the case when A and B have some common elements. Fig. 1.4 shows the case when $A \subseteq B$, and Fig. 1.5 shows the case when $B \subseteq A$.



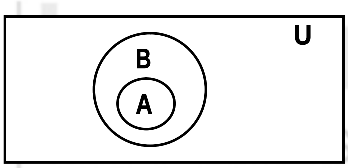
If $A \cap B = \phi$

Fig. 1.2



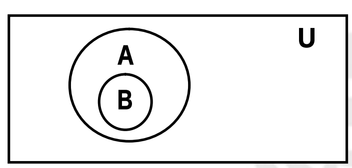
If $A \cap B \neq \phi$

Fig. 1.3



If $A \subseteq B$

Fig. 1.4



If $B \subseteq A$

Fig. 1.5

Remark 6:

(i) In general when nothing is mentioned about the common elements of A and B, presentation of Fig. 1.3 is used.

(ii) Sizes that we use to present A and B do not matter.

1.6 SET OPERATIONS

In school days, a child first learns counting numbers and then he/she learns how operations of addition, subtraction, multiplication and division are used on two numbers.

A similar type of approach is being used here. In sections 1.2 and 1.3 of this unit you have become familiar with the definition and types of sets respectively. In this section, we will learn about some commonly used operations on sets.

Union

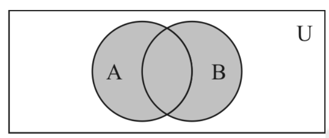
Let A be a set containing the persons getting salary (in Rs) between 10000 and 100000 per month and another set B containing the persons getting salary (in Rs) between 5000 and 20000 per month. For this example, if we are interested in finding those persons who are getting the salary within the range 10000-100000 or 5000-20000 then such persons will be those having salary between 5000 and 100000. The set of such persons is nothing but the union of two sets A and B.

Now, let us formally define the union of two sets.

Let A and B be two sets then union of A and B is denoted by $A \cup B$ and is defined as

$A \cup B = \{x: \text{either } x \in A \text{ or } x \in B\}$.

i.e. $A \cup B$ contains all the elements of A as well as of B (see Fig. 1.6).



Shaded region denotes $A \cup B$

Fig. 1.6

For example,

(i) If $A = \{2, 3, 5\}$, $B = \{3, 5, 7, 11\}$, then $A \cup B = \{2, 3, 5, 7, 11\}$.

(ii) If $A = \{a, b, c, d\}$, $B = \{d, e, f\}$, then $A \cup B = \{a, b, c, d, e, f\}$.

(iii) If Q = set of all rational numbers and I = set of all irrational numbers, then $Q \cup I = R$ = set of all real numbers.

That is, if all rational and irrational numbers are mixed then that mixture will be the set of real numbers.

Now, you can do the following exercises.

E 7) If $A = \{3\}$, $B = \{a, b, c\}$, then write $A \cup B$.

E 8) If $A = \{a, b\}$, $B = \{e, f\}$, then write $A \cup B$.

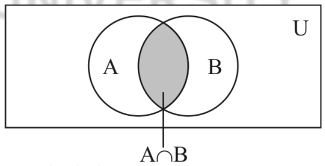
E 9) If $A = \{1, 2, 3\}$, $B = \{3, 4, 5, 6\}$, $C = \{3, 6, 7\}$, then write $A \cup B \cup C$.

Intersection

Let us again consider the example given above. For this example, if we are interested in finding those persons who are getting the salary within the common range then such persons will be those having salary between 10000 and 20000. The set of such persons is nothing but the intersection of two sets A and B.

Now, let us formally define the intersection of two sets.

Let A and B be two sets then intersection of A and B is denoted by $A \cap B$ and is defined as
$A \cap B = \{x: x \in A \text{ and } x \in B\}$.
i.e. $A \cap B$ contains common elements of A and B (see Fig. 1.7).



Shaded region denotes $A \cap B$

Fig. 1.7

For example,

(i) If A = {a, b, c}, B = {b, c, d, e}, then $A \cap B$ = {b, c}.

(ii) If A = {5, 7, 9}, B = {10, 11, 18}, then $A \cap B = \{\ \} = \phi$ = empty set, as there is no common element in the two sets.

Note: If $A \cap B = \phi$ then we say that two sets A and B are **disjoint.**

Here are some exercises for you.

E 10) If A = { }, B = {1, 4, 7}, then write $A \cap B$.

E 11) If A = {2, 4, 6, 8}, B = {2, 3, 5, 7, 11, 13}, then write $A \cap B$.

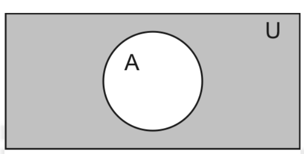
Complement of a Set

Suppose we have set of persons of a locality having voting right. Then set of those persons of the locality who do not have voting right is its complement, if the set of all persons of that locality is considered as a universal set.

Now, let us formally define complement of a set.

Let U be the universal set, then complement of a set A (where $A \subseteq U$) is denoted by A^c or $\overline{A}$ or A' and is defined as
$A' = \{x \in U: x \notin A\}$.

That is A' contains those elements of U which are not in A, i.e. A' contains all the elements of U other than A (see Fig. 1.8.).



Shaded region denotes the complement of the set A i.e A'

Fig. 1.8

For example, if U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and A = {1, 2, 4, 7, 9, 10}, then A' = {3, 5, 6, 8}, i.e. those elements of U which are not in A.

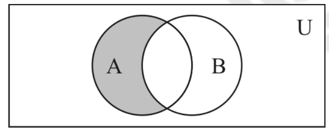
Here is an exercise for you.

E 12) If U = {x: x is an English alphabet} and
A = {x : x is an vowel of English alphabet}, then write A'.

Difference of Two Sets

Let A and B be two sets then difference of A and B is denoted by A – B and is defined as

$A - B = \{x: x \in A \text{ but } x \notin B\}$. See Fig. 1.9



Shaded region denotes A–B

Fig. 1.9

For example,

(i) If A = {1, 2, 3, 4, 5}, B = {4, 5, 6, 7}, then A – B = {1, 2, 3}

(ii) If A = {a, b, c}, B = {a, b, c, d, e}, then A – B = { }

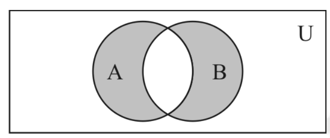
Now, you can do the following exercise.

E 13) If W = set of whole numbers and N = set of natural numbers then write W – N.

Symmetric Difference of Two Sets

Let A and B be two sets, then symmetric difference of A and B is denoted by $A \Delta B$ and is defined as

$A \Delta B = (A - B) \cup (B - A)$. See Fig. 1.10



Shaded region denotes AΔB

Fig. 1.10

For example, if A = {1, 2, 3, 4}, B = {3, 4, 5, 6, 7}, then A – B = {1, 2}, B – A = {5, 6, 7}

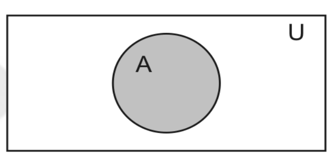
$\therefore \; A \Delta B = (A - B) \cup (B - A) = \{1, 2\} \cup \{5, 6, 7\} = \{1, 2, 5, 6, 7\}$.

Example 5: With the help of the Venn diagrams, show the following sets:

(i) A (ii) A' (iii) $A \cap B$ (iv) $A - B$ (v) $B - A$ (vi) $A \Delta B$ (vii) $A \subseteq B$ (viii) $B \subset A$

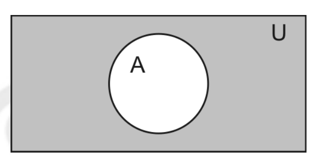
Solution:

(i)



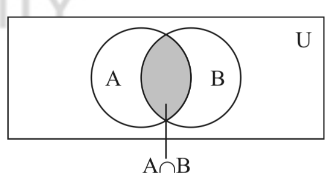
Shaded region denotes the set A

(ii)



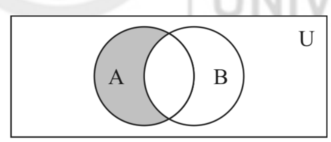
Shaded region denotes the complement of the set A i.e A'

(iii)



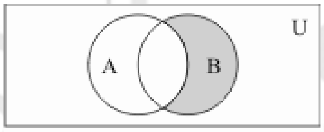
Shaded region denotes A∩B

(iv)



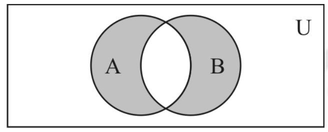
Shaded region denotes A–B

(v)



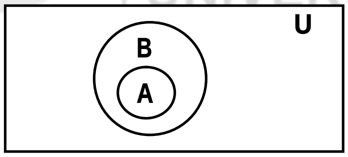
Shaded region denotes B–A

(vi)

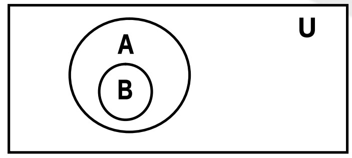


Shaded region denotes A∆B

(vii)



(viii)



Now, you can try the following exercise.

E 14) With the help of the Venn diagram justify the following

(i) $A \subseteq A \cup B$ (ii) $B \subseteq A \cup B$ (iii) $A \cap B \subseteq A$ (iv) $A \cap B \subseteq B$

(v) $(A \cap B)' = A' \cup B'$ (De-Morgan's law) (vi) $A \subseteq B \Rightarrow B' \subseteq A'$

1.7 SOME USEFUL AND IMPORTANT LAWS

Some commonly used laws of sets are listed below.

1. Idempotent Laws

For any set A

(i) $A \cup A = A$ (ii) $A \cap A = A$

2. Identity Laws

For any subset A of the universal set U

$A \cup \phi = A$, $A \cap U = A$, where ϕ is empty set

3. Commutative Laws

For any two sets A and B

(i) $A \cup B = B \cup A$ (ii) $A \cap B = B \cap A$

4. Associative Laws

If A, B, C are any three sets then

(i) $(A \cup B) \cup C = A \cup (B \cup C)$

(ii) $(A \cap B) \cap C = A \cap (B \cap C)$

5. Distributive Laws

If A, B, C are any three sets then

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

6. De-Morgan's Laws

For any two sets A and B

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

Example 6: If A = {1, 3, 5}, B = {3, 5, 7, 9}, C = {2, 6, 8, 9} are subsets of the universal set U = {1, 2, 3, 5, 6, 7, 8, 9} then verify

(i) De-Morgan's laws and

(ii) Distributive laws

Solution:

(i) De-Morgan's laws state that

(a) $(A \cup B)' = A' \cap B'$

(b) $(A \cap B)' = A' \cup B'$

To verify (a)

$A \cup B = \{1, 3, 5, 7, 9\}$

$\therefore (A \cup B)' = \{2, 6, 8\}$

$A' = \{2, 6, 7, 8, 9\}$, $B' = \{1, 2, 6, 8\}$

$\therefore A' \cap B' = \{2, 6, 8\}$

We see that here $(A \cup B)' = A' \cap B'$.

Hence verified.

To verify (b)

$A \cap B = \{3, 5\}$

$\therefore (A \cap B)' = \{1, 2, 6, 7, 8, 9\}$ and

$A' \cup B' = \{1, 2, 6, 7, 8, 9\}$

We see that here $(A \cap B)' = A' \cup B'$.

Hence verified.

(ii) Distributive laws state that

(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

To verify (a)

$B \cap C = \{9\}$

$\therefore A \cup (B \cap C) = \{1, 3, 5, 9\}$

$A \cup B = \{1, 3, 5, 7, 9\}$, $A \cup C = \{1, 2, 3, 5, 6, 8, 9\}$

$\therefore (A \cup B) \cap (A \cup C) = \{1, 3, 5, 9\}$

We see that here $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Hence verified.

To verify (b)

$B \cup C = \{2, 3, 5, 6, 7, 8, 9\}$

$\therefore A \cap (B \cup C) = \{3, 5\}$

$A \cap B = \{3, 5\}$, $A \cap C = \{\}$

$\therefore (A \cap B) \cup (A \cap C) = \{3, 5\}$

We see that here $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Hence verified.

Now, you can do the following exercise.

E 15) If the universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and $A = \{1, 3, 5, 6, 7\}$, $B = \{5, 6, 7, 8\}$, $C = \{1, 5, 7, 8\}$ are subsets of U, then verify (i) commutative laws and (ii) associative laws

---

Application of Sets

Venn diagrams are helpful in establishing many important relations between different sets, some of them are mentioned as under which are helpful in solving many practical problems too.

1. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

2. $n(A \cup B) = n(A - B) + n(A \cap B) + n(B - A)$

3. $n(A - B) = n(A) - n(A \cap B)$

4. $n(B - A) = n(B) - n(A \cap B)$

5. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C)$
$- n(B \cap C) + n(A \cap B \cap C)$

Let us consider an example based on above formulae.

Example 7: In a group of 500 persons, 400 can speak Hindi and 150 can speak English. Then how many can speak

(i) both Hindi and English

(ii) Hindi only

(iii) English only

Solution: Let A and B denote the set of persons who can speak Hindi and English, respectively. Then in usual notations, we are given
$n(A \cup B) = 500$, $n(A) = 400$, $n(B) = 150$

(i) We know that
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$\Rightarrow 500 = 400 + 150 - n(A \cap B \Rightarrow n(A \cap B) = 550 - 500 = 50$
$\therefore$ number of persons who can speak both Hindi and English = 50

(ii) We know that
$n(A - B) = n(A) - n(A \cap B) = 400 - 50 = 350$
$\therefore$ number of persons who can speak Hindi only = 350

(iii) We know that
$n(B - A) = n(B) - n(A \cap B) = 150 - 50 = 100$
$\therefore$ number of persons who can speak English only = 100

Here is an exercise for you.

E 16) Out of the 50 students in a class, 24 play cricket, 15 play hockey, 18 play football, 6 play cricket and hockey, 8 play cricket and football, 5 play hockey and football and 10 students do not play any of the three games. Then how many play (i) all the three games, (ii) hockey but not football and (iii) cricket and football but not hockey.

1.8 SUMMARY

Let us now summarise what we have covered in this unit.

1) Definition of a set with examples.
2) Two methods of writing a set.
3) Definition of various types of sets including empty set, singleton set, finite set, infinite set, equivalent sets, and equal sets.
4) Definition of subsets, proper subset, super set, universal set, power set.
5) Introduction of Venn diagrams.
6) Various operations on sets.
7) Idempotent, identity, commutative, associative, distributive and De-Morgan's laws.

1.9 SOLUTIONS/ANSWERS

E 1) (i) It is not a set, because a student may be intelligent according to someone while the same student may not be intelligent according to some other person.

(ii) It is not a set because a hockey player may be good in someone's view while the same player may not be good in view of some other person.

(iii) It is not a set because an actor may be good in someone's point of view while the same actor may not be good in the view of some other person.

(iv) It is a set having elements I, N, D, A $\begin{bmatrix} \because \text{repetition of elements} \\ \text{in a set is not allowed.} \end{bmatrix}$

E 2) (i) $x = 2n + 3,\ n \in W = \{0, 1, 2, 3, 4, \ldots\}$

$\therefore\ x = 3, 5, 7, 9, 11, \ldots$ $\begin{bmatrix} \text{Values are obtained on putting} \\ n = 0,1,2,\ldots \text{ in } x = 2n+3 \\ \text{e.g. when we put } n = 0, \text{ we get} \\ x = 2(0)+3 = 3 \end{bmatrix}$

And, therefore, A = {3, 5, 7, 9, 11,…}

(ii) $B = \{7^0, 7^1, 7^2, 7^3\} = \{1, 7, 49, 343\}$

(iii) N = {1, 2, 3, 4,…} and W = {0, 1, 2, 3, 4,…}
$\therefore$ if $x \in N$ and $x \in W$ then the elements which satisfy both are 1, 2, 3, 4, … and hence
C = {1, 2, 3, 4, 5, …}.

(iv) W = {0, 1, 2, 3, …}and Q = set of rational numbers,
$\therefore$ the elements which are common to both are 0, 1, 2, 3, … as no other rational number is a whole number and hence
D = {0, 1, 2, 3,…}.

E 3) (i) $\{x \in N : x = 5n,\ n \in N\}$

(ii) $\{x : x = \frac{1}{n},\ n \in N\}$

(iii) $\{x \in N : x = 2n,\ n \in N\}$

E 4) (i) A = {2, 9, 7, 5}, B = {5, 2, 9, 7} as repetitions in a set are not allowed.

We see that all the elements of A are in B and all the elements of B are in A.
$\therefore$ A = B. Hence the statement is true.

(ii) Here, n(A) = 3, n(B) = 3 and so the statement A ~ B is true.

(iii) $A = \{4, -4, 5\}$, $B = \{x:\ x^2 = 16,\ x^2 + x - 20 = 0,\ x \in Z\}$
$= \{x:\ x = \pm 4,\ (x+5)(x-4) = 0,\ x \in Z\}$
$= \{x:\ x = \pm 4,\ x = -5, 4,\ x \in Z\} = \{4\}$

Here, $-4 \in A$ but $-4 \notin B$
$\therefore\ A \neq B$, hence the statement is false.

(iv) Here, A = {a, b, c}, B = {d, e, f, g, h}.Thus, n(A) = 3, n(B) = 5.
$\therefore$ A ~ B is false because $n(A) \neq n(B)$.

E 5) $P(A) = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\},$
$\{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\}\}$

Notice that it has $2^4 = 16$ elements.

E 6) Smallest universal set is
{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 15, 17, 19, 20, 23, 25}.

E 7) $A \cup B = \{3, a, b, c\}$.

E 8) $A \cup B = \{a, b, e, f\}$.

E 9) $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7\}$.

E 10) $A \cap B = \{\ \} = \phi$.

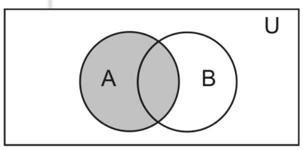
E 11) $A \cap B = \{2\}$.

E 12) A' = {x: x is a consonant of English alphabet}.

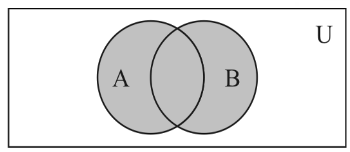
E 13) We know that

$W = \{0, 1, 2, 3, 4, \ldots\}$ and $N = \{1, 2, 3, 4, \ldots\}$
$\therefore W - N = \{0, 1, 2, 3, 4, \ldots\} - \{1, 2, 3, 4, \ldots\} = \{0\}$

E 14) (i)



Shaded region denotes A

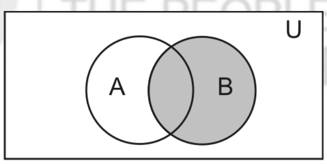


Shaded region denotes $A \cup B$

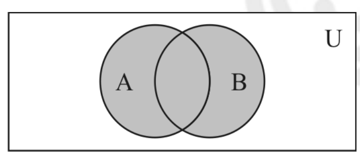
L.H.S. R.H.S.

Two Venn diagrams justify the relationship $A \subseteq A \cup B$.

(ii)



Shaded region denotes B

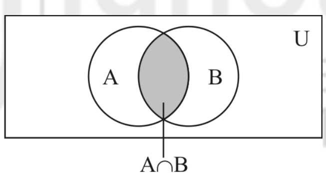


Shaded region denotes $A \cup B$

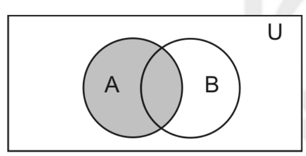
L.H.S. R.H.S.

Two Venn diagrams justify the relationship $B \subseteq A \cup B$.

(iii)



Shaded region denotes $A \cap B$

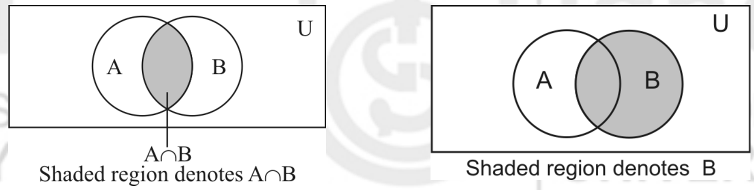


Shaded region denotes A

L.H.S. R.H.S.

Two Venn diagrams justify the relationship $A \cap B \subseteq A$.

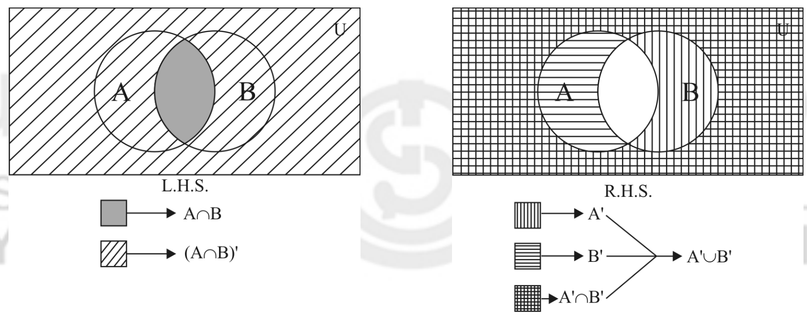
(iv)



L.H.S. R.H.S.

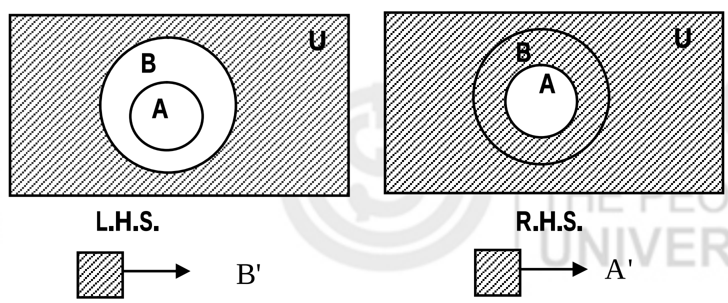
Two Venn diagrams justify the relationship $A \cap B \subseteq B$.

(v)



Two Venn diagrams justify the relationship $(A \cap B)' = A' \cup B'$.

(vi)



Two Venn diagrams justify the relationship $A \subseteq B \Rightarrow B' \subseteq A'$.

E 15) (i) Commutative laws state that

(a) $A \cup B = B \cup A$

(b) $A \cap B = B \cap A$

To verify (a)

$A \cup B = \{1, 3, 5, 6, 7\} \cup \{5, 6, 7, 8\} = \{1, 3, 5, 6, 7, 8\}$
$\therefore B \cup A = \{5, 6, 7, 8\} \cup \{1, 3, 5, 6, 7\} = \{1, 3, 5, 6, 7, 8\}$
$\therefore$ we see that here $A \cup B = B \cup A$.
Hence verified.

To verify (b)

$A \cap B = \{1, 3, 5, 6, 7\} \cap \{5, 6, 7, 8\} = \{5, 6, 7\}$
and $B \cap A = \{5, 6, 7, 8\} \cap \{1, 3, 5, 6, 7\} = \{5, 6, 7\}$
We see that here $A \cap B = B \cap A$
Hence verified.

(ii) Associative laws state that

(a) $(A \cup B) \cup C = A \cup (B \cup C)$

(b) $(A \cap B) \cap C = A \cap (B \cap C)$

To verify (a)

$A \cup B = \{1, 3, 5, 6, 7, 8\}$
$\therefore\ (A \cup B) \cup C = \{1, 3, 5, 6, 7, 8\}$
$B \cup C = \{5, 6, 7, 8\} \cup \{1, 5, 7, 8\} = \{1, 5, 6, 7, 8\}$
$\therefore A \cup (B \cup C) = \{1, 3, 5, 6, 7\} \cup \{1, 5, 6, 7, 8\} = \{1, 3, 5, 6, 7, 8\}$
We see that here $(A \cup B) \cup C = A \cup (B \cup C)$.
Hence verified.

To verify (b)

$A \cap B = \{5, 6, 7\}$
$\therefore (A \cap B) \cap C = \{5, 6, 7\} \cap \{1, 5, 7. 8\} = \{5, 7\}$
$B \cap C = \{5, 6, 7, 8\} \cap \{1, 5, 7, 8\} = \{5, 7, 8\}$
$\therefore A \cap (B \cap C) = \{1, 3, 5, 6, 7\} \cap \{5, 7, 8\} = \{5, 7\}$
We see that here $(A \cap B) \cap C = A \cap (B \cap C)$.
Hence verified.

E 16) Let C, H, F, denote the set of students who play cricket, hockey and football respectively. Then in usual notations, we are given.
$n(C) = 24$, $n(H) = 15$, $n(F) = 18$, $n(C \cap H) = 6$, $n(C \cap F) = 8$,
$n(H \cap F) = 5$, $n(C' \cap H' \cap F') = 10$

(i) Before finding the required number of students, we are to first obtain the number of students who play at least one of the three games which is given as
$= n(C \cup H \cup F) = 50 - n(C' \cap H' \cap F') = 50 - 10 = 40$

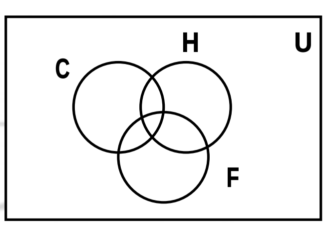
Now, we know that

$$n(C \cup H \cup F) = n(C) + n(H) + n(F) - n(C \cap H) - n(C \cap F) - n(H \cap F) + n(C \cap H \cap F)$$
$$\Rightarrow 40 = 24 + 15 + 18 - 6 - 8 - 5 + n(C \cap H \cap F)$$
$$\Rightarrow 40 = 57 - 19 + n(C \cap H \cap F) = 38 + n(C \cap H \cap F)$$
$$\Rightarrow n(C \cap H \cap F) = 40 - 38 = 2$$

$\therefore$ number of students who play all the three games = 2

(ii) We know that

$$n(H - F) = n(H) - n(H \cap F)$$
$$= 15 - 5 = 10$$



$\therefore$ number of students who play hockey but not football = 10

(iii) $n((C \cap F) - H) = n(C \cap F) - n((C \cap F) \cap H) = 8 - 2 = 6$

$$\begin{bmatrix} \because n(A - B) = n(A) - n(A \cap B) \\ \text{Here } A = C \cap F,\ B = H \end{bmatrix}$$

$\therefore$ number of students who play cricket and football but not hockey = 6