UNIT 8 FRESNEL DIFFRACTION

Structure

8.1 INTRODUCTION

We know from our day-to-day experience that we can hear persons talking in an adjoining room whose door is open. This is due to the ability of sound waves to bend around the comers of obstacles in their way. You are also familiar with the ability of water waves to propagate around obstacles. You may now ask: Does light, which is an electromagnetic wave, also bend around comers of obstacles in its path? In the previous block you have learnt manifestation of wave nature of light in the form of interference: Light from two coherent sources interfers to form fringed pattern, But what may puzzle you is the fact that light casts shadows of objects, i.e. appears to travel in straight lines rather than bending around comers. This apparent contradiction was explained by Fresnel who showed that the ease with which a wave bends around comers is strongly influenced by the size of the obstacle (aperture) relative to its wavelength. Music and speech wavelengths lie in the range 1.7 cm to 17m. A door is about 1 m aperture so that long wavelength waves bend more readily arcund the door way. On the other hand, wavelength of light is about 6×10^{-7}m and the obstacles used in ordinary experiments are about a few centimetres in size, i.e. 10^4-10^5 times bigger. For this reason, light appears to travel along straight lines and casts shadows of objects instead of bending around their comers. However, it does not mean that light shows no bending, it does so under suitable conditions where size of obstacles is comparable with the wavelength of light. You can get a feel for this by closely examining shadows cast by objects. You will observe that the edges of shadows are not sharp. The deviation of waves from their original direction due to an obstruction in their path is called diffraction.

You may have seen TV tower in Delhi. It is 235m high and almost three times taller than Qutub Minar. Have you ever thought: Why is TV transmission beamed from a height? The TV transmission involves short wavelength signals, $\lambda \cong$ 1cm. These arc blocked by hills, buildings and the curvature of Earth. It is only to avoid blockage that TV signals are transmitted from high towers. The radio signals are reflected by the ionospheric layers before reaching us. In contrast to this, the T.V. signals which are microwaves, do not gel reflected by the ionosphere. Their transmission takes place along the line of sight. To get the T.V. signals transmitted over long distances, geostationary satellites are used, which when placed at suitable heights reflect these signals.

The phenomenon of diffraction finds great use in daily life. You will learn that diffraction places a fundamental restriction on optical instruments, including human eye, in respect of resolution of objects. In this block you will learn that a lot of good physics is involved in diffraction limited systems.

The phenomenon of diffraction was first observed by Grimaldi, an Italian mathematician. And a systematic explanation of diffraction was given by Fresncl on the basis of Huygens-Fresncl principle. According to it, diffraction is attributed to the mutual interference of secondary wavelets from different parts of the same wavefront by taking phase difference into account. (The interference phenomenon involves two coherent wave trains.)

Fraunhofer diffracdon and Fresnel diffraction are also called far field diffraction and near field diffraction, respectively.

For mathematical convenience and ease in understanding, diffraction is classified in two categories: Fraunhofer diffraction and Fresnel diffraction. In Fraunhofer class of diffraction, the source of light and the observation screen (or human eye) are effectively at infinite distance from the obstacle. This can be done most conveniently using suitable lenses. It is of particular practical importance in respect of the general theory of optical instruments. You will learn about it in the next unit.

In Frcsncl class of diffraction, the source or the observation screen or both are at finite distances from the obstaclc. You will recognise that for Fresnel diffraction, the expcrimental arrangement is fairly simple. But its theoretical analysis is more difficult than that of Fraunhofcr diffraction. Also, Fresnel diffraction is more general; it includes Fraunhofcr diffraction as a special case. Moreover, it has importance in historical perspective in that it led to the development of wave model of light. You will learn these details in this unit.

You may be aware of the preliminaries of diffraction phenomenon from your school physics curriculum. Or you may have opted PHE-02 course on Oscillations and Waves. In whatever situation you are placed, you should refresh your knowledge.

Objectives

After studying this unit, you will bc able to

- stale simplc cxperimcnts which illustrate diffraction phenomenon
- dcscribc an cxperimcntal sct-up for diffraction at an aperture e.g. a circular aperture
- cxplain that Fraunhofcr diffraction is a special case of Fresnel diffraction
- discuss thc concept of Frcsnel half-period zones and apply it to zone plate
- discuss diffraction pattcrn duc to a circular aperture and a straight edge, and
- solvc numcrical problcms.

8.2 OBSERVING DIFFRACTION: SOME SIMPLE EXPERIMENTS

As you know, the wavelength of visible light ranges from 400 nm to 700 nm. And to see diffraction, careful observations have to be made. We will now familiarise you with some simple situations and experiments to observe diffraction of light. The pre-requisits for thesc arc: (i) a source of light, preferably narrow and monochromatic, (ii) a sharp edged obstacle and (iii) an observation screen, which could be human retina as well.

1. Look at a distant street-light at night and nearly close the eye lashes. The light appears to streak out from the bulb. This is because light has bent around the corners of your eyelids.
2. Stand in a dark room and look at a distant light bulb in another room. Now move slowly until the doorway blocks half of the light bulb. The light appears to streak out into the umbra region of the dark room due to diffraction around the doorway.
3. Take a piece of fine cloth, say fine handkerchief or muslin cloth. Stretch it flat and keep it close to the eye. Now focus your eye on a distant street lamp (atleast 100 m away) through it. Do it at night. Do you observe a regular pattern of spots arranged along a rectangle? On careful examination you will note that the spots on the outer part of the pattern appear coloured. Now rotate the handkerchief in its own plane. Does the pattern rotate? You will be excited to see that the pattern rotates and the speed of rotation of the pattern is same as that of the handkerchief.

   We are now tempted to ask: Do you know why is this pattern of spots obtained? You will agree that the handkerchief is a mesh (criss-cross) of fine threads in mutually perpendicular directions. Obviously, the observed pattern is formed by the diffraction of light (from the lamp) by the mesh of the handkerchief.

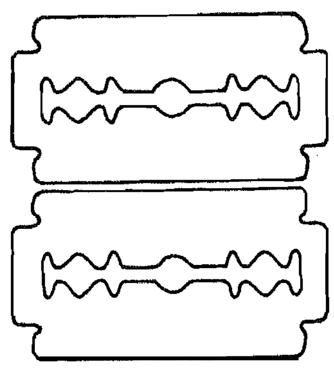


Fig.8.1: Observing diffraction using a pair of razor blades

4. Take a pair of razor blades and one clear glass electric bulb. Hold the blades so that the edges arc parallel and have a narrow slit in-between, as shown in Fig. 8.1. Keep the slit close to your eye and parallel to the filament. (Use spectacles if you normally do.) By carefully adjusting the width of the slit, you should observe a pattern of bright and dark bands which show some colours. Now use a blue or red filter. What do you observe? Does the pattern become clearer?

5. Mount a small ball bearing carefully on a plate of glass with a small amount of beeswax so that no wax spreads beyond the rim of the ball. Place this opaque obstacle in a strong beam of light (preferably monochromatic) diverging from a pinhole. Under suitable conditions, you will observe a bright spot, called Poisson spot at the centre of the shadow cast by the ball bearing. This exciting observation proved unchallengable evidence for diffraction of light.

$.3 PRODUCING A DIFFRACTION PATTERN

You will recall that in the Fresnel class of diffraction, the source of light or the screen or both are, in general, at a finite distance from the diffracting obstacle. On the other hand, in Fraunhofer diffraction, this distance is effectively infinite. This condition is achieved by putting suitable lenses between the source, the obstacle and the screen. A large number of workers have observed and studied Fresnel and Fraunhofer diffraction patterns. A systematic study of Fresnel diffraction pattern from obstacles of different shapes e.g. small spheres,, discs and apertures – circular, elliptical, square, or triangular – of different sizes was done by Indian Physicist *Y.V.* Kathvate under the guidance of Prof. C.V. Raman. Their

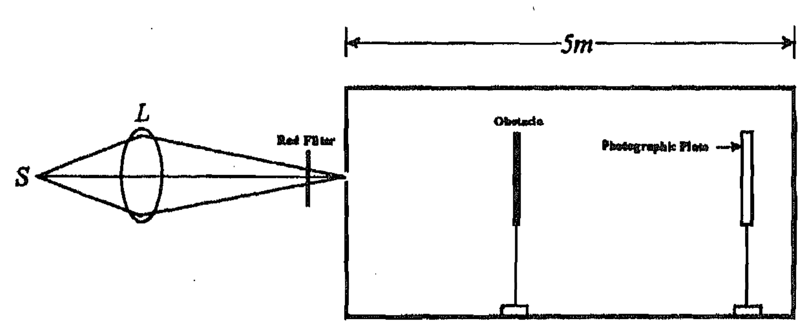


Fig.8.2: Schematics of experimental arrangement used by Kathvate to observe Fresnel diffraction

experimental set up for photographing these patterns is shown in Fig. 8.2. It consists of a light tight box (length nearly 5 m) with a fine pinhole at one end. The light on the pinhole from a 100 W lamp was focussed using a convex lens. A red filter was used to obtain monochromatic light of wavelength 6320 Å. The obstacle was placed at a suitable distance (about 2 m) from the pinhole. The photographic plate was mounted on a movable stand so that its distance from the obstacle could be varied. They used steel ball bearings of radii 1.58mm, 1.98mm, 2.37mm and 3.17mm as spherical obstacles. They also worked with discs of the same sizes. (As such, you should not attach much significance to the exactness of these sizes.) These obstacles, in turn, were mounted on a glass plate, which was kept at a distance of about 2 m from the pinhole.

The photographic plate was kept at distances of 5cm, 10cm, 20cm, 40cm and 180cm from the mounted glass plate (obstacle). For the last case, the diffraction patterns obtained from these spheres are shown in Fig.8.3 (a). These patterns clearly show the distribution of light intensity in the region of geometrical shadow of the obstacles.

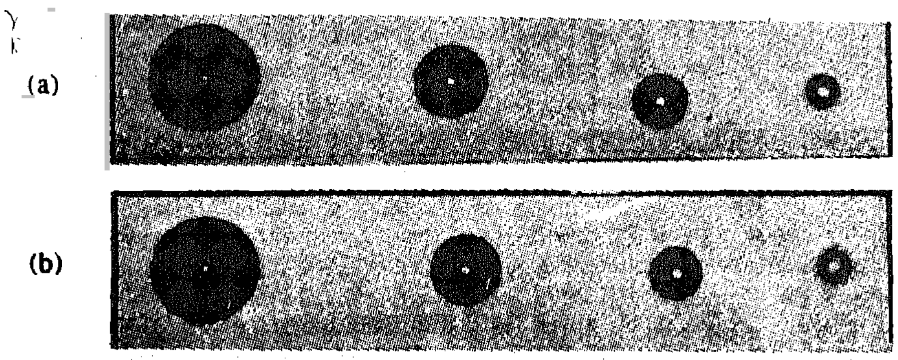
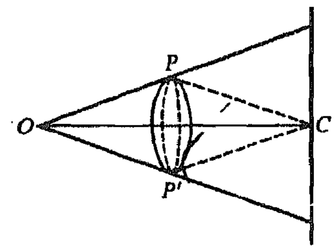


Fig.8.3: Fresnel diffraction patterns: Kathvate experiments with (a) spheres and (b) circular discs of four sizes

Poisson was a member of the committee which was appointed to judge Fresnel's dissertation. To disprove Fresnel, and hence wave theory, Poisson argued that a central bright spot should appear in the shadow of a circular obstacle. His logic, called reductio ad absurdum, goes as follows: Consider the shadow of a perfectly round object being cast by a point source (O) shown below.



According to the wave theory all the waves at the periphery will be in phase. This is because they have covered the same distance from the source. So the waves starting from the rim PP' and reaching C should all be in phase at the centre of the shadow. This implies that there should be a bright spot at the centre of the shadow. This was considered absurd by Poisson. He was definitely not aware that the bright spot in question had already been discovered by Maraldi almost a century ago. Soon after Poisson's objection, Arago carried out the experiment using a disk of 2mm diameter. To his surprise, he rediscovered the central bright spot.

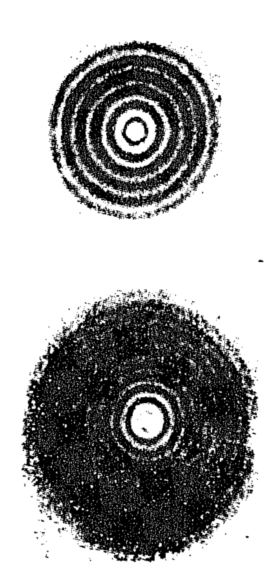


Fig.8.4: Enlarged view of fringe for discs of radii 1.58mm (upper) and 1.98 mm (lower)

The diffraction patterns for circular discs of the same size are illustrated in Fig.8.3(b). You will find that these patterns are almost similar to those for spheres. Moreover, the diffraction patterns on the left half of this figure, which correspond to bigger spheres and discs (radii 3.17mm and 2.37mm), show the geometrical shadow and a central bright spot within it. On the other hand, in the diffraction patterns corresponding to the smaller sphere and disc of radius 1.98mm, the geometrical images are recognisable but have fringes appearing on edges. The fringe pattern around the central spot becomes markedly clearer for the sphere and disc of radius 1.58mm. An enlarged view of this pattern is shown in Fig. 8.4. The formation of the bright central spot in the shadow and the rings around the central spot are the most definite indicators of non-rectilinear propagation of light This suggests that light bends in some special way around opaque obstacles. These departures from rectilinear propagation come under the heading of diffraction phenomenon.

Let us pause for a minute and ask: Are these diffraction patterns unique for a given source and obstacle? The answer to this question is: Fresnel pauerns vary with the distance of the source and screen from the obstacle. Let us now study how this transition evolves.

8.3.1 Spatial Evolution of a Diffraction Pattern: Transition from Fresnel to Fraunhofer Class

To observe transition in the Fresnel diffraction pattern with distance, we have to introduce a small modification in Kathvate's experimental arrangement, as shown in Fig. 8.5(a). The point source is now located at the focal point of a converging lens L. The spherical waves originating from the source O are changed into plane waves by this lens and the wavefront is now parallel to the diffracting screen with a narrow opening in the form of a long narrow slit (Fig. 8.5(b)). These waves pass through the slit. The diffracted waves are also plane and may have an angular spread. You may now like to know the shape, size and intensity distribution in the diffraction pattern on a distant observation screen.

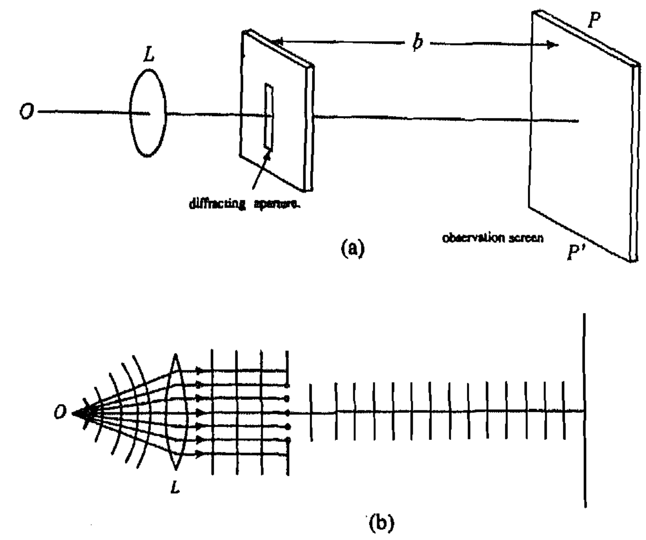


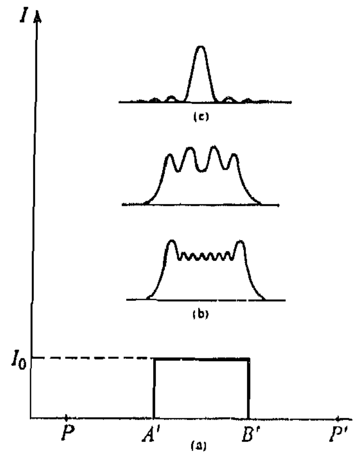
Fig. 8.5 *(a)*: Arrangement to observe transition in Fessnel diffraction pattern
(b) Cross-sectional view of the geometry shown in *(a)* above.

A slit is *a* rectangular opening whose width *(0.1 mm or so)* is much smaller than its length (1 an or more).

1. When the incident wavefront is strictly parallel to the diffracting screen, we get a vertical patch of light when the observation screen is immediately behind the aperture. That is, we get a region AB' of uniform illumination on the observation screen. The size of this region is equal to the size of the slit both in width and height. The remaining portion of the screen is absolutely dark. A plot of this intensity distribution is shown in Fig. 8.6 (a). From P to A', the intensity is zero. At A', it abruptly rises to I_0 and remains constant from A' to B'. At B', it again drops to zero. We can say that AB' represents the edges of the geometrical shadow (and the law of rectilinear propagation holds).
2. As the screen is moved away from the aperture, a careful observation shows that the patch of light seen in (1) above begins to lose sharpness. If the distance between the obstacle and the observation screen is large compared to the width of the slit, some fringes start appearing at the edges of the patch of light. But this patch resembles the shape of the slit The intensity distribution shows diffraction rippling effect somewhat like that shown in Fig. 8.6(b). From this we can say that the intensity distribution in the pattern depends on the distance at which the observation screen is placed.

3. When b (–1m) is much greater than the width of the slit (– 0.1 mm), the fringes seen in (2) above – close to edge of the patch – now spread out and the geometrical image of the slit can no longer be recognised. As distance is increased further, diffraction effects become progressively more pronounced.

4. When b is very large, i.e. once we have moved into the Fraunhofer region, ripples no longer change character. You can observe this pattern by putting a convex lens after the slit. The observation screen should be arranged so that it is at the second focal plane of the lens. These variations in Fraunhofer diffraction are shown in Fig. 8.6(c). You will learn the details of the Fraunhofer pattern in the next unit.



Pig. 8.6: Spatial evolution of a diffraction pattern

From this we may conclude that the Fresnel diffraction can change significantly as the distance from the aperture is varied. You must now be interested to understand these observations atleast qualitatively. First systematic efforl in this direction was made by Fresnel. Let us learn about it now.

8.4 FRESNEL CONSTRUCTION

Let us consider a plane wave front represented by WW' propagating towards the right, as shown in Fig. 8.7(a). First we calculate the effect of this plane wavefronl at an external point P_0 on the screen at a distance b. Then we will introduce an obstacle like a straight edge and see how intensity at P_0 changes.

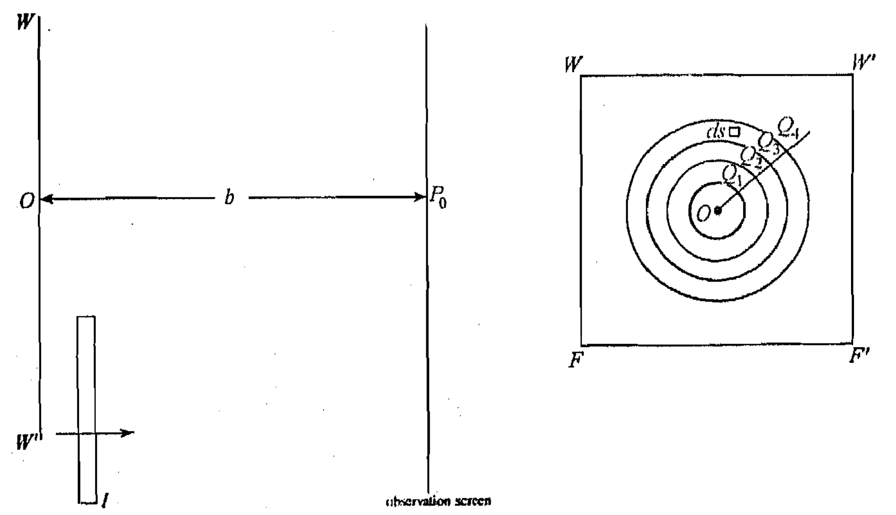


Fig.8.7: Fresnel construction (a) Propagation of a plane wavefront and (b) Division of wavefrnnt into annular spaces enclosed by concentric circles

We know that every point on the plane wavefront may be thought of as a source of secondary wavelets. We wish to compute the resultant effect at P_0 by applying Huygens-Fresnel principle. One way would be to write down the equations of vibrations at P_0 due to each wavelet and then qdd them together. This is a cumbersome proposition. The difficulty in mathematical calculation arises on two counls: (i) There arc an infinite number of points and each acts as a source of secondary wavelets and (ii) Since the distance travelled by the secondary wavelets arriving at P_0 is different, they reach the point P_0 with different phases. To get over these difficulties, Fresnel devised a simple geometrical method which provided useful insight and beautiful explanation of diffraction phenomenon from small obstacles. He argued that (i) It is possible to locate a series of points situated at the same distance from P_0 so that all the secondary wavelets originating from them travel the same distance. (ii) We can, in particular, find the locus of those points from where the wavelets travel a distance $b + \frac{h}{2}$, $b + \frac{2\lambda}{2}$, $b + \frac{3h}{2}$, and so on.

The Fresnel construction consists of dividing the wavefront into annular spaces enclosed by concentric circles (Fig. 8.7(b)). The effect at P_0 will be obtained by summing contributions of wavelets from these annular spaces called **half period elements.** When an obstacle is inserted in-between the wavefront WW' and the point P_0, some of these half period elements will be obstructed depending upon the size and shape of the obstacle. The wavelets from the unobstructed parts only will reach P_0 and their resultant can be calculated easily by Fresnel's method. Let us now learn about Fresnel construction, half period elements and the method of summation of the contributions of secondary wavelets.

8.4.1 Half Period Elements

To discuss the concept of Fresnel's half-period elements we assume, for simplicity, that light comes from infinity so that the wavefront is plane. Refer to Fig. 8.8. It shows a plane wavefront WW' $F'F$ of monochromaric light propagating along the z-direction. We wish to

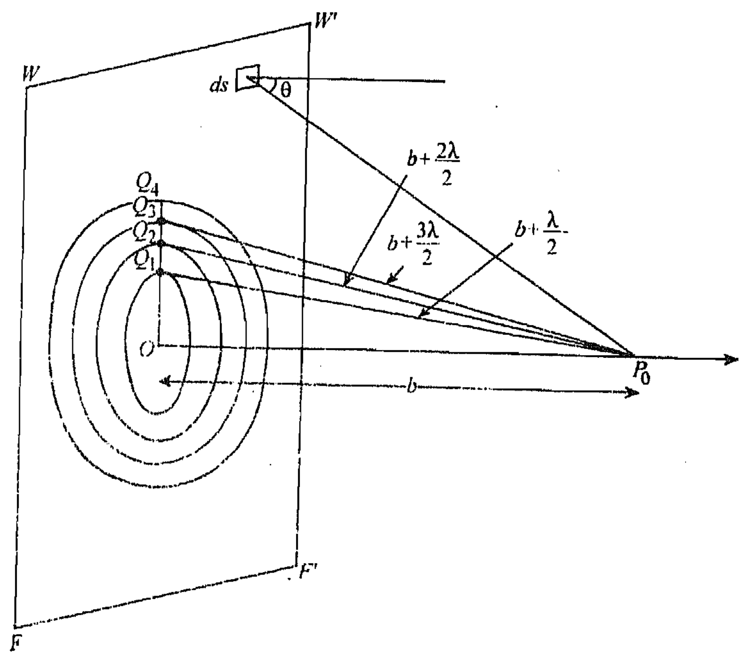


Fig. 8.8: Half period zones on a plane wavefront: A schematic construction

calculate the resultant amplitude of the field at an arbitrary point P_0 due to superposition of all the secondary Huygens' wavelets originating from the wavefront. To do so, we divide the wavefront into half-period zones using the following construction: From the point P_0 we drop a perpendicular P_0O on the wavefront, which cuts it at O. The point O is called the pole of the wavefront with respect to the point P_0. Suppose that b is the distance between the foot of the perpendicular to P_0, i.e. $OP_0 = b$. Now with P_0 as centre, we draw spheres of radii $b + \frac{\lambda}{2}$, $b + \frac{2h}{2}$, $b + \frac{3\lambda}{2}$, and so on. You can easily visualise that these spheres will intersect the plane wavefront in a series of concentric circles with centre O and radii OQ_1, OQ_2, OQ_3, ..., as shown in Fig.8.8. This geometrical construction divides the wavefront into circular strips called **zones.** The first zone is the space enclosed by the circle of radius OQ_1, the second zone is the annular space between the circles of radii OQ_2 and OQ_1. The third zone is annular space between the circles of radii OQ_3 and OQ_2 and so on. These concentric circles or annular rings are called **Fresnel zones** or **half** period elements. This nomenclature has genesis in the fact that the path difference between the wavelets reaching P_0 from corresponding points in successive zones is $\lambda/2$.

To compute the resultant amplitude at P_0 due to all the sccondary wavclcts cmanating from the entire wavefront, we first consider an infinitesimal area dS of thc wavefront. We assume that the amplitude at P_0 due to dS is (i) dircctly proportional to thc arca dS since it determines thc number of sécondary wavelets, (ii) invcrscly proportional to thc distance of dS from P_0 and (iii) directly proportional to thc obliquity factor $(1+\cos\theta)$, where θ is the angle between the normal drawn to the wavefront at dS and the linc joining dS to P_0. θ is zero for the central point O. As we go away from O, thc value of θ increases until it becomes 90° for a point at infinite distance on thc wavcfront. Physically, it cnsures that wavefront moves forward. That is, therc is no rcvcrsc (or backward) wave. The obliquity factor takes the value 2 for forward direction. 1 for θ = 90° and zero for θ = 180°.

If we denote the resultant amplitudes at P_0 duc to all thc sccondary wavclcts from the fust; second, third, fourth. ...., nth zone by a_1, a_2, a,, a_4, ..., a, , then wc can write

$$a_n = \text{Const} \times \frac{A_n}{b_n} \; (1+ \cos\theta) \tag{8.1}$$

where A_n is the area of the nth zone and b_n is the avcrage distancc of thc nth zone from P_0.

Eq. (8.1) shows that to know the amplitude of secondary wavclcts arriving at P_0 from any zone, we must know A_n. This, in turn, requires knowlcdge of thc radii of thc circlcs defining the boundaries of the Fresnel zones. To calculate radii of various half pcriod zoncs in terms of known distances, let us denote $OQ_1 = r_1$, $OQ_2 = r_2$, $OQ_3 = r_3$, ..., $OQ_n = r_n$. From Pythagoras' theorem we find that the radius of thc first circlc (zonc) is given by

$$r_1 = \left[\left(b + \frac{\lambda}{2}\right)^2 - b^2\right]^{1/2} \cong \sqrt{b\lambda + \frac{1}{4}\lambda^2}$$

$$\cong \sqrt{b\lambda} \tag{8.2a}$$

The approximation $\lambda \ll b$ holds for practical systcms using visible light. Similarly, the radius of the nth circle (zone) is given by

$$r_n = \left[\left(b + \frac{n}{2}\lambda\right)^2 - b^2\right]^{1/2}$$

$$\cong \left[nb\lambda + \frac{n^2\lambda^2}{4}\right]^{1/2}$$

$$\cong \sqrt{nb\lambda} \tag{8.2b}$$

whcrc we have neglected the term $\frac{n^2\lambda^2}{4}$ in comparison to $nb\lambda$. This approximation holds for all diffraction problems of interest to us hcre.

It readily follows from Eqs. (8.2a) and (8.2b) that thc radii of the circlcs arc proportional lo the square root of natural numbcrs, i.c. 1, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$ ... . Thcrcforc, if thc first zone has radius r_1, the successive zoncs have radii 1.41 r_1, 1.73 r_1, 2 r_1 and so on. For He-Ne laser light (λ = 6328 Å), if we takc P_0 to bc 30 cm away (b = 30 cm), thc radius of rhe first zone will be **0.436** mm.

Lct us now calculate the area of each of the half-pcriod zoncs. For the first zonc

$$A_1 = \pi r_1^2 = \pi\left[\left(b + \frac{\lambda}{2}\right)^2 - b^2\right]$$

$$= \pi b\lambda + \frac{\pi}{4}\lambda^2$$

$$\cong nbh \tag{8.3a}$$

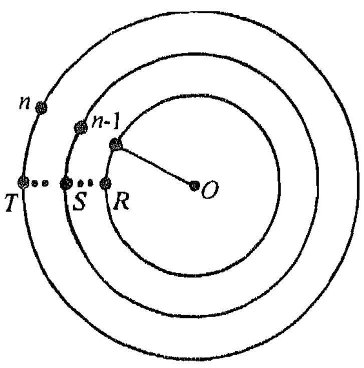
The area of the second zone, i.e. thc annular rcgion bctwccn thc first and thc second circlcs is

$$\pi\left(r_2^2 - r_1^2\right) = \pi[(b+\lambda)^2 - b^2] - \pi\, b\, \lambda$$

$$\cong 2\pi\, b\, \lambda - \pi\, b\, \lambda = \pi\, b\, \lambda \qquad (8.3b)$$

Similarly, you can readily verify that the area of the nth zone

$$A_n = \pi\left(r_n^2 - r_{n-1}^2\right) \cong \pi\, b\, \lambda \qquad (8.3c)$$



Refer to figure above and consider the contributions from the $(n-1)$th and nth zones. Firstly, the areas of the two annular regions are approximately equal, i.e., the amplitudes of secondary wavelets starting from both the zones are equal. Secondly, the points on the innermost circle of $(n-1)$th zone e.g. points like R are situated at a distance of $b+(n-2)\lambda/2$ from P_0, whereas the points on the innermost circle of nth zone e.g. points like S are situated at a distance of $b + (n-1)\lambda/2$ from P_0. The path difference between the secondary wavelets reaching P_0 from R and S is $\lambda/2$. This means that the waves reaching P_0 are out of phase by $\lambda/2$ and cancel each other. Similarly for every point between R and S in the $(n-1)$th zone we have a corresponding point between S and T in the nth zone which have a path difference of $\lambda/2$ or phase difference of π and hence cancel each other. Since the areas of the two zones are approximately equal, we arrive at the result that for every point in the $(n-1)$th zone we have a point in the nth zone which is out of phase by π or half of a period. Hence the name half-period zones.

That is, all individual zones have nearly the same area. The physical implication of the equality of zone areas is that the amplitude of secondary wavelets starting from any two zones will be very nearly equal. You must however remember that the result contained in Eq. (8.3) is approximate and is valid for cases where $b \gg n\lambda$. A more rigorous calculation shows that the area of a zone gradually increases with n:

$$A_n = \pi\lambda\left[b + \left(n - \frac{1}{2}\right)\frac{\lambda}{2}\right] \qquad (8.3d)$$

In this equation $b + \left(n - \frac{1}{2}\right)\frac{\lambda}{2}$ denotes the average distance of the nth zone from P_0. The geometry of the nth zone is shown in Fig. 8.9. It is important to point out here that the effect of increase in A_n with n is almost balanced by the increase in the average distance of the nth zone from P_0. That is, the ratio A_n/b_n in Eq. (8.1) remains $\pi\lambda$, which is constant,

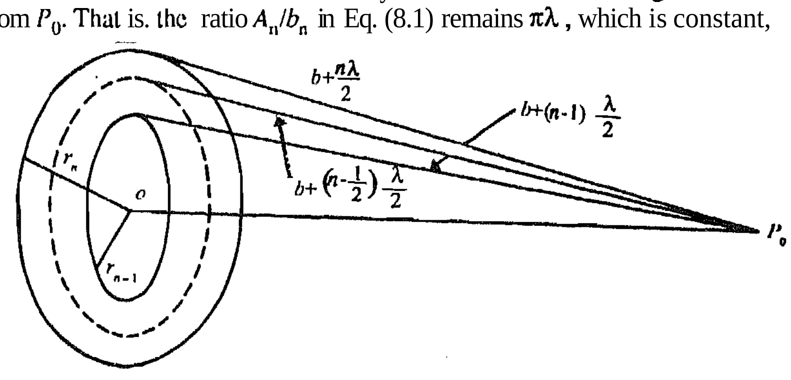


Fig. 8.9 : Half period zones and their average distance from the point of observation.

independent of n. This means that the amplitude due to any zone will be influenced by the obliquity factor; it is actually responsible for monotonic decrease in the amplitudes of higher zones ($a_1 > a_2 > a_3, \ldots > a_n$). Also, it is important for our computation to note that phases of wavelets from consecutive zones differ by one-half of a wavelength. Therefore, the secondary waves from any two corresponding points in successive zones [nth and $(n-1)$th or $(n+1)$th] reach P_0 out of phase by π or half of a period.

Suppose that the contribution of all the secondary wavelets in the nth zone at P_0 is denoted by a_n. Then, the contribution of $(n-1)$th zone a_{n-1}, will tend to annihilate the effect of nth zone. Mathematically we write the resultant amplitude at P_0 due to the whole wavefront as a sum of an infinite series whose terms are alternately positive and negative but the magnitude of successive terms gradually diminishes; that is,

$$\xi = a_1 + a_2\, e^{i\pi} + a_3\, e^{i\,2\pi} + a_4\, e^{i\,3\pi} + \ldots$$

$$= a_1 - a_2 + a_3 - a_4 + \ldots + (-1)^{n+1}\, a_n \qquad (8.4)$$

We will show that the sum of this infinite series is equal to $a_1/2$. In words, the resultant amplitude at P_0 due to all the secondary wavelets emanating from the entire wavefront is equal to one-half of the contribution of secondary wavelets from the first zone.

There are several methods of deriving this result. Here we will describe a simple graphical construction. (The mathematical method is given as TQ.) Let AB, CD, EF, GH, ... respectively denote the amplitudes of resultant vectors a_1, a_2, a_3, a_4 ... due to the first, second, third, fourth,...nth zone. (We know that a_1, a_2, a_3, a_4, ... a_n are alternately positive and negative.) These vectors are shown separately in Fig. **8.10(a)** to show their magnitudes and positions. But their true positions are along the same line, as shown in Fig. 8.10 (b). The resultant of the first two zones will be the small vector **AD.** But the resultant of the first three zones is the large vector AF; of the four zones the smaller vector AH and so on.

Refer to Fig. 8.10(a) again. You will note that the resultant of infinitely large number of zones is equal to $a_1/2$. If we consider a finite number of zones, say n, the resultant is given by

$$\xi(n) = \frac{a_1}{2} + \frac{a_n}{2} \tag{8.5}$$

where n is any number (odd or even).

To see this, you closely reexamine Fig. 8.10(b). You will note that all vectors representing a_1, a_2, a_3, a_4, ... are line segments whose midpoint coincides with the midpoint of a_1 (marked as—).

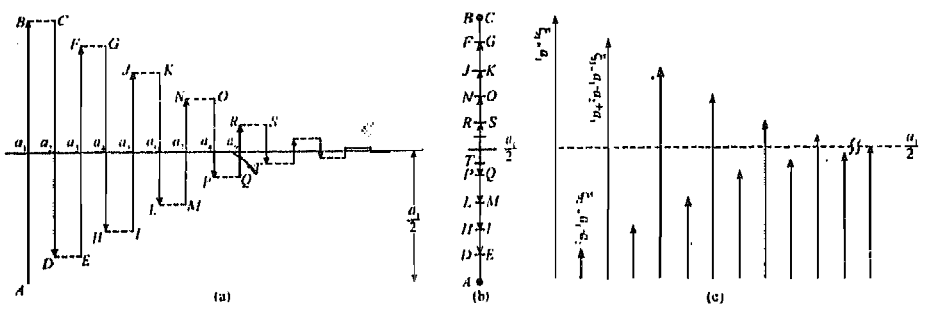


Fig. 8.10: Phasor diagram for Fresnel (half-period) zones. Individual amplitudes are shown in (a). Actually all vectors arc along the same line. This is shown in (b). The resultant amplitudes due to n(= 2, 3, ...) zones are shown in (c).

(You must convince yourself about this.) In other words, the vector representing a_n is a line, half of which is above the horizontal line passing through the midpoint of a_1 and the other half is below this line. The resultant of n zones is a vector joining A to the end of the vector representing a_n. When n is odd, the end point of the vector representing a_n will be above the horizontal line by $a_n/2$, which proves the required result.

If n is even, the end point will be below this horizontal line by $a_n/2$. Added vectorially, we have the same result. We thus see that the resultant amplitude at P_0 due to n zones is half the sum of amplitudes contributed by the first and the last zone. ξ will be numerically greater than $a_1/2$ when n is odd and smaller than $a_1/2$ when n is even. For example, the resultant contribution due to 7 zones is AO, which is equal to $\frac{a_1}{2} + \frac{MN}{2}$. On the other hand, for 8 zones the resultant is AQ = $\frac{a_1}{2} - \frac{OP}{2}$.

It may be emphasized that in this graphical method of summation of the series, we have used three properties: (i) vectors representing a_1, a_2, ... are all along the same straight line (ii) alternate vectors are oppositely directed and (iii) the magnitudes of a_1, a_2, ... decrease gradually. We now consider a simple example to illustrate these concepts.

Example 1

Consider a series with n = 100 in which each term is equal to the arithmetic mean of the preceding and the following terms. Calculate the resultant.

Solution

As a special case, we can take the terms of the series as 100, 99, 98, ... 3, 2, 1.

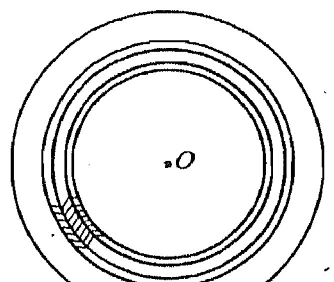
$$\begin{aligned} \therefore \quad \xi &= (100-99) + (98-97) + (96-95) + \dots (4-3) + (2-1) \\ &= 1 + 1 + 1 \dots 50 \text{ terms} \\ &= 50 \end{aligned}$$

which is half of the first term. Now consider the relation

$$\xi = \frac{a_1}{2} + \frac{a_n}{2}$$

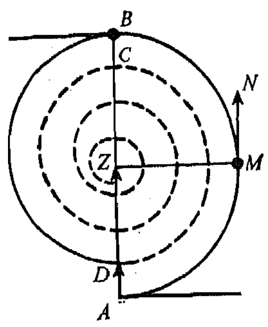
and take different number of terms in this arithmetic series. If we have only one term, (a_1 = 100), we take the first term as also the last term as 100. Then we get

So far we have considered the effect of a whole number of half period elements at a given point. The sum of the amplitudes due to all the secondary wavelets starting from the nth zone was represented by a_n. But so far we have not computed the magnitude and phase of this vector. An obvious related problem is to calculate the effect at P_0 due to a fraction of a given half period element. We can do this easily by the vector summation method. We divide a Fresnel zone into a series of n sub-zones of equal areas. Refer to figure below. It shows such a division for the annular space between $(n-1)$th and nth circles. O is taken as centre and circles of slightly differing radii have been drawn such that the annular space between two consecutive circles encloses equal area. Now within the area covered by a sub-zone, we can neglect variation in inclination factor. Since all these sub-zones have been drawn so that they have equal areas, the amplitude at P_0 due to these small equal areas will be the same. But the phases will change continuously from one sub-zone to the next sub-zone by $\lambda/2n$ since the phase difference between the secondary wavelets starting from the innermost to the outermost sub-zone of any one Fresnel half period zone is $\frac{\lambda}{2}$ or π. If we make n very large, we will have infinitesimally small but equal areas and phases of wavelets from these vary continuously and uniformly.



Thus we have a set of disturbances of equal amplitude but uniformly changing phase such that the phase difference between the two extreme disturbances is π. These extreme vectors are represented by AA* and BB′ in the figure shown on the next page. We know that in such a case the vector diagram is a semicircle and the resultant of the summation of amplitudes is the diameter AB.

Now we will compute magnitude and phase of the resultant AB. If all the disturbances from the subzones were in the same phase, the resultant would have been a line along AA' and equal to the length of the arc of the semicircle AB $(=\pi r)$ of radius r. But we find that the actual resultant amplitude is AB = $2r$. Thus the resultant amplitude is $\frac{2}{\pi}$ times the value which would be obtained if all the wavelets within a Fresnel half period element had the same phase. Since the line AB is parallel to the line MN, we see that the resultant phase of vector AB is the some as that of the vector MN representing the disturbance starting from the middle point (M) of the zone. In other words, AB is perpendicular to AA'. That is, it is a quarter-period behind the wavelet starting from the innermost sub-zone. We can find, in a similar manner, the resultant contribution due to the next half-period zone. It is given by CD and differs from AB by π. The resultant of the sum of these two zones is the smaller vector AD. The magnitudes of vectors and their phases for succeeding zones give rest of the figure. The resultant curve is the vibration spiral with gradually smaller and smaller semicircles until eventually it coincides with Z. The resultant of all the half-period elements is AZ which is half of that which would be produced by the first zone alone. It is equal to $\frac{1}{2}\times\frac{2}{\pi}=\frac{1}{\pi}$ times that which would be produced by all the wavelets from the first zone acting together in the same phase.



$$\xi = \frac{a_1}{2} + \frac{a_n}{2} = 100$$

Next we take two terms. Then

$$\xi = (100 - 99) = 1$$

Also

$$\frac{a_1}{2} - \frac{a_n}{2} = \frac{100}{2} - \frac{99}{2}$$

$$= 50 - 49.5 = 0.5$$

For three terms $\quad \xi = (100 - 99) + 98 = 99$

and $\quad \frac{a_1}{2} + \frac{a_3}{2} = 50 + 49 = 99$

For four terms, $\quad \xi = (100 - 99) + (98 - 97) = 2$

and $\quad \frac{a_1}{2} + \frac{a_4}{2} = 50 - 48.5 = 1.5$

For five terms $\quad \xi = (100 - 99) + (98 - 97) + 96 = 98$

and $\quad \frac{a_1}{2} + \frac{a_5}{2} = 50 + 48 = 98$

For six terms $\quad \xi = (100 - 99) + (98 - 97) + (96 - 95) = 3$

and $\quad \frac{a_1}{2} + \frac{a_6}{2} = 50 - 47.5 = 2.5$

and so on. Thus we see that ξ is given by $\frac{a_1}{2} + \frac{a_n}{2}$ to a fairly good degree of accuracy.

8.4.2 Rectilinear Propagation

Refer to Fig. 8.11. Light originates from a point source and propagates towards the right. Suppose that the source is 1 m away from the aperture. We may take the spherical wave falling on the aperture as nearly a plane wave. (The radius of curvature of the incident

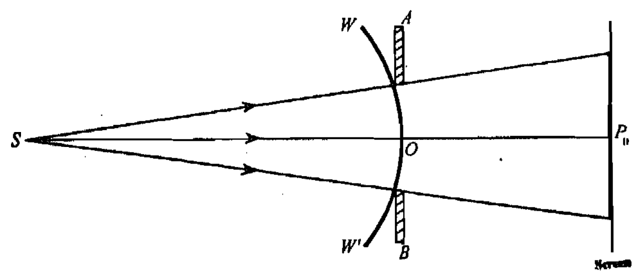


Fig. 8.11: Fresnel construction and rectilinear propagation of light

spherical wave will not qualitatively change the argument.) Let us work out the sizes of Fresnel half period elements for the typical case where the screen is 30 cm away from the aperture. Taking $\lambda = 5 \times 10^{-5}$ cm, we get $r_1 = \sqrt{(30\text{ cm}) \times (5 \times 10^{-5}\text{ cm})} = 3.87 \times 10^{-2}$ cm. This means that the diameter of the first zone is less than 1 mm. Let us consider the 100th zone. Its radius $r_{100} = \sqrt{30\text{ cm} \times 100 \times 5 \times 10^{-5}\text{ cm}} = 3.87 \times 10^{-1}$ cm so that the diameter will be a little less than 1cm. Therefore, if the aperture is about 1 cm in diameter, it will accomodate over 100 Fresnel zones and the amplitude at P_0 due to the exposed part of the wavefront will be $\frac{a_1}{2} + \frac{a_{100}}{2}$. Since $a_{,,}$ will be fairly small, the intensity at P_0 will be essentially half of that due to the first half period zone, which is the intensity expected at P_0 when the aperture is completely removed. We thus find that even through a small aperture we get the original intensity at P_0. That is, light travels in a straight line for all practical purposes.

Let us now understand the formation of shadows and illuminated regions due to an obstacle (Fig. 8.12). Consider the point P_2 whose pole is O_2. If the distance between O_2

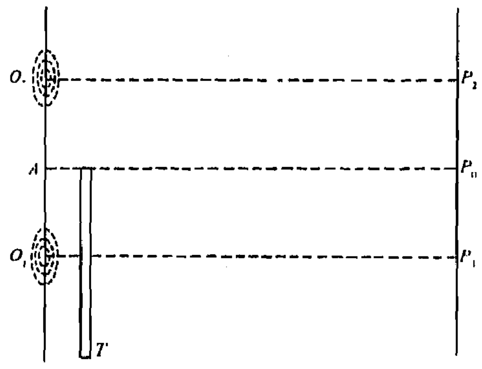


Fig. 8.12: Fresnel construction and formation of shadows/illuminated regions

and the edge A of the obstacle is nearly 1 cm, over 100 half period elements will be accomodated in it. And as seen above, the intensity at P_2 will be nearly equal to $\frac{a_1}{2}$. In other words, he obstacle Twill have no effect at the point P,. Similarly, at P_1, which is taken 1 cm inside the geometrical edge of the shadow, over 100 half period elements around O_1 are obstructed and the intensity at P_1 will be less than $\frac{a_{100}}{2}$, which is almost negligible. This implies almost complete darkness at P_1. In other words, the obstacle has completely obstructed the light from the source and the region around point P, is in the shadow. Only around P_0, which signifies the geometrical edge of the shadow, we find fluctuations in intensity depending on how many half period elements have been allowed to pass or have been obstructed. This explains the observed rectilinear propagation of light since Fresnel zones are obstructed or allowed through by obstacles of the size of a few mm for these typical distances.

A special optical device, designed to obstruct light from alternate half-period elements is known as Zone plate. It provides experimental evidence in favour of Fresnel's theory. Let us learn about it now.

8.4.3 The Zone Plate

The zone plate is a special optical device designed to block light from alternate hall-period zones. You can easily make a zone plate by drawing concentric circles on a white paper, with their radii proportional to the square roots of natural numbers and shading alternate zones. Fig.8.13 shows two zone plates of several Fresnel zones, where

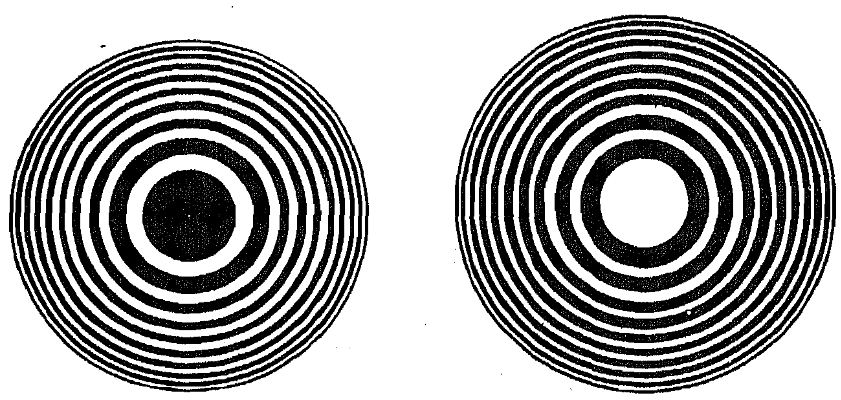


Fig. 6.13: Zone plate : (a) positive (b) negative

all even numbered or odd numbered zones are blacked out. Now photograph these pictures. The photographic transparency (negative) in reduced size acts as a Fresnel zone plate. (Recently, Gabor has proposed a zone plate in which zones change transmission according to a sinusoidal wave.) Lord Rayleigh made the first zone plate in 1871. Today zone plates are used to form images using X-rays and microwaves for which conventional lenses do not work.

If you now pause for a while and logically reflect upon the possible properties of a Fresnel zone plate, you will reach the following conclusions:

1. A zone plate acts like a converging lens (see Example 2) and produces a very bright spot. To understand the formation of the spot, let us suppose that the first ten odd zones are exposed to light. Then, Eq.(8.4) tells us that the resultant amplitude at P_0 is given by

$$\xi_{20} = a_1 + a_3 + a_5 + ... + a_{19} \tag{8.5}$$

If the obliquity factor does not produce much change, we may write $\xi_{20} = 10\, a_1$, which means that the amplitude for an aperture containing **20** zones is twenty times and intensity is 400 times that due to a completely unobstructed wavefront. This is illustrated in Example 2.

Example 2

Show that a zone plate acts like a converging lens.

Solution

Refer to Fig. 8.14. It shows the section of the zone plate perpendicular to the plane of the paper. S is a point source of light at a distance u from the zone plate and emits spherical waves. We wish to find the effect at P_0 at a distance v from the plane of the zone plate.

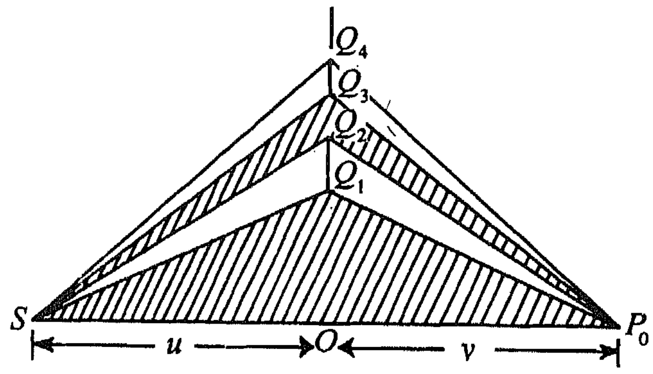


Fig.8.14: Action of a zone plate as a converging lens

In this zone plate we divide its plane into zones bounded by circles with centre at O and radii OQ_1, OQ_2, OQ_3, ... OQ_n such that the path of the ray from S to P_0 increases by $\frac{\lambda}{2}$ from successive zones. Then we can easily write

$$SQ_1 + Q_1P_0 = u + v + \frac{\lambda}{2}$$

$$SQ_2 + Q_2P_0 = u + v + \frac{2\lambda}{2}$$

$$\vdots$$

$$SQ_n + Q_nP_0 = u + v + \frac{n\lambda}{2}$$

By Pythagoras' theorem we can write

$$SQ_n = \sqrt{SO^2 + OQ_n^2}$$

$$= \sqrt{u^2 + r_n^2} = u + \frac{r_n^2}{2u} + ...$$

where r_n is the radius of the nth zone.

Similarly, you can convince yourself that

$$Q_nP_0 = v + \frac{r_n^2}{2v} + \ldots$$

, If $r_n << u$ or v, we can ignore terms higher than $\frac{r_n^2}{2u}$ or $\frac{r_n^2}{2v}$. Hence

$$SQ_n + Q_n P_0 = u + \frac{r_n^2}{2u} + v + \frac{r_n^2}{2v} = u + v + \frac{n\lambda}{2} \text{ (by construction)}$$

$$\therefore \quad r_n^2 \left(\frac{1}{u} + \frac{1}{v}\right) = n\lambda$$

That is, the radii of the circles are proportional to the square root of natural numbers, as before.

If we identify $\frac{r_n^2}{n\lambda}$ as f_n, the focal length of he zone plate, we find that

$$\frac{1}{u} + \frac{1}{v} = \frac{n\lambda}{r_n^2} = \frac{1}{f_n}$$

which is identical to the lens equation. Therefore, this device behaves like a converging lens with a focal length f = r_n^2 *lnh* and forms *a* real image of S at P_0.

---

2. The zone plate has several foci. To understand this, we assume that the observation screen is at a distance of one focal length from the diffracting aperture. Then it readily follows from the above example that the most intense (first order) focal point is situated at $f_1 = r_1^2/\lambda$. To give you a feel for numerical values, let us calculate f_1 for a zone plate with radii $r_n = 0.1\sqrt{n}$ cm and illuminated by a monochromatic light of wavelength $\lambda = 5500$ Å. You can easily see that

$$f_1 = \frac{r_1^2}{\lambda} = \frac{(0.1 \text{ cm})^2}{5500 \times 10^{-8} \text{ cm}} = 182 \text{ cm}$$

To locate higher order focal points, we note from Eq. (8.2b) that for r_n fixed, n increases as b decreases. Thus for b $= f_1/2$, $n = 2$. That is, as P_0 moves towards the zone plate along the axis, the same zonal area of radius r_1 encompasses more half-period zones. At this point, each of the original zones covers two half-period zones and all zones cancel. When b $= f_1/3$, $n = 3$. That is, three zones contribute from the original zone of radius r_1. Of these, two cancel out but one is left to contribute. Thus other maximum intensity points along the axis are situated at

$f_n = \frac{r_1^2}{n\,h}$ for n odd. For the above numerical example, $f_3 = \frac{182}{3}$ cm, $f_5 = \frac{182}{5}$ cm, $f_7 = \frac{182}{7}$ cm and so on. Between any two consecutive 7 foci, there will be dark points.

8.5 DIFFRACTION PATTERNS OF SIMPLE OBSTACLES

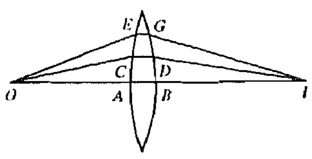
From Sec. 8.3 you will recall that by utilizing Kalhvatc's experimental arrangement, the Fresnel diffraction pattern of various apertures and obstacles could be photographed by varying distances between the source, the object and lhe photographic plate. We will now use results derived in Sec. *8.4* to explain the observed diffraction pattern of simple obstacles like circular aperture and straight edge.

We begin by studying the Fresnel diffraction pattern of a circular aperture.

8.5.1 A Circular Aperture

Refer to Fig. *8.15.* It shows a sectional view of the experimental arrangement in which a plane wave is incident on a thin metallic sheet with a circular aperture. You will note that the plane of the wavefront is parallel to the plane of the metal plate; both being perpendicular to the plane of the paper.

Let us calculate the intensity at a point P_0 situated on the line passing through the centre of the circular aperture and perpendicular to the wavefront. Suppose that the distance between the point P_0 and the circular aperture is ***b***. As discussed earlier, the intensity at the observation point due to the entire uninterrupted plane wavefront is given by Eq. (8.4),



It is instructive to compare the action of a converging lens and a zone plate in forming a real image of an object. Refer to figure below which shows a converging lens. Consider two rays *OABI* and *OCDI*. Taking the refractive index for air and glass as μ_a and μ_g, the optical path lengths of these rays will be $\mu_a OA + \mu_g AB + \mu_a BI$ and $\mu_a OC + \mu_g CD + \mu_a DI$. *The* lens is so designed that these optical paths are equal. This is true for dl other rays, e.g. *OEGI*. Thus different rays starting from *O* reach I in the same phase and form a bright image.

In a zone plate (Fig. 8.14), alternate zones are blocked. Therefore, rays fmm the source *S* after passing through the first, third, fifth, etc. wncs reach the point P_0 with a path difference of X, 2X , 3X , ... and hence reinforce each other. This results in the formation of a bright image at P_0. Obviously, we will get several positions of bright images if the path difference between the successive exposed zones is $n\lambda$.

where a_1, a_2, ... etc. give the contributions due to successive Fresnel zones. Our problem here can be solved by constructing appropriate Fresnel zones and finding out as to how many of these half period elements are transmitted by the aperture. However, it is important to note that for an aperture of a given size, the number of half period elements transmitted may not always be the same. This is because the radii of the Fresnel zones depend upon the distance of point P_0 from O ($r_n = \sqrt{n \lambda b}$). You can easily convince yourself that if the point P_0 is far away from the aperture (b is very large), the radius of the first zone, equal to $\sqrt{\lambda b}$, may be larger than the radius of the aperture. In such a situation, all the secondary wavelets starting even from the entire first zone alone may not be transmitted. That is, the wavelets from a small portion of the first Fresnel zone only are transmitted.

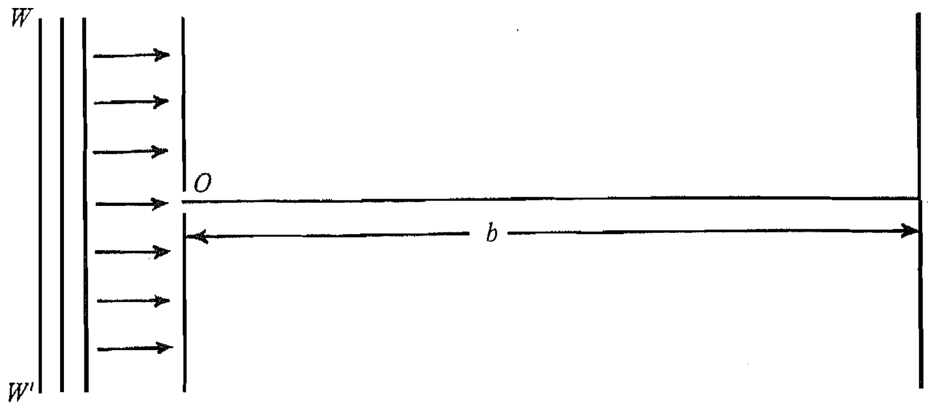


Fig. 8.15: Diffraction by a circular aperture: A cross-sectional view of the experimental arrangement

The next question we have to answer is: How to calculate the amplitude at P_0 when the aperture has transmitted only a fraction of the first Fresnel zone? As a first approximation, we assume that the wavelets arrive at P_0 in phase. (This is quite justified because the path difference between the extreme wavelets within anyone half period elements is $\lambda/2$ and . since only a fraction of the first zone may be transmitting here, the net phase difference will be correspondingly less.) Further, the inverse square law for intensity tells us that the amplitude at P_0 will be inversely proportional to b. Hence, the effect at P_0, which is at a large distance, will be quite small.

As the point P_0 moves towards the aperture (b becomes smaller), the zone size shrinks and a greater part of the central zone is transmitted. As a result, the intensity increases gradually. As the observation point comes closer and closer, with the shrinking of the sizes of zones, a stage may reach when the first zone exactly fills the aperture. Then $\sqrt{b\lambda}$, the radius of the first zone is also the radius of the aperture. We know that the first zone contributes a_1 to the amplitude at P_0. Compare it with the situation where the obstacle with circular aperture is not present. The entire wavefront contributes but the amplitude at P_0 is $\frac{}{2}$. Since intensity is proportional to the square of amplitude, the intensities at P_0 with and without the aperture are respectively a_1^2 and $\frac{a_1^2}{4}$. That is, the intensity at a given point is four times as large when the aperture is inserted in the path than when it is completely removed. This surprising result is not apparent in the realm of everyday experience dominated by rectilinear propagation of light

As the observation point P_0 comes still closer, the circular aperture may transmit first two zones. The amplitude will then be $(a_1 - a_2)$ which is expected to be very small. The additional light produces practically zero amplitude, hence darkness, at P_0. Bringing the point P_0 gradually closer will cause the intensity to pass through maxima and minima along the axis of the aperture depending on whether the number of zones transmitted is odd or even. If we continue to bring the point P_0 closer to O, the number of Fresnel zones transmitted by the aperture goes on increasing. The value $\frac{a_1}{2}$ is finally reached when the point P_0 is so close that an infinitely large number of zones contribute to the amplitude.

The same variation in intensity should be experienced if the point P_0 is kept fixed and the radius of the aperture is varied continuously. This can be done experimentally but is somewhat more difficult.

We have calculated the intensity at points on the axis but the above considerations do not give any information about the intensity at points off the axis. A detailed and complex mathematical analysis which we shall not discuss here, shows that P_0 is surrounded by a system of circular diffraction fringes. Photographs of these fringe patterns have been taken by several workers and we referred to Kathvate's experiments earlier in this unit.

We now illustrate the concepts developed here by solving an example.

Example 3

In an experiment a big plane metal sheet has a circular aperture of diameter 1 mm. A beam of parallel light of wavelength $\lambda = 5000$ Å is incident upon it normally. The shadow is cast on a screen whose distance can be varied continuously. Calculate the distance at which the aperture will transmit 1,2,3, ... Fresnel zones.

Solution

Let $b_1, b_2, b_3, ..., b_n$ be the distances at which 1,2,3, ..., n zones are transmitted by an aperture of fixed radius r. From Eq. (8.2 b) we know that

$$n\, b_n \lambda = r_n^2$$

so that

$$b_n = \frac{r^2}{n\lambda}$$

since r is fixed. Hence

$$b_1 = \frac{r^2}{\lambda} = \frac{(.05\text{ cm})^2}{5 \times 10^{-5}\text{ cm}} = 50\text{ cm}$$

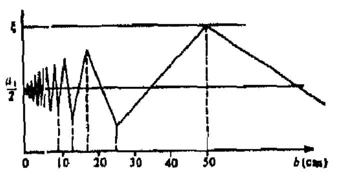


Fig. 8.16: Variation of amplitudes at axial points at different distances from the aperture

Similarly, we find that

$$b_2 = \frac{r^2}{2\lambda} = \frac{50\text{ cm}}{2} = 25\text{ cm},\ b_3 = \frac{50}{3}\text{ cm} = 16.7\text{ cm},\ b_4 = \frac{50\text{ cm}}{4} = 12.5\text{ cm}$$

$$b_5 = 10\text{ cm},\ b_6 = 8.3\text{ cm},\ b_7 = 7.1\text{ cm},\ b_8 = 6.2\text{ cm and so on.}$$

The amplitudes corresponding to these distances are plotted in Fig. 8.16.

Another conclusion of historic interest follows if we substitute the aperture by a circular disc or a round obstacle just covering the first Fresnel zone. The light reaching the point of observation P_0 will be due to all zones except the first. The second zone is therefore the first contributing zone and the intensity of light spot at the centre of the shadow of the obstacle will be almost equally bright as when the first zone was unobstructed.

You may now ask: Why is the bright spot at the centre only? This is because there is no path difference and hence phase difference between waves reaching an axial point. *At* any other off-axis point, waves will reach with different phases and may tend to cancel mutually. The existence of this spot was demonstrated by Arago, though Poisson gave his theoretical arguments to disprove wave theory of light.

You may now like to answer an SAQ.

SAQ 1

Spend 2 min

A coin has a diameter of 2 cm. How many Fresnel zones does it cut off if the screen is 2 m away? Do you expect to see a bright spot at the centre? If we move the screen to a distance of 4 m, how many zones will it cut off? Will the bright spot now look brighter? Why? Take **$\lambda = 5 \times 10^{-7}$ m.**

So far we have discussed diffraction patterns which had axial symmetry: the object or aperture was circular and the plane wavefront originated from a point source. We now wish to consider the case wherein source is a slit source. This source will emit cylindrical waves with the slit as axis. We wish to study the diffraction pattern of a straight edge.

The slit has a very small width compared to its length. Or we may say that in comparison to its width, it has an infinite length.

8.5.2 A Straight Edge

Let S be a slit source perpendicular to the plane of the paper. This sends a cylindrical wavefront towards the obstacle which is a straight edge perpendicular to the paper. You can take a thin metal sheet or a razor blade with the sharp edge parallel to the slit. Fig. 8.17(a) shows a section perpendicular to the length of the slit. The line joining S and E, the point on the wavefront touching the edge of the straight edge, when produced meets the screen at P_0, which is the geometrical boundary of the shadow. Consider any point P on the screen. A line joining it to S cuts the wavefront at R. We wish to know how intensity varies on the screen. This calculation is somewhat complicated because, unlike the previous case, we now have a cylindrical wavefront. Moreover, the obstacle does not possess an axial symmetry.

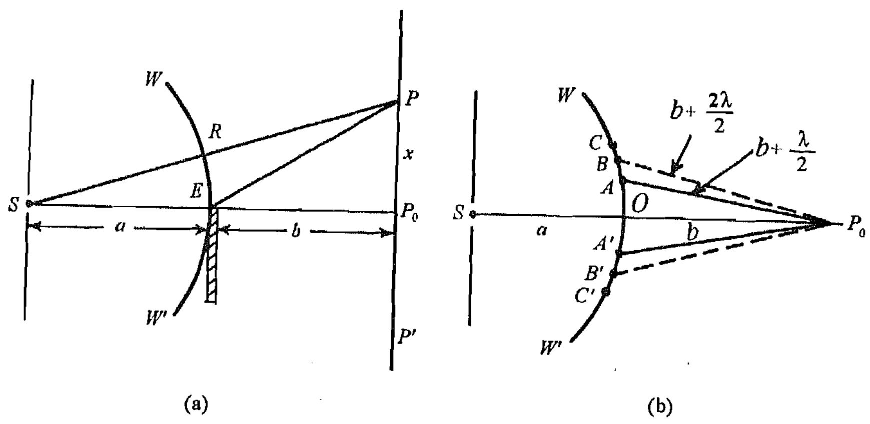


Fig. 8.17: (a) Cross sectional view of the geometry to observe diffraction due to a straight edge and (b) Fresnel construction divides the cylindrical wavefront in half period strips

For a plane wave and obstacles with axial symmetry you know how to construct Fresnel zones. To construct half period elements for a straight edge, we divide the cylindrical wavefront into strips. As before, we make sure in the construction that the amplitudes of the wavelets from these strips arrive at P_0 out of phase by π so that alternate terms are positive and negative. This is achieved by drawing a set of circles with P_0 as centre and radii $b, b+\frac{\lambda}{2}, b+\frac{2\lambda}{2}$,... cutting the circular section of the cylindrical wave at points O, AA', BB', CC', ... Fig. 8.17(b). If lines are drawn through A, A', B, B' etc. normal to the plane of the paper, the upper as well as the lower half of the wavefront gets divided into a set of half-period strips. These half period strips stretch along the wavefront perpendicular to the plane of the paper and have widths OA, AB, BC, ... in the upper half and OA', $A'B'$, B'C', ... in the lower half. You may recall that Fresnel zones are of nearly equal area. For half period strips, this does not hold. The areas of half-period strips are proportional to their widths and these decrease rapidly as we go out along the wavefront from O.

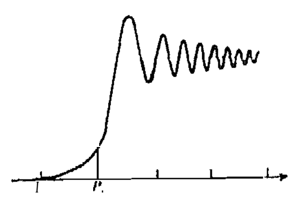


Fig. 8.18: Intensity distribution in the diffraction pattern due to a straight edge

From the geometry of the arrangement it is obvious that on the screen there will be no intensity variation along the direction parallel to the length of the slit. Therefore, the bright and dark fringes will be straight lines parallel to the edge.

A plot of theoretically calculated intensity distribution on the screen is shown in Fig. 8.18. You will note the following salient features:

(i) As we go from a point P' deep inside the shadow towards the point P_0 defining the edge of the shadow, the intensity gradually rises. At P', the intensity is almost zero.

(ii) At P_0, the intensity is one-fourth of what would have been the intensity on the screen with the unobstructed wavefront.

(iii) On moving further towards P, the intensity rises sharply and goes through an alternating series of maxima and minima of gradually decreasing magnitude before approaching the value for the unobstructed wave. This is expected since effect of the edge at far off distances will be almost negligible.

(iv) The intensity of the first maximum is greater than the intensity of unobstructed wave, i.e. it is greater than 4 times the intensity at P_0. Beyond these alternate maxima and minima, there is uniform illumination.

(v) The diffraction fringes are not of equal spacing (as in interference experiments); the fringes gradually come closer together as we move away from the point P_0.

You may now like to know atleast qualitative explanation of these results. From Fig. 8.17 we note that the line joining P and S divides the wavefront into two parts. The amplitude of wave at P is due *to* the part WE of the wavefront, which is completely unaffected by the straight edge. The amplitude at P will be maximum if RE contains odd number of half strips. This will happen if EP $-$ RP = (2n + 1) $\lambda/2$. (When EP - $RP = n\lambda$, the portion RE will contain even number of strips.) As pointed out earlier, the amplitudes due to strips are alternately positive and negative. Therefore, as point P moves away from P_0, the illumination on the screen will pass alternately through maxima and minima when the number of half period strips in RE is 1,2,3,4, ... At P_0, only half of the wavefront EW contributes. Therefore, the amplitude is halved and the intensity is one-fourth of the unobstructed wavefront.

It is worthwhile to ponder as to what pattern the geometry of the experimental configuration throws. We expect dark and bright bands parallel to the edge. However, the dark bands will not be completely dark since the upper half of the wavefront RW always contributes light to this part of the screen.

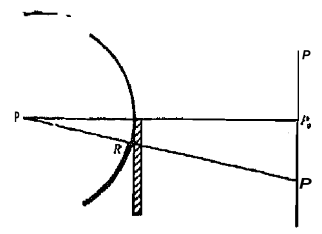


Fig. 8.19: Cross-sectional view of geometry shown in Fig. 8.17(a) when the observation point is in the geometrical shadow

Let us now consider the situation for the point P' inside the geometrical shadow. Refer to Fig. 8.19. You will note that the corresponding point R is shifted below the edge so that the illumination at P' is due entirely to the wavelets from the upper half of the wavefront; the lower portion having been blocked by the edge. Even the upper half' is exposed only in part. If the edge cuts off r strips of the upper half of the wavefront, the effect at P' will be due to (r + 1), (r + 2), (r + 3) etc. strips which may be taken to be equal to one-half of that due to (r + 1)th strip. This will rapidly diminish to zero as shown in Fig. 8.18 because the effectiveness of higher order strips goes on decreasing.

Let us now deduce the width of the diffraction bands. Again refer to Fig. 8.17(a). Suppose that we have the nth dark band at P. Then

$$EP - RP = n\lambda \qquad (8.6)$$

From the ΔEPP_0, if $PP_0 = x$, we have

$$EP = \left(b^2 + x^2\right)^{1/2} = b\left(1 + \frac{x^2}{b^2}\right)^{1/2}$$

$$\cong b\left(1 + \frac{1}{2}\frac{x^2}{b^2}\right) = b + \frac{1}{2}\frac{x^2}{b} \qquad (8.7)$$

where we have retained only first two terms in the binomial series.

From the ΔSPP_0, we can similarly write

$$SP = (a + b) + \frac{1}{2}\frac{x^2}{(a + b)}$$

Hence,
$$RP = SP - SR = b + \frac{1}{2}\frac{x^2}{(a + 0)} \qquad (8.8)$$

and
$$EP - RP = \left(b + \frac{1}{2}\frac{x^2}{b}\right) - \left(b + \frac{1}{2}\frac{x^2}{a + b}\right)$$

$$= \frac{1}{2}\left(\frac{x^2}{b} - \frac{x^2}{a + b}\right) = \frac{x^2\, a}{2b\,(a + b)} \qquad (8.9)$$

For the nth dark band, we get

$$\frac{x^2 a}{2b\,(a + b)} = n\lambda$$

or
$$x = \sqrt{\frac{2b\,(a + b)}{a}\, n\lambda} \qquad (8.10)$$

We therefore find that the distances of the dark bands from the edge of the geometrical shadow are proportional to the square root of natural numbers. Consequently, the bands will get closer together as we go out from the shadow. This fact distinguishes the diffraction bands from the interference bands, which are equidistant.

To enable you to grasp these ideas, we now give a solved example.

Example 4

If in the above experiment a = 30 cm, b = 30 cm and $\lambda = 5 \times 10^{-5}$ cm, calculate the position of the 1st, 2nd, 3rd and 4th minima from the edge of the shadow.

Solution

From Eq. (8.10) we know that the distance of nth minima from the edge of the shadow is given by

$$x = \sqrt{\frac{2b\,(a + b)}{a}\, n\lambda}$$

If we substitute given values of a, b and h and take n = 1.2, 3 and 4, we find that

$$x_1 = \left[\frac{2 \times (30\,\text{cm}) \times (60\,\text{cm})}{30\,\text{cm}} \times (5 \times 10^{-5}\,\text{cm})\right]^{1/2}$$

$$= 7.75 \times 10^{-2}\,\text{cm}$$

$$x_2 = \sqrt{2}\, x_1 = 1.09 \times 10^{-1}\text{cm}$$

$$x_3 = \sqrt{3}\, x_1 = 1.34 \times 10^{-1}\text{cm}$$

$$x_4 = 2\, x_1 = 1.55 \times 10^{-1}\text{cm}.$$

From these values we find that the distance between consecutive minima decreases continuously as we move away from the edge of the shadow.

You may now like to answer an SAQ.

Spend 2 min

SAQ 2

Instead of the straight edge we keep a narrow obstacle, say a wire of diameter 1 mm. What will be the intensity on the screen?

Let us now summarise what you have learnt in this unit.

8.6 SUMMARY

- When the distance between the source of light and the observation screen or both from the diffracting aperture/obstacle is finite, the diffraction pattern belongs to Fresnel class.
- When the screen is very close to the slit aperture/obstacle, the illumination on the screen is governed by rectilinear propagation of light.
- The Fresnel diffraction pattern represents fringed images of the obstacle. Depending on the distance, there can be an infinite number of Fresnel diffraction patterns of a given obstacle/ aperture.
- When plane wavefronts are incident on a diffracting slit and the pattern is observed on a screen effectively at an infinite distance, the diffraction pattern belongs to Fraunhofer type. Unlike the Fresnel diffraction, there is only one Fraunhofer diffraction pattern.

- Fresnel construction for the diffraction pattern from a circular obstacle, when a plane wavefront is incident on it, consists of dividing the wavefront into half period zones.
- The area of each Fresnel half-period zone is nearly equal to $\pi b\lambda$.
- The resultant amplitude due to nth zone at any axial point is given by

$$a_n = \text{Constant} \times \frac{A_n}{b_n} (1 + \cos\theta)$$

- The magnitude of resultant amplitude AB due to the first half period element is $\frac{2}{\pi}$ times the value which would be obtained if all the wavefronts within the half-period element had the same phase.
- The phase of the resultant vector of the first hall. period zone is $\frac{\pi}{2}$ behind the phase of light from the centre of the zone.
- A zone plate is an optical device in which alternate half-period zones are blackened.
- In Fresnel diffraction pattern due to a circular aperture the intensity at an axial point goes through a series of maxima and minima as we vary the distance of the point of observation.
- The diffraction pattern of a straight edge consists of alternate bright and dark bands. The spacing between minima (or maxima) decreases as we move away from the edge of the shadow:

$$x = \sqrt{\frac{2b(a+b)}{a} n\lambda}$$

8.7 TERMINAL QUESTIONS

1. Starting from Eq. (8.4) prove the result contained in Eq. (8.5). Assume that the obliquity factor is such that each term in Eq. (8.4) is less than the arithmetic mean of its preceding and succeeding terms.
2. The eighth boundary of a zone plate has a diameter of 6mm. Where is its principal focal point located for light of wavelength 5000 Å?
3. How many Fresnel zones will be obstructed by a sphere of radius 1 mm if the screen is 20 cm away? Take λ = 5000 Å. If the distance of the screen is increased to 200 cm, what will be the size of the sphere which will cut off 10 zones?

8.8 SOLUTIONS AND ANSWERS

SAQs

1. The radius of the coin is equal to 1 cm. To know the number of zones being obstructed, we use the relation

$$n = \frac{r_n^2}{b\lambda}$$

where $r_n = 1$ cm, $b = 200$ cm and $\lambda = 5 \times 10^{-5}$ cm.

$$\therefore \quad n = \frac{(1\text{ cm})^2}{(200\text{ cm}) \times (5 \times 10^{-5}\text{ cm})}$$

$$= 100$$

You should expect to see a very dim spot at the centre.

When the screen is moved to 4 m, the number of zones being obstructed is given by

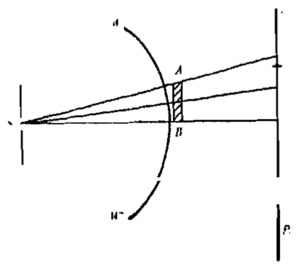


Fig. 8.20: A cross-sectional view of the arrangement for producing diffraction due to a narrow obstacle

$$n = \frac{(1}{(400\ \text{cm}) \times (5 \times 10^{-5}\ \text{cm})}$$

$$= 50$$

The central spot is expected to be somewhat brighter. Does it not appear to contradict the inverse square law?

2. Refer to Fig. 8.20 A point P_1 outside the geometrical shadow is similar to such a point in the straight edge. So we will have unequally spaced bright and dark fringes parallel to the wire on each side of the shadow. What is the intensity at Q inside the shadow? It is simply half the effect of the first half period strip on either side of the thin wire. It will show equally spaced fringes inside the shadow.

TQs

1. We rewrite Eq. (8.4) as

$$a(P_0) = \frac{a_1}{2} + \left(\frac{a_1}{2} - a_2 + \frac{a_3}{2}\right) + \left(\frac{a_3}{2} - a_4 + \frac{a_5}{2}\right) + \frac{a_5}{2} + \ldots \qquad \text{(i)}$$

When n is odd. the last term would be $\frac{a_n}{2}$. We are told that obliquity is such that each term is less than the arithmetic mean of its preceding and succeeding terms i.e., $a_n < \frac{1}{2}(a_{n-1} + a_{n+1})$. Then the quantities in the parentheses in (i) will be positive. So when n is odd, the minimum value of the amplitude produced by consecutive zones is given by

$$a(P_0) > \frac{1}{2}(a_1 + a_n)$$

To obtain the upper limit, we rewrite Eq. (8.4) as

$$a(P_0) = \left(a_1 - \frac{a_2}{2}\right) - \left(\frac{a_2}{2} - a_3 + \frac{a_4}{2}\right) - \left(\frac{a_4}{2} - a_5 + \frac{a_6}{2}\right) - \ldots - \frac{a_{n-1}}{2} + a_n \qquad \text{(ii)}$$

Following the argument used in obtaining the lower limit on the amplitude, we find that the upper limit is

$$a(P_0) < a_1 - \frac{a_2}{2} - \frac{a_{n-1}}{2} + a_n \qquad \text{(iii)}$$

Since the amplitudes for any two adjacent zones are nearly equal, we can take $a_{n-1} \cong a_n$. Within this approximation

$$a(P_0) < \frac{a_1 + a_n}{2} \qquad \text{(iv)}$$

The results contained in (ii) and (iv) suggest that when n is odd, the resultant amplitude at P_0 is given by

$$a(P_0) = \frac{a_1 + a_n}{2} \qquad \text{(v)}$$

Following the same method, you can readily show that if n were even

$$a(P_0) = \frac{a_1 - a_n}{2} \qquad \text{(vi)}$$

2. $D_8 = 0.6$ cm so that $r_8 = 0.3$ cm. We know that

$$f_n = \frac{r_n^2}{n\lambda}$$

$$f_8 = \frac{r_8^2}{8\lambda} = \frac{(0.3\ \text{cm})^2}{8 \times (5 \times 10^{-5}\ \text{cm})}$$

$$= 2.25 \times 10^2\ \text{cm}$$

$$= 225\ \text{cm}$$

3.a) The radius of a Fresnel zone is given by

$$r_n = \sqrt{n \lambda b}$$

Here we are told that $r_n = 0.1$ cm, b = 20 cm and $\lambda = 5 \times 10^{-5}$cm.

$$\therefore \quad n = \frac{r_n^2}{b\lambda} = \frac{10^{-2}\ \text{cm}^2}{(20\ \text{cm}) \times (5 \times 10^{-5}\ \text{cm})} = 10$$

(b) In this part we have to calculate r_n for given values of n = 10, b = 200 cm and $\lambda = 5 \times 10^{-5}$cm:

$$r_n = \sqrt{10 \times (200\ \text{cm}) \times (5 \times 10^{-5}\ \text{cm})}$$

$$= 0.32\ \text{cm}$$