UNIT 11 VECTORS IN MECHANICS

Structure **Page Nos.**

11.1 INTRODUCTION

In this unit we have focussed on various aspects related to studying vectors that students meet in 'mechanics', like displacement, velocity, acceleration and force. We start with Sec. 11.2, in which we discuss strategies for explaining to learners the difference between scalars and vectors. One point made here is that though a vector remains unchanged when shifted parallel to itself, its physical effect can change when its point of application shifts.

Next, in Sec. 11.3, we have devoted quite some space and time to ways of presenting the concept of relative velocity. This is an important fundamental concept for mechanics and is confusing to many students.

Finally, in Sec. 11.4, we have focussed on ways of introducing Newton's laws of motion to the students. We suggest, for example, that all three laws must not be introduced in one go, but as and when the related concepts come up. For instance, the first law can be presented when you discuss about the change in velocity and what brings about this change, i.e., the external force. Then, once the concepts of acceleration, linear momentum and its rate of change with respect to time have been introduced, we suggest that Newton's second law could be introduced. Finally, through several examples of bodies coming into contact, the forces of action and reaction could be introduced, leading to the introduction of the third law of motion.

We hope that this unit would help you to teach your learners vectors in the context of mechanics in a way that would make them comfortable with the basic concepts involved.

The discussion in this unit will be continued in the next unit on moments and couples.

Objectives

After reading this unit, you should be able to improve the ability of your learners to

- explain the difference between scalars and vectors;
- identify situations in which the operations of addition and multiplication of vectors are to be applied, and to apply them correctly;
- solve problems related to relative velocity;
- solve problems related to the applications of Newton's laws of motion.

11.2 VECTORS AND SCALARS

Let us start with what most of your students would be familiar with. They have come across vectors like force, displacement and velocity. However, many students are unclear about **the difference between vectors and scalars**. They need to realise that a scalar is a quantity like the amount of time spent, the distance travelled, the pressure applied, the amount of interest paid, and so on. These quantities require only magnitude to describe them. But a vector is a quantity that needs two things to describe it — a magnitude, and a direction. Here you could give them an example of speed vs. velocity to help them see the difference.

Many students tend to use 'speed' and 'velocity' interchangeably. How can you help them realise that velocity is a vector, whose **magnitude is the speed**? For one, you could expose your learners to several real-life examples and situations involving these two concepts. For instance, ask them to consider a situation of a car moving from one point to another, and back. In this case, ask them to find the average speed and the average velocity. Ask them the difference between the two. Are they able to tell you that the average speed is not zero while the average velocity is zero?

You could then ask your students to write other examples of vectors and scalars. You could also give them a list of entities, like mass, weight, temperature, etc., and ask them to identify which of these are scalars and which are vectors.

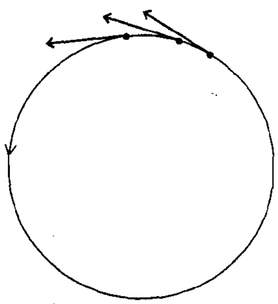


Fig. 1

As another example, you could ask them to consider a particle describing a circle with uniform speed, i.e., constant speed. Although the particle is moving along the circle with constant speed, the direction of motion keeps changing. Therefore, the velocity of the particle is not uniform (constant) (see Fig. 1). The students must understand why we say that the direction keeps changing. Do they realise that at each instant the velocity of the particle is along the tangent to the circle at that point? Therefore, it changes its direction from one instant to the next. The point here is that the velocity is changing all the time while the speed remains the same.

Yet another example to show the students the relationship between speed and velocity is the slowing down of a car moving along a straight road. In this case, the direction of the car does not change but its speed slowly reduces. You can also ask your learners to think of other examples and situations, as suggested below.

E1) Give examples of real-life situations in which:

- i) the speed is constant, but the velocity is not constant;
- ii) the direction is constant, but the speed is changing, so that the velocity is not constant;
- iii) both speed and direction are changing, so that the velocity is also changing.

E2) In the following situations, is the velocity uniform? Why, or why not?

- i) The movement of the earth around the sun;
- ii) The earth rotating about its axis.

E3) A stone is thrown up and allowed to come down. At the mid-point, P, of its ascent and descent, is the velocity the same? What about speed?

E4) Which of the following quantities are scalars, and which are vectors?

i) Mass, ii) weight, iii) pressure, iv) temperature,
iv) displacement of the hands of a clock.

Give reasons for your answers.

A major difference between scalars and vectors is regarding the operations on them. The addition and multiplication of scalars obey the rules of the corresponding operations of numbers. So, students don't err in this respect. However, when it comes to the **addition of vectors**, students make many mistakes. For instance, if $\mathbf{v}_1$ is 40 km/hr southwards and $\mathbf{v}_2$ is 35 km/hr eastwards, many students write one of the following for $\mathbf{v}_1 + \mathbf{v}_2$:

75 km/hr southwards
75 km/hr eastwards
75 km/hr in the southeast direction

We need to give them real-life situations, and models, to explore and apply the triangle (or parallelogram) law of addition of vectors. To start with, you could help them recall movement along the number line as

$(+5) + (+3) = +8, (+5) + (-9) = ?, \ldots.$

Then you could ask them to draw diagrams of situations like the following, and then try and answer them.

If you walk 10 steps northwards, followed by 5 steps eastwards, how far from the original point are you, and in what direction?

How did they solve this? Did they use the Pythagoras theorem and/or trigonometric functions? You can now explain what is happening in the problem in terms of vector addition.

In this context, you could also ask your learners to explain why any vector can be thought of as the resultant of 3 vectors orthogonal to each other along the axes. You could give them several exercises to write the resultants of vectors, and to resolve them into components along the axes.

Why don't you do the following exercises now?

E5) Why do students need to understand when two vectors are equal before they can understand why the addition of vectors follows the parallelogram law?

E6) List three situations you would give students for exploring the operation of addition of two **or more** vectors.

E7) If a student answers that the resultant of 10 km west and 5 km south is in the direction $\tan^{-1}(\frac{1}{2})$, what would your reaction be?

Let us now come to **vector multiplication**. As you know, students don't seem to realise that this can be done in two ways, one resulting in a scalar and the other in a vector. Then, they don't know which is to be used when. And, to compound the confusion, they don't see why either should be applied at all. Let us consider both the products, and the related issues one by one.

Scalar (or **Dot) Product** : How do you introduce this form of vector multiplication to your students? Do you present it as a natural extension of scalar multiplication?

When it is defined as $\mathbf{a.b} = a_x b_x + a_y b_y + a_z b_z$, in terms of the components along the axes, it seems a very obvious extension.

But, when it is also defined as $\mathbf{a.b} = ab \cos\theta$, where θ is the angle between a and b, the students do not see how the two are equal. Here is where you need to remind them of direction cosines, and then ask them to try and prove that the two definitions are equal.

Once the students see that the dot product **a.b** = ab cos θ, this gives them an easy way to find the angle θ. What you need to stress here is that though **two vectors are being multiplied, the product is not a vector.**

There is a lot more you need to do to help students become comfortable with dot products. For instance, they wonder why they should study it at all. Of course, while studying physics, they would have come across the scalar product being used for calculating work done. But they just apply the formula without understanding why it is appropriate. How could you help them understand the physical meaning of work done by a body? Do they see that it is the amount of energy spent in displacing the body through a distance by a force acting on it?

For work to be done, the displacement and the force have to be in the same direction. For example, if you are holding two bricks in your hand and standing, you are doing no work against gravity. If you are holding up the bricks and walking on a flat ground, then the only work you are doing is against the force of friction. Only when you lift up a brick, you do work against gravity. When force and displacement are in the same direction, the magnitude of the work done is the product of the magnitudes of the force and displacement. Similarly, when the force is orthogonal to the displacement, the work done is zero. This is why physicists define the amount of work done to be the product of the collinear components of force and displacement. This is what the dot product **F.s** is, where **s** is the displacement vector. If no component of **F** is along **s**, then the work done is 0. This is because the dot product gives the components of displacement along the direction of the force.

You could give your students varied exercises for finding the components of force and displacement, and finding their dot product. A good source for such exercises is the webpage *http://hyperphysics.phy_astr.gsu.edu/hbase/vsca.html*

Try this exercise now.

E8) How would you explain to a student the need for the dot product, and its utility?

Let us now consider the other kind of multiplication of vectors.

Vector (or **Cross**) **Product** : This is a major source of confusion and misunderstanding for students. They usually rote-learn the formula, without understanding why this kind of product has been thought of. You would need to show them how this is merely an extension of multiplication in **C**. Also, the relationship of the direction of $\mathbf{a} \times \mathbf{b}$ with the directions of **a** and **b** has been chosen so as to extend $\mathbf{R}^2$ to $\mathbf{R}^3$. The students know that the direction of the axes in $\mathbf{R}^3$ are chosen according to the movement of a right-handed screw. In the same way, we define the direction of $\mathbf{a} \times \mathbf{b}$ to be perpendicular to the plane of **a** and **b** in a direction given by the right-hand rule. If students see these connections, it is easy for them to see why several properties like $\mathbf{a} \times \mathbf{b} \neq \mathbf{b} \times \mathbf{a}$ hold.

Another source of confusion about this product is remembering the components of $\mathbf{a} \times \mathbf{b}$ along the axes. This could be sorted out if the students write it according to the determinant rule, namely,

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix},$$ where a_x, etc., have the obvious meanings.

Students do wonder about what use this product is. Think about this while doing the following exercise.

E9) Which applications of the vector product can you motivate your students with?

Another problem related to the difference between scalars and vectors is related to finding average and instantaneous change in velocity (a vector) with respect to time (a scalar). We have discussed this in Block 2, Calculus. Here we shall look at a related concept.

11.3 RELATIVE VELOCITY

When a person usually talks of the movement of a vehicle, she says, for instance, that the bus was going at a **speed** of 40 km/hr. Does she realise that she's only referring to the magnitude of the motion, and not its direction? Does she also realise that this speed (or velocity) is w.r.t. the surface of the earth?

In fact, our students have quite a bit of difficulty with understanding that the velocity of a particle or a body is always with reference to some other body, which may be stationary or in motion. For example, when we talk of a train moving with a velocity of 80 km/hr eastwards, we mean the velocity of the train **with reference to the surface of the earth**. And, the earth is also moving. However, we usually do not take this into consideration when we give the velocity of the train.

To help your learners understand relative motion, you could give them a variety of exercises and activities to do. For instance, you could ask them to consider situations like the following one, and answer questions like the ones given after it.
Suppose you are sitting in a bus which is moving with a velocity of 60 Km/hr northwards. Then your velocity relative to the bus is zero, because you are sitting in it. However, you are travelling, along with the bus. So, you are moving in relation to the earth with a velocity of 60 Km/hr northwards, the same as the velocity of the bus relative to the earth.

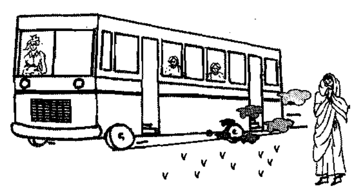


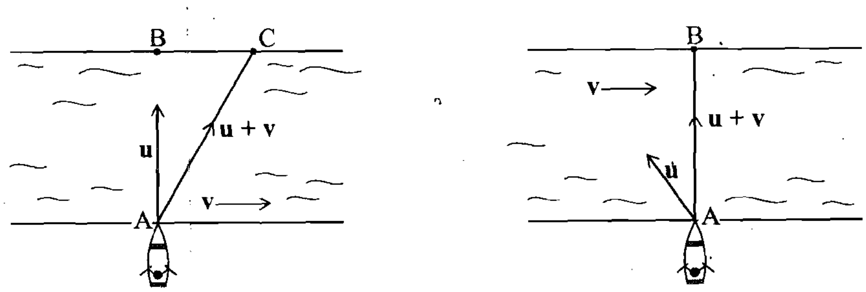
Fig. 2

Further, suppose there is a person standing in a field by the side of the road. She sees that the bus, and you sitting in it, are moving northwards at 60 Km/hr. But sitting in the bus, your perception will be that she is moving at the rate of 60 Km/hr ***southwards****. In other words, although the person in the field is stationary w.r.t. the earth, she is in motion relative to the bus or you sitting in the bus.*

Q1) What is your velocity with respect to a person sitting in a stationary car on a road which the bus is going on? What is the velocity of this person relative to the person in the field?

Q2) If another bus is moving at a velocity of 30 Km/hr northwards, what is its velocity relative to you?

Another example you could give your learners is to consider the situation in which they are going for a walk, and a strong wind is blowing. Ask them which way it is easier for them to walk — against the wind, or in the same direction. Why is this so? Can they explain to you that if **u** is the velocity with which a person is walking when there's no wind, and the velocity of the wind is **v**, then her velocity relative to the earth will be **u** + **v** in the direction of the wind, and **u** – **v** in the opposite direction?
Yet another example could be that of a boat crossing a flowing river. The velocity of the river is **across the movement of the boat** and is pushing it in the direction of the water (see Fig. 3a). Ask your learners what would happen to the rower who tries to go straight across from one bank to the other. If the boat is rowed in a direction at right angles to the current of the river, then it will not reach the point B but will land up at some point C further downstream.

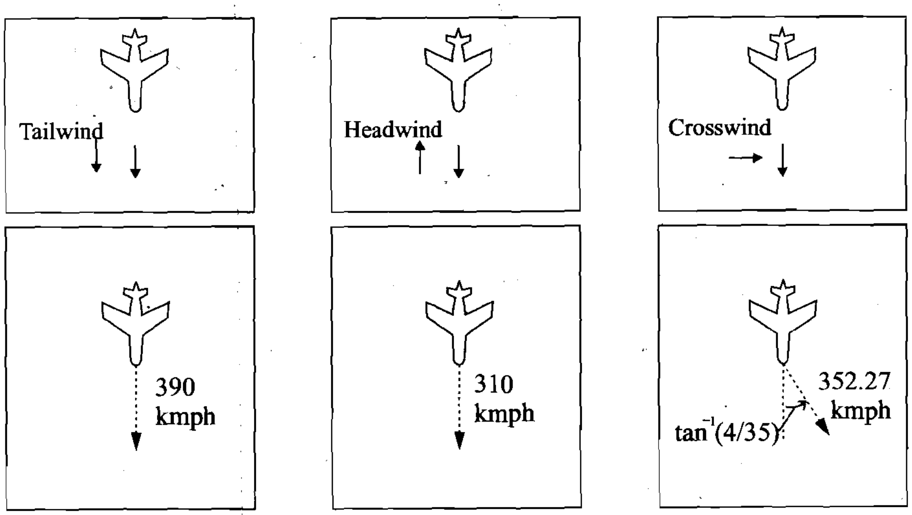


(a) (b)

Fig. 3

Again, if the rower has to reach the point B, then in which direction should she row? Can the students tell you that it should not be directly across, but across and upstream, as shown in Fig. 3b? The downstream push received by the boat would be balanced by the upstream movement. In this context, some student remarked that the velocity needed by the boat to go straight across would be **u** – **v**. Ask your students if this is so.

Another example of relative motion is that of an aeroplane being affected by the velocity of the wind. Observe the three planes in Fig. 4. Each plane is heading south with a velocity of 350 km/hr. Each plane flies amidst a wind which blows at 40 km./hr. In Fig. 4(a), the plane encounters a tailwind (from behind) of 40 km/hr. The combined effect of the tailwind and the velocity of the plane is a resultant velocity of 390 km/hr. In Fig. 4(b), the plane encounters a headwind (from the front) of 40 km/hr. The combined effect of the headwind and the velocity of the plane is a resultant velocity of 310 km/hr. In Fig. 4(c), the plane encounters a crosswind (from the side) of 40 km/hr. The combined effect of the headwind and the velocity of the plane is a resultant velocity of 352.27 km/hr = $\sqrt{(350)^2 + (40)^2}$ km/hr (directed at an angle of $\tan^{-1}(40/350)$ degree east of south). These three resultant velocities can be determined using simple rules of vector addition, and the Pythagorean Theorem. Because of the effect of the wind, the pilot of the plane has to adjust the speed and direction of the plane to account for this factor.



(a) (b) (c)

Fig. 4

You could also give your learners some exercises like the following ones to do.

E10) A river flows at the rate of 5 km/hr. Find the velocity of the boat in still water, if the time taken by the boat to travel some distance upstream is double the time taken by the boat to travel the same distance downstream.

E11) Two vehicles are moving in the same direction with velocities **u** and **v**, respectively. What is the velocity of the first vehicle relative to tne second one? If the two vehicles were moving with velocities **u** and **v** in opposite directions, then what would the velocity of the first be relative to the second one?

E12) A river is flowing at a velocity of 2 km/hr. If a boat is rowed at a speed of 4 km/hr so as to reach a point on the other bank opposite to the initial point, find the direction in which the boat must be rowed.

E13) A wind is blowing with a velocity of 40 km/hr westwards. At what velocity should a plane fly eastwards, so as to cover a distance of 300 km. in 30 minutes?

What we have discussed here can be supplemented with the interactive webpage *physicsclassroom.com/mmedia/vectors.*

Let us now come to another area of mechanics that tends to be treated very mechanically!

11.4 APPLYING NEWTON'S LAWS OF MOTION

Most of us introduce our learners to the three laws of motion by 'laying down the laws'! It may be better to introduce them through situations in which the laws apply and by getting students to analyse the situations to get a feel for the laws. Also, all three needn't , and shouldn't, be introduced to the students at the same time. They could be brought in as and when the concepts involved are discussed.

For instance, when we discuss velocity and acceleration, we find that many students tend to confuse the two. Over here, you could bring in **Newton's first law**, also called **the law of inertia**. It states that
"A body continues in its state of rest or of uniform motion along a straight line unless it is acted upon by an external force."

You could use this law to ask your students **whether zero acceleration means that a body has zero velocity**. Many students think that this is so. A good way to help them to realise their misconception is to explore situations like a car moving with uniform velocity and then stopping. This is a situation of positive velocity and negative acceleration. Here, you could ask questions which help your students focus on what it is that brings about a change in the velocity.

Negative acceleration is also referred to as **retardation.**

From this, via Newton's first law of motion, you could lead them towards looking at force as a vector. The students need to think about why force is a vector, and what the implications are of this being a vector. You could discuss with the students their pre-conceived notions about force, and match them with the concept of force in physics. You could ask them to explore the forces acting in different situations like a stone being thrown, or a stone hanging at the end of a string, etc.

While they're understanding 'force', your students could be asked questions designed to focus their attention on the relationship between the force applied, the mass of the particle or the body it is applied on and the change in the velocity produced as a result of this application. They could do activities like applying the same force to two bodies of different masses, and recording any consequent changes in velocity. Similarly, they could consider what happens to the velocity when they apply the same force to different points of the same body. In this way, you could lead your students to the statement of **Newton's second law of motion**, which states that

"The rate of change of momentum of a body is directly proportional to the force applied, and takes place in the direction of the force."

The students need to note that **Newton assumed that the mass of the body on which force is applied is constant.** Also, they must realise that a body with constant linear momentum is not acted upon by any net force, and is also not exerting any net force.

If the students have understood what the law means, you could ask them to try and formulate the law mathematically, rather than giving them the formulation. This would be very useful for helping them to check their own understanding. Are they able to arrive at

$$\mathbf{F} \propto \frac{d}{dt}(m\mathbf{v}),$$

where **F** is a force applied on a body of mass m moving with a velocity **v** at time t?

As we've mentioned earlier, in Newtonian mechanics, the mass m is assumed to be constant. You can explain to your students how, with appropriate units of force, (constant) mass, velocity and time, we get

$$\mathbf{F} = \frac{d}{dt}(m\mathbf{v}) = m\frac{d\mathbf{v}}{dt} = m\mathbf{a}.$$

Many students may be familiar with this statement. In fact, when asked to state Newton's second law of motion, many students answer 'Force equals mass into acceleration'. They need to realise that this is not the law, but **a derivation** from the law.

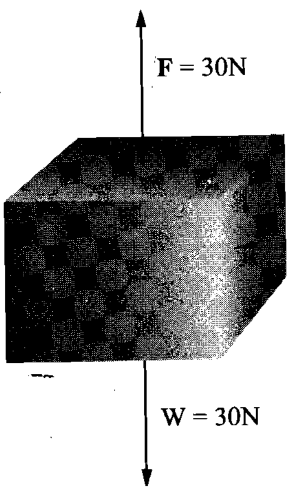


Fig. 5

Also, many students don't look at both the laws together. What they need to see is that Newton's laws say that

i) a net force causes acceleration (**note** that the force doesn't **cause** motion); and
ii) the acceleration is directly proportional to the force, in the same direction as the net force.

Now let us see the basic problem students face when trying to solve problems involving the second law. They are confused about which forces are acting, and the outcome. For instance, when I asked some Class 12 students if the block shown in Fig. 5 could be moving , all except two of them said it had to be at rest. The two exceptions explained that it could be moving in a horizontal direction. But the other students argued that since only vertical forces were acting on the block, it could not be moving horizontally!

E14) Which of the two arguments do you agree with? How would you convince the students about your argument?

When students are doing problems related to the application of Newton's laws, the first thing is to ask them to identify all the forces acting on the body under consideration. Of course, it is useful if they are able to draw a schematic diagram showing the body as a particle, and showing all the forces as vectors acting on the particle. This will help them visualise, analyse and solve the problem. Consider the following example.

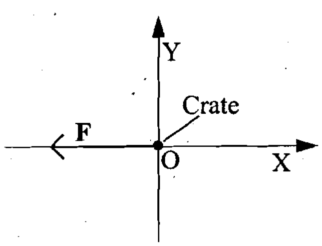


Fig. 6

Problem 1 : A crate with mass 300 kg lies in the back of a tempo that is moving at a speed of 60 km./hr. The driver applies brakes so that the vehicle slows down to 30 km./hr. in 10 seconds. What force (assumed constant) acts on the crate during this time? (Assume that the crate does not slide.)

Solution : First, let us draw the situation schematically, as in Fig. 6. Here $\mathbf{v}_0 = 60$, $\mathbf{v} = 30$, $t = 10$. Therefore, the acceleration,

$$\mathbf{a} = \frac{\mathbf{v} - \mathbf{v_0}}{t} = -3\left(\frac{1}{3600}\right)(1000)\,\mathrm{m/s^2} = \frac{-5}{6}\,\mathrm{m/s^2}.$$

So, applying Newton's second law, we get

$$\mathbf{F} = 300\left(\frac{-5}{6}\right)\mathrm{N} = -250\mathrm{N}.$$

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You could give them exercises like the following ones to do also.

E15) Suppose you push a crate with mass 240 kg for a distance of 2.3 m over a frictionless surface, exerting a constant horizontal force of 130 N. Show this situation diagrammatically. Further, if the crate starts from rest, what is its final velocity?

E16) a) Give at least two examples where the mass of a moving body does not remain constant.

b) In the case of an object losing mass, what equation relating **F**, m and **a** would we get?

Having discussed with your learners the two laws of motion, you could give them a few examples of daily life for leading them on to **Newton's third law of motion**, namely,

"To every action there is an equal and opposite reaction."

For instance, ask them what happens if a ball is thrown vertically downwards with some force. They know that it will hit the ground and bounce back to some height. Next, ask them what happens when it is thrown down with more force. Do they say that it is likely to bounce back to a greater height? Ask them, "Why does the ball bounce back? What are the various forces acting on the ball?"

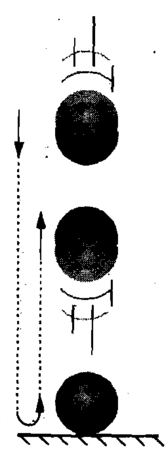

Fig. 7 :The ball bouncing away from the ground due to a reaction.

One of the teachers explained the situation to her students by saying, "You know that the ball was thrown to the ground with force. It is further accelerated by the gravitational force of the earth. As it comes into contact with the ground it exerts a force on the ground. To **this action** of the ball hitting the ground and exerting a force, **a reaction** takes place with the ground forcing the ball away (see Fig. 7). As a result of this reaction the ball bounces up again. If you throw the ball at the ground with a greater force, it will bounce back to a greater height. So, a greater force of action of the ball on the ground produces a greater reaction of the ground on the ball. But, a **warning**! This is as long as we can consider the ball and the ground on which it falls to be hard elastic surfaces. What do you think will happen if the ball is soft? Will it bounce back?" She gave them a few days to think about why it would not. After this she had a discussion with them, in which she explained to them the reason for it not bouncing was that the area of contact with the ground is large and a lot of energy gets dissipated.

As an activity regarding action and reaction, you could ask your learners to press the top of their desks with their hand. As they do it, ask them what they experience. Do they feel that the top of the desk, in turn, presses their hand?

You could also ask your students to consider a bullock pulling a cart, and find out who is applying the action, and on whom, and where the reaction is. Similarly, ask them to consider the situation of a boat being pushed by a person on to the shore and find out the action and reaction.

To check their understanding of forces acting in pairs, you could do exercises like the following one with them

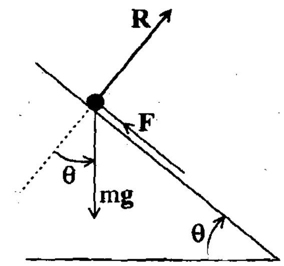

Fig. 8

E17) A ball of mass m is kept at rest on a smooth inclined plane by an external force **F** applied to it in an upward direction of the plane (see Fig.8). Find **F**.

There is yet another way in which the forces of action and reaction are seen — in connection with the concept of tension in a string. Students have studied these forces in Class 11. However, there are a few misconceptions they have about them. These are:

1) They think that the action and reaction are on the same body. The point is that if A applies a force $\mathbf{F}_{AB}$ on B, then B will apply the force $\mathbf{F}_{BA}$ on A, where $\mathbf{F}_{AB} = -\mathbf{F}_{BA}$. Therefore, if the motion of only one of these bodies, say A, is being considered, then we only consider the force $\mathbf{F}_{BA}$, which affects the motion of A. Students make an error here because they add up both $\mathbf{F}_{AB}$ and $\mathbf{F}_{BA}$, which results in a zero net force.

2) Students wonder which force is action, and which is reaction. This is not important. What is important is that when the two bodies exert mutual forces on one another, the two forces are equal and opposite.

3) Students wonder where, and when, tension comes into the picture — is it a reaction to the force by which the string or spring is pulled?

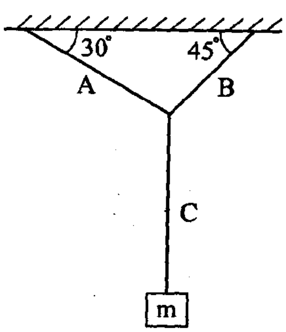

Fig. 9(a) : A block hangs from three strings, A, B and C.

All these confusions can be sorted out by asking students to discuss the forces in different situations — a book placed on a table, a person walking, a crate sliding on a smooth surface, objects hung by a string, and so on. You could do some problems of the following kinds with them too.

Problem 2 : A block of mass 15 kg is hung by 3 strings, as shown in Fig. 9(a). Find the tension in each string.

Solution : We consider the knot at the junction of the three strings to be 'the body'. Fig. 9(b) shows the free-body diagram of the knot, which remains at rest under the action of the three forces T_A, T_B and T_C, which are the tensions in the three strings. (We assume that, like the string, the knot is massless, so that its weight does not appear in the diagram.) Choosing the x and y axes as shown in Fig. 9(b), we can resolve the forces into their x and y components. The acceleration components are zero, so we can write:

$$\Sigma \mathbf{F}_x = \mathbf{T}_{Ax} + \mathbf{T}_{Bx} = m\mathbf{a}_x = \mathbf{0},$$
$$\Sigma \mathbf{F}_y = \mathbf{T}_{Ay} + \mathbf{T}_{By} + \mathbf{T}_{Cy} = m\mathbf{a}_y = \mathbf{0}.$$

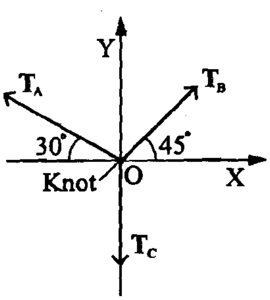

Fig. 9(b) : The free-body diagram of the knot in Fig. 9(a).

From Fig. 9(b), we see that

$$\mathbf{T}_{Ax} = \mathbf{T}_A \cos 30^\circ = -0.866\mathbf{T}_A,$$

$$\mathbf{T}_{Ay} = \mathbf{T}_A \sin 30^\circ = 0.5\,\mathbf{T}_A,$$

$$\mathbf{T}_{Bx} = \mathbf{T}_B \cos 45^\circ = 0.707\,\mathbf{T}_B$$

$$\mathbf{T}_{By} = \mathbf{T}_B \sin 45^\circ = 0.707\,\mathbf{T}_B,$$

$$\mathbf{T}_{Cx} = \mathbf{0},$$

$T_{Cy} = -T_C$.

Let us examine the free-body diagram of the mass m, shown in Fig. 9(c). Only the y components are involved, and again the acceleration is zero. So, we get

$T_{Cy} - mg = ma_y = \mathbf{0}.$

Also, $T_C = mg = (15.0 \text{ kg})(9.8 \text{ m/s}^2) = 147$ N.

So, equating the x and y components of the forces on the knot, we get

$-0.866\, \mathbf{T}_A + 0.707\, \mathbf{T}_B = \mathbf{0};$
$0.5\, \mathbf{T}_A + 0.707\, \mathbf{T}_B - \mathbf{T}_C = \mathbf{0}.$

Substituting the value for T_C and solving the two equations simultaneously, we get

$\mathbf{T}_A = 108$ N, $\mathbf{T}_B = 132$ N.

Check to see if the vector sum of the three forces is indeed zero.

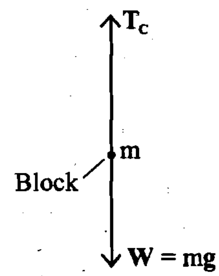

Fig. 9(c) : The free-body diagram of the mass in Fig. 9(a).

Please train your students to check the results in all problems.

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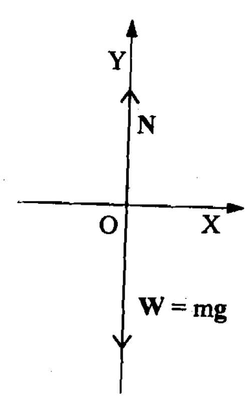
Problem 3 : A passenger of mass 50 kg is riding in an elevator while standing on a platform scale (see Fig. 10(a)). What does the scale read when the elevator is

i) descending with constant velocity?
ii) ascending with acceleration 3 m/s^2?

Solution : Let us first see what happens in the most general case, i.e., for any vertical acceleration **a**. Let us choose our frame of reference to be outside the elevator (the fixed elevator shaft of the building, for instance), because an accelerating elevator is not an inertial reference frame. Both **g** and **a** are measured with respect to this external frame of reference. Fig. 10(b) shows the free-body diagram of the passenger.

(a)



(b)

Fig. 10

There is the downward force of the weight, mg and the upward normal force, **N**, exerted by the scale. The normal force is exerted **by** the scale **on** the passenger; the scale reads the downward force exerted **by** the passenger **on** the scale. By Newton's third law, these are equal in magnitude. So, if we can find **N**, we know the scale reading.

From Fig. 10(b), we have

$$\Sigma \mathbf{F}_y = \mathbf{N} - m\mathbf{g} = m\mathbf{a}\text{, i.e., } \mathbf{N} = m(\mathbf{g}+\mathbf{a}).$$

Now, let us consider the situation (i). Since it is descending with constant velocity, $\mathbf{a} = \mathbf{0}$. So,

$$\mathbf{N} = m\mathbf{g} = (50\text{ kg})(9.8\text{ m/s}^2) = 490\text{ N}.$$

When $\mathbf{a} = 3\text{ m/s}^2$, as in (ii), we have

$$\mathbf{N} = m(\mathbf{g} + \mathbf{a}) = (50\text{ kg})(9.8\text{ m/s}^2 + 3\text{ m/s}^2) = 640\text{ N}.$$

The scale reading, which indicates the normal force with which the floor is pushing up on the passenger, increases when the elevator is accelerating upwards (**a** is positive according to the coordinate system we have defined) and decreases when it is accelerating downwards. In free fall (i.e., $\mathbf{a} = -\mathbf{g}$) the scale reading is zero (there is no normal force).

———×———

Why don't you try some problems now, yourself, and with your students?

E18) Sketch a diagram showing the forces in each of the following situations. Clearly show which force acts on which object, and their magnitudes and directions.

i) A freely falling 1000 kg rock;
ii) A 1000 kg rock resting on the ground;
iii) A car crashing into a wall;
iv) A person standing on a weighing machine.

You could also look at Chapter 5 of the Physics textbook for Class 11, NCERT, 2002, where issues related to this section are discussed.

Let us now end our discussion on vectors in mechanics, with a brief overview of what we have covered in it.

11.5 SUMMARY

In this unit we have stressed the following points.

1. The difference between a scalar and a vector is not clear to students. Suggestions have been given about how to remove their confusion.

2. Some strategies have been suggested for helping students remove their misconceptions regarding the addition, scalar product and vector product of vectors.

3. Examples, activities and exercises for explaining 'relative velocity' have been discussed.

4. Ways of presenting Newton's laws of motion and their applications have been suggested. In particular, we have focussed on the difficulties learners have about identifying the forces to be considered in a given situation. Some suggestions for helping your students overcome these difficulties have also been given.

11.6 COMMENTS ON EXERCISES

E1) i) For instance, a cyclist moving eastwards, and then westwards after some time.

ii) Some examples are:

- a ball falling under the force of gravity;
- a ball thrown vertically upwards, and reaching a certain height;
- a cylinder rolling down an inclined plane.

iii) Some examples are :

- a shotput thrown in a direction inclined to the horizontal;
- an object dropped from a moving vehicle;
- a tennis ball hit by a racket, over the net.

E2) The direction keeps changing in both cases. Therefore, the speed is constant, but not the velocity.

E3) When the stone is thrown up, it slows down. Its speed and velocity reduce. The velocity at P, however, is still upward. After it reaches the maximum height, the stone starts coming down. At P the velocity is now in the downward direction. The speed increases from the zero speed it has at the maximum height, but is less than the speed at which it had left the hand. If you calculate it, the speed at P will turn out to be the same as the speed of the stone going up at P.

E4) (i) and (iii) are scalars because they only have magnitude. The others are vectors because they also require direction to define them.

E5) Understanding the equality of vectors is necessary, because the resultant of the two vectors that are being added has to be identified. They have to understand that not just the magnitude, but also the direction, must be identical. Otherwise they would, for instance, not realise the importance of the direction of the diagonal in the parallelogram.

Can you think of at least one other reason?

E6) For example, in a combination of pulleys, movement of the weight or of the pulley requires addition of vectors to find the resultant. When you are having a tug-of-war contest, both sides pull from opposite directions, and the resultant is the effect of the forces applied.

You can think of many other examples requiring addition of vectors.

E7) Here, they need to be careful because $\tan^{-1}\left(\frac{1}{2}\right)$ is in the 1^{st} and 3^{rd} quadrants.

In this case, they would need to take the angle in the 3^{rd} quadrant. So, they must learn to be clear about which angle they mean.

E8) Several points are given below:

- The dot product formula is easy to compute and gives you information about the angle between vectors.

- Many times you don't need an exact angle — it's often enough to know whether or not two vectors are perpendicular. (Ask them how they can tell this from the dot product).
- The dot product formula extends directly to vectors in three-dimensional space (and even higher!).
- You could explain how the dot product gives the work done by a body.
- Explain how the scalar product is used for exercising magnetic potential energy and the potential of an electric dipole.
- Look up examples of use in economics, chemistry, etc.

E9) For instance, geometrically the cross product is useful as a method for constructing a vector perpendicular to two given vectors. Secondly, it is useful for finding out whether three given vectors are coplanar or not. (How?) Thirdly, it appears in the calculation of moment of a force, and of a magnetic force on a moving charge.

Think of other examples of its use in other subjects that your students are studying.

E10) Ask your students to recall that they have done problems like this in their mathematics class while solving linear equations. Some of them may offer a solution as follows:

Let the velocity of the boat in still water be x km./hr. Then the velocity downstream will be (x+5) km per hour, and the velocity upstream will be (x–5) km/hr. As the time taken to travel a distance upstream is double the time taken to travel the same distance downstream,

$2(x-5) = x+5$, yielding $x = 15$.

So, the velocity of the boat in still water is 15 km/hr.

Ask your students to actually **verify that** with this velocity of the boat in still water, the conditions given in the problem are satisfied.

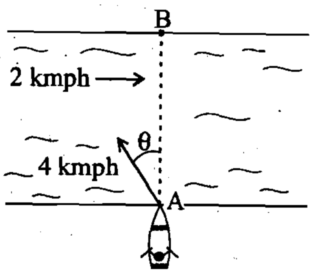


Fig. 11

E11) The velocity of the first vehicle relative to the other will be $\mathbf{u} - \mathbf{v}$ in the first case, and $\mathbf{u} + \mathbf{v}$ in the second case.

E12) Ask your class why the angle should be 30° (see Fig.11). Do they realise that the velocity of the boat has to be at an angle θ to the direction directly across so as to take into account the velocity of the river downstream?

Then $\frac{2}{4} = \sin\theta$, yielding $\theta = 30^\circ$.

E13) Some student may say, the net velocity of the plane will be $(\mathbf{u} - 40)$ km/hr, if its velocity is $\mathbf{u}$ km/hr when there's no wind.

The condition of the problem gives:

$$(\mathbf{u} - 40) \times \frac{1}{2} = 300 \Rightarrow \mathbf{u} = 640 \text{ km/hr.}$$

E14) This is a classical case of students not separating their own common experience, which is conditioned by friction in the situation presented. If there is no friction, the block would continue to be at rest if it was already at rest. But if it was in motion initially, it would continue to move horizontally even if there is no force on the block. The forces in the vertical direction could either lift the block, if they are in the upward direction, or they could press down on the surface on which the block is placed. If there is no surface on which the block lies, and therefore no reaction to balance the downward forces, the block would fall down.

E15) The schematic diagram could be as in Fig. 12. Note that for the purpose of this problem, we are ignoring the vertical forces of weight, etc. By Newton's second law,

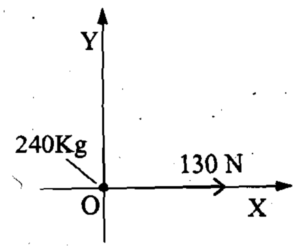


Fig. 12

$$\mathbf{a} = \frac{\mathbf{F}}{m} = \frac{130}{240} \mathrm{m/s^2} = 0.54 \mathrm{m/s^2}.$$

$\therefore$ The final velocity, $\mathbf{v} = \sqrt{v_o^2 + 2a(x - x_o)} = \sqrt{2ad}$

$$= \sqrt{2(0.54)(2.3)} \mathrm{m/s} = 1.6 \mathrm{m/s}.$$

E16) a) You may give them the example of a rocket taking off. It accelerates by throwing material in the reverse direction, and also by burning fuel to generate energy and give itself a thrust (force) to more forward.

b) $\mathbf{F} = \frac{d}{dt}(mv) = \frac{dm}{dt} v + m\frac{dv}{dt} = \frac{dm}{dt} v + ma$.

E17) The three forces acting on the body are its weight m**g** acting vertically downward, the reaction **R** of the inclined plane on the ball, which is along the perpendicular to the plane, and the force **F** keeping it at rest. The three must be in equilibrium. So,

$R = mg\cos\theta$ and $F = mg\sin\theta$,

where θ is the angle of inclination of the plane with the horizontal.

E18) The forces have to be drawn keeping in mind the fact that there may be an action and a reaction in many cases. We need to also think about the factors we will take into account, and those we will ignore.

For example, in (i) we may ignore the resistance and buoyancy of air. In this situation there is only the force on the rock due to gravity, which is pulling it down. So the only force is the weight acting downwards.

In (ii), for the rock to rest on the ground there has to be a force which supports it on the ground. This force would come from the reaction of the surface on which the rock is placed. You could think of other forces and their directions.

You should analyse (iii) and (iv) in the same way.