UNIT 12 MOMENTS AND COUPLES

12.1 INTRODUCTION

One major area of difficulty identified by students of Class 12 mathematics is the concept of **moment** of a force or of a couple. Looking at the reasons for confusion, and how to remedy the situation, is the focus of this unit.

To start with, in Sec. 12.2, we consider some basic concepts related to rotational motion. These are analogous to displacement, velocity and acceleration in the context of rectilinear motion. Here, we particularly ask whether these are vector or scalar quantities. Further, if they are vectors, then what direction do they have? Once the answers to such questions have been understood by the students, they would be in a position to clarify their doubts regarding moments.

In Sec. 12.3 and Sec. 12.4, we look at problems related to understanding moments and couples. You will find some suggestions here that have been useful for improving the understanding of several students. We suggest that you try them out with your students too.

Finally, in Sec. 12.5, we have considered problems about bodies in equilibrium.

Throughout this unit, we have followed the notational convention that the magnitude of a vector **v** will be denoted by v.

Objectives

After going through this unit, you should be able to help your learners improve their ability to

- describe what a rigid body, and a rigid-body transformation, is;
- explain what the moment of a force (torque) about a point is;
- recognise situations where moments come into play;
- explain the mathematical representation of a moment as a cross product;
- describe what a couple is, physically and mathematically;
- solve simple problems involving moments and couples.

12.2 ROTATIONAL CONCEPTS

To begin with let us look at some concepts that need to be understood by a student before she comes to moments. The first of these concepts is that of a rigid body and the forces that come into play when it is rotated. Students do wonder about what a rigid body is and why we only talk of a rigid body when discussing moments and couples. A good way to help them understand this is to give them examples of rigid and non-rigid bodies around them. For example, water is not rigid, jelly-like substances are non-rigid, a human being who is changing the position of her limbs is a non-rigid body; a duster, a blackboard, table are rigid. Then ask them: what could the defining criterion be for a rigid body? After discussions and refinements, they may come out with some form of the following definition — "A rigid body is a collection of particles such that the distance between any two points of it always remains constant."

What the student needs to realise is that a rigid body is an object which has a definite shape. It does not change even when a deforming force is applied. In nature there is no perfectly rigid body, as all real bodies experience some deformation when forces are exerted. So, a perfectly rigid body can only be idealised. However, in several cases we choose to treat the objects under focus as rigid bodies when the deformation is negligible. For example, the deformation of a cricket ball as it bounces off the ground can be ignored. Again, if an object is dragged along a plane, a frictional force acts on it. But its deformation due to the frictional force can be neglected. However, you cannot neglect the deformation of a railway track due to the weight of the train. Likewise, the deformation of the fibreglass pole used by a pole-vaulter can also not be neglected. So, in the last two situations we cannot apply the rigid body model.

Here's a related exercise for your learners.

E1) Give a list of a few objects that a layperson may call a rigid body, but are not rigid bodies according to the definition.

Now, what happens to a rigid body under the action of forces that induce translation or rotation? Does a rigid body remain the same? For instance, if you translate (push) a book across the table, does it change shape? Or, if you rotate it about a fixed point, does it change shape? Under such forces, the distance between any two particles remains the same. So, the rigid body's position changes but not its shape. Because of this, translation and rotation are called rigid-body transformations. Can your students give you examples of forces acting on rigid bodies that are not rigid-body transformations, that is, those that result in changing their shapes? Ask them to think about this.

A problem some students have is that though they can identify a rotation or translation when carried out in front of them, they can't explain the difference between these motions clearly. How would you explain the difference between a rotation and a translation? One teacher demonstrated several examples to her class of both these motions. Then, she described the difference between them by saying, "For a rigid body in translation all its points are in linear motion. Their distances from each other remain fixed, but all of them move from their earlier positions through the same distance in the same direction in the same time interval. None of them remain fixed or stationary. Rotation, on the other hand, is motion in which the position of a few points of the body remain fixed and all the others move." After a bit of thought she added, "Either a few points in the body are fixed or the distance of the body from a fixed point remains constant while it is in motion." What do you think about this formulation? Is this sufficient? Discuss this with your learners too. Ask them to analyse the motion of a book that is pushed, a rotating door, a spinning top, wheels of a cycle, etc., and suggest changes in the teacher's explanation, if necessary. A few teachers that I have had such discussions with, have suggested that "A rigid body is said to rotate if each of its points moves in a circular path, the centres of these paths lying on a straight line called the axis of rotation." You could discuss this statement, and other such formulations, with your students to help them clarify their understanding of rotational motion.

Now, rotation can be with respect to a fixed axis or a moving axis. We shall analyse the motion of a rigid body about a **fixed** axis of rotation. Let us start with some exercises.

E2) How would you help your students analyse the rotations of a rigid body about a fixed line? Which kinds of situations do they need to be exposed to for this?

E3) Give examples of motions that have rotation as a component, but cannot be considered as the motion of a rigid body about a fixed axis of rotation.

Other basic concepts students need to understand are the rotational analogues of translational concepts like displacement, velocity, acceleration and momentum. For instance, analogous to linear displacement is angular displacement. Linear displacement is a vector. So, students assume that angular displacement must be a vector too. If this is so, what is its direction? Regarding its magnitude, the students know how and why the angular displacement, $\Delta\theta$, is $\theta_2 - \theta_1$, where θ_1 and θ_2 are the angles of the particle before and after rotation (see Fig. 1). They are also able to accept and use the convention that angular displacement in the anti-clockwise direction has a positive sign, and in the clockwise direction has a negative sign. So, the magnitude of $\Delta\theta$ is not a problem. Finding the direction of this displacement is a problem. This problem also exists for angular velocity and angular acceleration. Their magnitudes are:

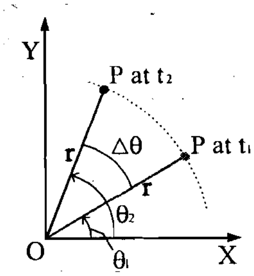

Fig. 1 : Angular displacement, Δθ.

angular velocity, $\omega = d\theta / dt = \lim_{h \to 0} \dfrac{\theta(t+h) - \theta(t)}{h}$, and

angular acceleration $\alpha = d\omega/dt$.

However, by convention, the direction of these vectors is parallel to that of the fixed axis of rotation. The sense of the rotation tells us the direction along this line. By convention, rotation in the anti-clockwise direction has a positive sign, and that in the clockwise direction has a negative sign. For instance, when we open and close a door, the directions of the angular displacement and angular velocity are in the upward or downward direction, depending on whether the door's movement is in the anti-clockwise or clockwise direction. The source of the students' confusion is that they see the rotation taking place in the horizontal direction, but the vectors that come into play are in the vertical direction. The students need to be helped to understand why this convention has been chosen. (Any such convention must also obey tne laws of vector addition, and the chosen convention does so.)

You could give the students several exercises like the following one to help them internalise these ideas.

Problem 1 : Consider a grindstone which starts from rest at time t = 0. Assume that its axis of rotation is fixed (see Fig. 2), and the grindstone rotates with a constant angular acceleration of 3 rad/sec^2. After 2.5 seconds, what is

i) the angular displacement of any point on the grindstone?
ii) the angular speed of the grindstone?

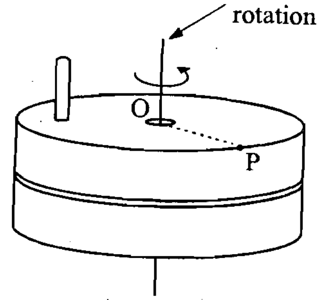

Fig. 2 : A rotating grindstone.

Solution:

i) α and t are given. We need to find $\theta = \theta_0 + \omega_0 t + ½\, \alpha t^2$, where we know that $\theta_0 = 0$, $\omega = \omega_0 = 0$ and $\alpha = 3$ rad/sec^2.
So, after 2.5 seconds,
$\theta = ½\,(3)(6.25)$ radians $= 9.38$ radians.

Since 2π radians is 1 revolution, any point P will be displaced through 1.49 revs. So, the final position will be at 0.49^{th} of a revolution in the anti-clockwise direction from its position of P at time $t = 0$.

ii) Since $\omega = \omega_0 + \alpha t$, after 2.5 seconds ω will be 7.5 rad/sec.

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It is important for students to consider the parallels, and the differences, between translational and rotational variables. They need to be given exercises, including numericals, enabling them to become familiar with the concepts concerned. For example, they could be asked to check whether the equations of motion remain valid for rotational motion as well. Can they see angular displacement $\theta(t)$ as

$$\theta(t) = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2,$$

and from here develop other possible relationships? You could also ask your students to check if there are rotational analogues of other relationships of linear motion.

Why don't you try some exercises yourself, and with your students, now?

E4) Is there any other way of defining the directions of the angular variables so that the laws of vector addition are obeyed?

E5) If the axis of rotation is not fixed, then angular displacement is not a vector. Can you prove or disprove this?

There is one difficulty that students face regarding any of these concepts. This originates from the fact that some of the analysis that is done in this part of mechanics lies at the edge of physics and mathematics. So, there are many problems asked which appear to have no relevance to their physical and immediate experience. For example, a question like "Derive the relationship $\omega = \omega_0 + \alpha t$" can be answered relatively easily. But the student doesn't see why this problem is to be done. How does it help her understand things better? We teachers need to relate such problems to the implications they have for concrete physical examples and to their daily experience, if possible. This would make the problems, and the concepts, more interesting to the students.

In this section we have considered some basic concepts students need to understand before they attempt to understand moments. Now we shall look at some ways of removing difficulties many learners have when dealing with moments.

12.3 UNDERSTANDING MOMENTS

The moment of a force, as you know, is the turning tendency of the force. It is the rotational analogue of force in the context of linear motion. A force acting on a body can alter its state of motion. Similarly, the moment of a force about a point can change the state of rotational motion of a body. In what other ways is moment analogous to force? Are they different in some respects too? How could we encourage students to explore these questions?

Physicists, as usual, like to be different! They use the term 'torque' for 'moment of a force'.

At a few workshops with Class 12 students, I asked them what a moment of a force is. A few couldn't answer at all. But many of them told me that $M = pF$, where M is the moment of the force **F** about a point O and p is the perpendicular distance of O from the line of action of **F**. However, when I tried to probe them regarding the physical and geometrical interpretation and sign of a moment, they were at sea. This is a common situation. Students do not understand what moment really is. Is it a scalar?

Is it a vector? If so, what is its direction? To answer all these questions, they need to do several activities. One useful activity is given below:

Activity : Ask your student to hold a long rod (say one metre long) in her hand so that it is horizontal (see Fig. 3). Another student can hang a weight from the rod. What does the student who is holding the rod feel? Doesn't she feel a turning effect on the rod and her wrist? What happens to this turning effect if the weight is increased or if the point from which it is hung is moved further away from the hand along the rod?

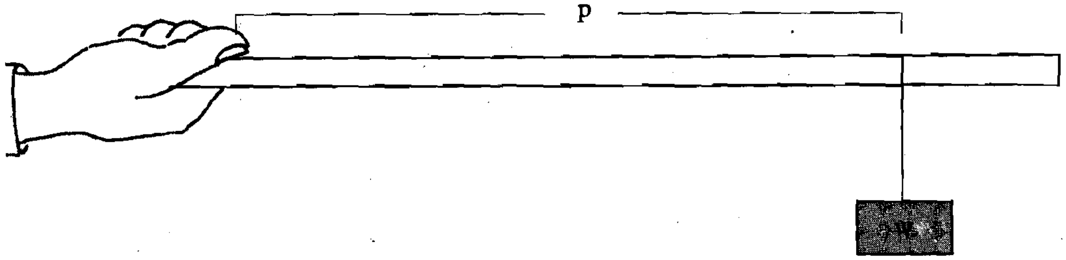


Fig. 3 : The moment of W is Wp

Does she feel the turning effect increase? What happens if the weight and its distance from the hand is not changed, but the angle of the rod to the horizontal is changed? Does the hand feel a change in the turning effect? How is the turning effect altered as the rod approaches the vertical position? Does she realise that it becomes smaller and smaller? Does she require more turning exertion to hold the rod horizontally straight, or to hold it vertically, i.e., perpendicular to the ground?

Once the student analyses these changes, she would be able to relate the turning effect to the weight (the force here). You could tell her that this turning effect of a force is called the moment of that force.

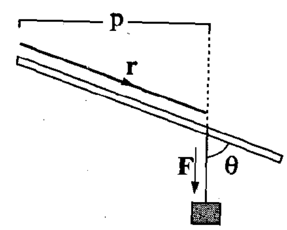


Fig. 4 : M = Fr sinθ

Such activities help the student to realise that the angle of the rod to the horizontal (or vertical) makes a difference to the moment. Here is where you could explain the theory behind this — that the distance to be taken is **the perpendicular distance** between the fixed point and the line of action of the force. This, in general, is r $\sin\theta$ (in Fig. 4). So, the moment of **F** about the point is F r $\sin\theta$. Note that this is also the magnitude of the cross product $\mathbf{r} \times \mathbf{F}$.

Apart from such activities, your students need to observe and analyse situations in which moments come into play. They could be asked to explore the motion of the common balance, the opening of a door, the closing of an open book, etc. For example, let us consider the balance. When we use a balance, rotation takes place about an axis passing through a fixed point (the fulcrum) (see Fig. 5). In this case, the moments depend upon the weights in the pan and the length of the arm of the balance. The longer the arm, the greater is the turning moment. Similarly, the more the weight in a pan, the greater is the turning moment of the balance.

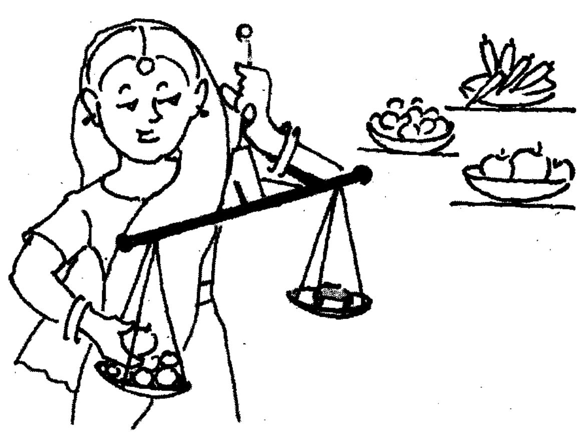


Fig. 5 : Using the balance

You could also ask the students to see what happens when they apply a force on a door (see Fig. 6). What happens if they apply a force **F** to the door at the hinge line AB ($\mathbf{F}_1$ in Fig. 6)? The door won't move. What happens if **F** is applied along a line that intersects AB ($\mathbf{F}_2$ in Fig. 6)? Again, it won't move. So, when does it move? They would soon be able to describe the points and directions in which **F** would be able to move the door — points not on AB, and **F** applied along a line that is skew with respect to AB. When is **F** the least, and most effective? With experimenting, they would find that this happens when **F** is applied at a point on the door which is farthest from AB and is in a direction perpendicular to the line connecting its point of application with AB ($\mathbf{F}_3$ in Fig. 6).

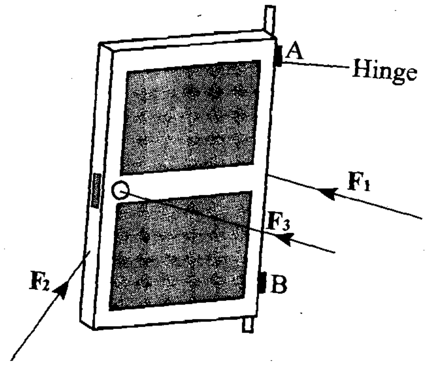


Fig. 6

What your students need to realise is that when they apply a force for moving the door, there is no motion along the fixed axis because the net displacing force on the door is balanced. However, the turning effect is not balanced.

You could also ask your students to identify the force, the point of its application and the point of rotation in the case of a hand pump. In the hand pump in Fig. 7, the force **F** is applied to the end A of a long lever which is attached to a shaft at O.

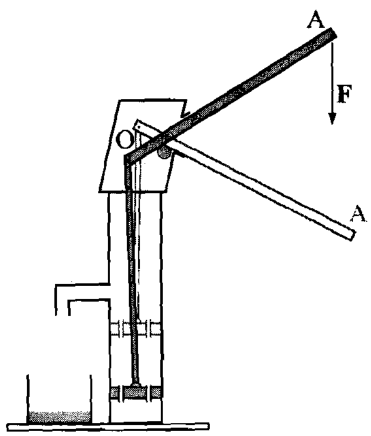


Fig. 7 : A hand pump

Why don't you try an exercise with your students now?

The SI unit of moment is Nm, short for Newton metre.

E6) a) A 100 N vertical downward force is applied to the end A of a 1.2m long lever which is attached to a shaft at O. If OA makes 60° with the horizontal, then determine

i) the moment of the force about O;

ii) the magnitude of the horizontal force applied at A which creates the same moment about O;

iii) the smallest force applied at A which creates the same moment about O.

b) How far from O must a 240N vertical force act to cause the same moment about O as in (i) above?

E7) Analyse the rotational motion of a centrifuge or the opening of a screw regarding the forces and moments that come into play.

E8) What is the moment of the force **F** about O if the line of action of the force **F** passes through O?

E9) If the line of action of **F** doesn't contain O, and it is reversed, then what is the moment of **F** about O?

The answer to E8 is simple. From the definition, the students should realise that the moment of **F** about O is zero if either the line of action of **F** passes through O, that is, if p = 0, or if **F** itself is zero. But the answer to E9 involves the sign of the moment, which they need to know to be able to distinguish between the two cases shown below in Fig.8.

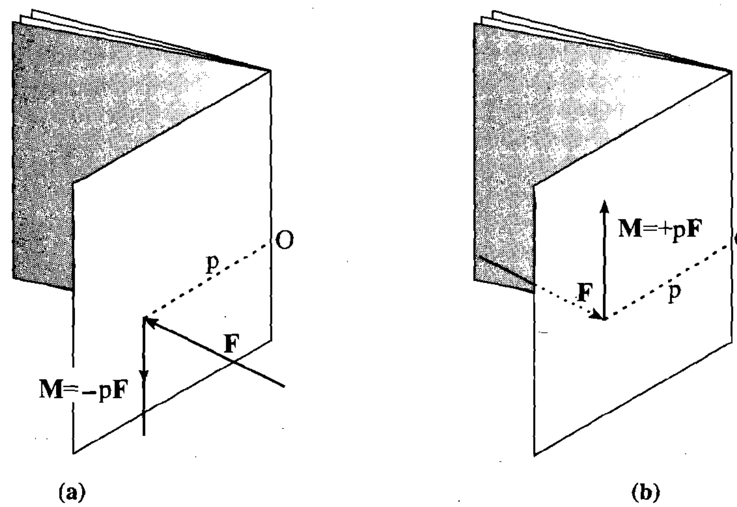
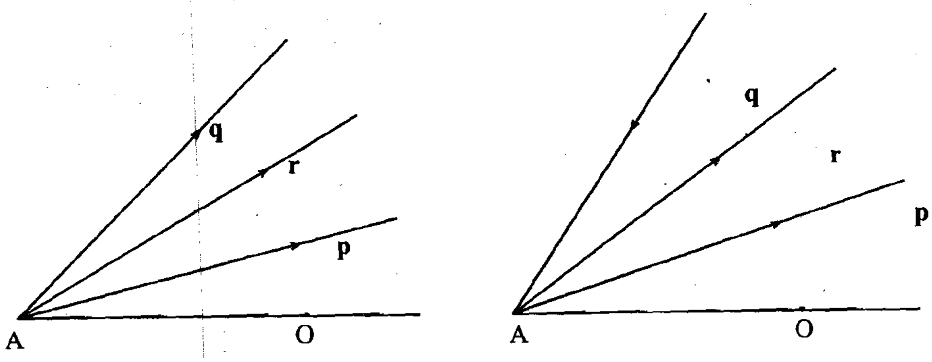


Fig. 8

Let us suppose the line of **M** is drawn perpendicular to the plane of the paper, intersecting it at the point O. The moment is +p**F** when it indicates an anti-clockwise rotation around O in the plane and −p**F** when a clockwise rotation is indicated. This is a convention in keeping with treating the moment as the vector product **r** × **F**, where P is the point at which **F** acts and **r** is the vector **OP**. So, the direction of **M** is normal to the plane of **r** and **F**, given by **the right-hand rule** for the cross product of two vectors — if we rotate the curled fingers of our right hand from **r** to **F** through the smaller angle between them, then the direction of the extended thumb gives the direction of **M**. **The reason for choosing this convention is that the magnitudes agree, and the laws of vector addition are also obeyed.**

Giving students exercises like the following one to do, would also help them in realising the vectorial convention for **M**.

E10) In Fig. 9(a) why do both the forces **p** and **q** tend to rotate the body about O in the same sense? In Fig. 9(b) why do **p** and **q** tend to cause rotation about O in opposite senses?



(a) (b)

Fig. 9

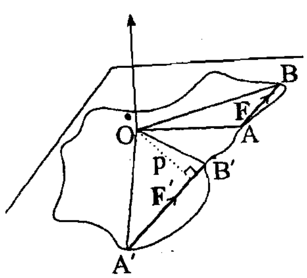


Fig. 10

Now, more often than not, we find that we have an intuitive understanding of a concept or process. However, a thorough conceptual understanding requires more — in this case, the mathematical reasoning behind the features of the concept. For example, consider the case when two forces **F** and **F′** have the same line of action (see Fig.10). A student who has some understanding "knows" that the moments of these forces about O are equal if **F** = **F′**. But how to prove it? This requires mathematical arguments, one of which is to use the geometrical interpretation of the moment.

Since **F** and **F′** have the same line of action and the same direction, the moment of **F** about O = 2(area of ΔOAB) = pF, where p is the length of the perpendicular from O to the line of action of the force **F** (and **F′**).

Similarly, the moment of **F′** about O = 2(area of ΔOA′B′) = pF′.

Hence, the result.

Do your learners relate to this line of reasoning? For this, they would first need to understand why the moment of a force **F** about the point O has magnitude equal to twice the area of the triangle whose base is the magnitude of the force **F** and height is p. This is demonstrated very nicely in Fig. 15.34 and Fig. 15.35 of the Mathematics Class 12 (Part-II) textbook by NCERT.

The same arguments can be used to show that the moment of **F** about O is unchanged even if the point of application is shifted to any other point of the line of action of **F**. Next, you could ask your students to analyse the situation when **F** and **F′** have the same magnitude but opposite direction. What kind of arguments do they use for doing so? Mathematical reasoning, or physical intuition, or a mix? You need to find out from them through discussions with them and through peer-group interactions.

Why don't you do an exercise now with your students?

E11) Ask your students what happens to the direction of **M** if the direction of **F** is reversed. Do they realise that this is also reversed? What arguments did they use to prove this?

One source of error for many students is the inability to use the appropriate sign of the moment of each force when a number of forces act on a body. The algebraic sum of the moments of all the forces about some fixed point in the body is obtained by giving the value of each moment its proper sign and adding them together. This is where the students often err. You could give them exercises like the following one to do, and ask them to discuss their reasoning behind their solutions. This will help you to correct any misunderstandings.

Problem 2: Compute the moment of the 1000 N force about points A, B and C (see Fig. 11).

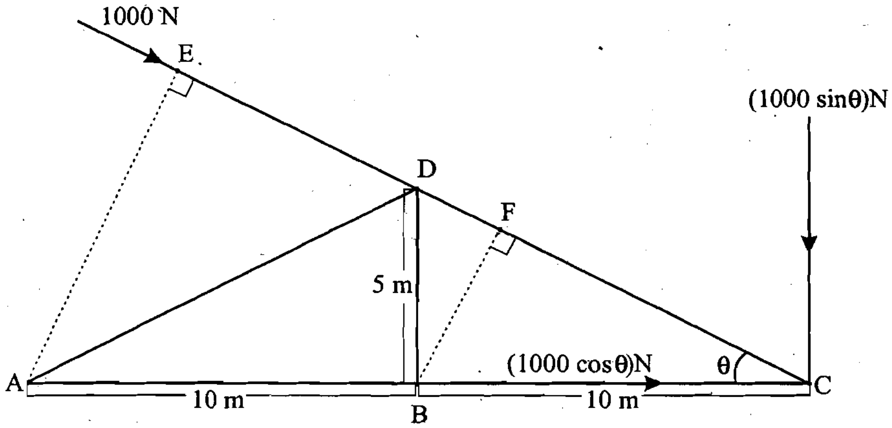


Fig. 11

Solution : We first need to find the length of the perpendiculars from A and B to the 1000 N force. Using Fig. 11, we know that AE = 20 sin θ, $BF = 10\sin\theta$ and $\sin\theta = \frac{1}{\sqrt{5}}$.

The moment of the force about A = $AE \times 1000 = \frac{20 \times 1000}{\sqrt{5}}$ Nm.

The moment of the force about $B = BF \times 1000 = \frac{10}{\sqrt{5}} \times 1000$ Nm.

Since the point C lies on the line of action of the force, the moment of the force about C is zero.

We must encourage our students to develop as many different solutions as possible.

Another solution: Sometimes, it is convenient to use the rectangular components (1000 cos θ) N and (1000 sin θ) N of the force 1000 N.

The moment of the force 1000 sin θ N about A is $20 \times 1000 \sin\theta = \frac{20 \times 1000}{\sqrt{5}}$ Nm,

and its moment about B is $10 \times 1000 \sin\theta = \frac{10 \times 1000}{\sqrt{5}}$ Nm.

Since A,B and C lie along the line of action of the force 1000 cos θ N, this force has zero moments about these points.

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The following exercises may help your learners to get used to dealing with the different signs of moments of different forces. Try them yourself, and with your learners.

E12) Use the concept of vector moment of a force about a point to prove Varignon's theorem when the forces act at a point.

E13) Suppose forces 3,5, 3 and 5 N act along the sides AB, CB, DC,DA, respectively, of a square of side 4 m. Find the sum of their moments

i) about B,
ii) about C,
iii) about the centre of the square.

In each case, in which direction will the body finally turn?

E14) ABCD is a square of side 10 m. A certain force **P** passes through A and has moments of 20 and 30 Nm, in opposite senses, about B and D, respectively. Find the magnitude of **P**, and the moment of **P** about C.

E15) What kind of errors did the students make while doing the problems above? What is their reasoning behind these mistakes?

Before ending this section, let us mention another source of confusion for students — moment of a force about a line. Here, they need to be clear that **this is a scalar quantity** because it is merely the component of the **vector moment about a point on that line** in the direction of that line.

Let us now look at problems related to dealing with equal unlike parallel forces acting on a body.

Fig. 12 : An understanding couple!

12.4 UNDERSTANDING COUPLES

Let us start this section by recalling a few situations in your student's experience in which couples come into play. How many can you think of? At least one is the use of a screwdriver. Ask your students which forces they apply when using this gadget. If they only apply one force to the handle, what happens? Do they realise that to turn it two forces of equal size, which are in opposite directions, need to be applied, one on either side of the handle (see Fig. 13)?

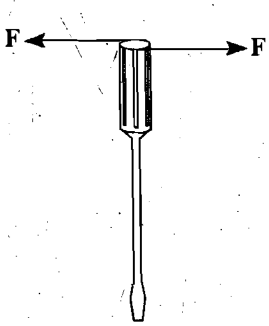


Fig. 13 : Two forces acting as a couple for turning a screwdriver

Which other examples can you, or they, think of? What about using a key in a lock or spinning a top? The key and the top rotate due to two such forces, which are generated by our fingers. Through such examples, you could the students to experience a **couple**. While trying to understand the concept, they need to focus on three things:

i) **two forces** are involved;
ii) they are **equal** in magnitude, but **opposite** in direction;
iii) their lines of action are **distinct.**

To help them realise the necessity of each of these components, ask them what happens if any one aspect is missing — is there still a turning tendency? Doing several activities and exercises involving these aspects of couples helps the students to understand the concept.

Let us discuss some problems related to finding the moment of a couple. Firstly, the students are confused about how to find this. Most of them think it is pF – p′F, where p and p′ are the lengths of the perpendiculars from the fixed point to the lines of action of **F** and –**F**, respectively (see Fig. 14).

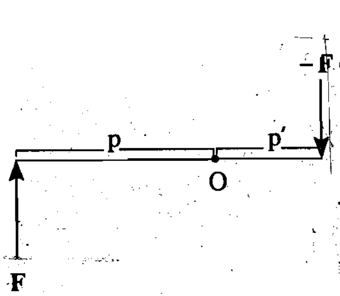


Fig. 14

However, the students need to be clear that both forces in a couple tend to turn the body in the same direction. This is because the forces are in opposite directions. So, the total turning effect is in the same direction. So, for instance, consider the moment of the couple in Fig. 13. Each force produces a moment of magnitude Fr about the centre of the disc in a clockwise direction. Therefore, the total moment of the couple

ii) They fasten a 20 m long 20 cm by 40 cm board to the merry-go-round so that the middle of the board is at the middle of the merry-go-round. What is the resulting couple moment if they push on either end of the board in opposite directions?

iii) What moment about the merry-go-round axis would they generate by fastening one end of the board to the middle of the merry-go-round and if both push in the same direction on the other end?

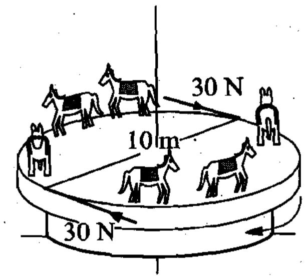


Fig. 19

E20) Show that three forces acting along the sides of a triangle, represented in magnitude and direction by the three sides of a triangle taken in order, constitute a couple whose moment is twice the area of the triangle. Does this contradict the triangle law of forces, which states that
"Three forces acting at a point, represented in magnitude and direction by the three sides of a triangle taken in order, are in equilibrium."?

So far we have considered the moment of a force and the moment of a couple. Let us see the behaviour of these moments when a body is in equilibrium.

12.5 EQUILIBRIUM

In the previous unit we discussed aspects of forces of reaction and tension. Here we shall consider situations in which these forces come into play in the context of equilibrium.

First, let us see what students understand by equilibrium. When we asked students of Class 12 questions with this focus, we discovered that most of them thought that a body in equilibrium is a body at rest. Do you agree with them? If a body is at rest, it is certainly in equilibrium; but not conversely. Why is this so? What 'equilibrium of a body' means is that the linear acceleration of its centre of mass is zero, and its angular acceleration α about any fixed axis is zero. So, there are certain restrictions imposed on the forces and moments acting on a body to bring it into a condition of equilibrium. These two requirements are:

i) the sum of the forces **in any direction** must be zero, and
ii) the sum of the moments of the forces **about any point** must be zero.

Let us consider a few questions that we can give students for helping them understand this concept.

Problem 5 : Investigate the state of equilibrium of a woman weighing W Newtons on a weightless ladder, as shown in Fig.20. Here AB = 4a and BC = a.

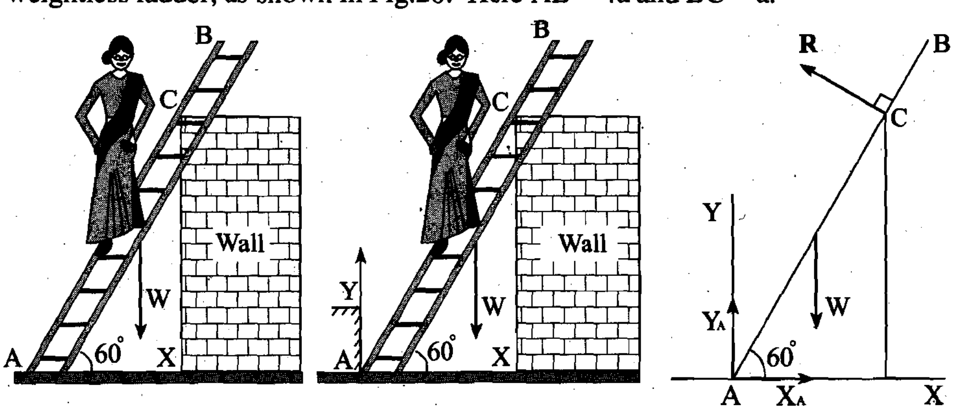


Fig. 20

Solution : To help your learners get a better insight, you could first discuss the reactions due to constraints. When the end A of the ladder moves along the surface XA (Fig. 20 (a)), then the reaction Y_A is directed normal to XA. In Fig. 20(b), we have a supporting corner at A. This kind of constraint prevents a body from moving in the shaded region of the plane AXY. Its reaction cannot be defined uniquely. As a rule, it is resolved into two components X_A and Y_A along the coordinate axes (see Fig. 20(c)). Ask your students what the reaction at C is. If they are clear about the situation, they would explain why the reaction at C is along the normal to the ladder.

Let us investigate the rotational equilibrium of the woman and the ladder. The external constraints are the supports at points A and C. We replace them with appropriate reactions. The reaction R at C acts at right angles to the ladder. If we take the moment of the forces about A, then the moment of the reaction Y_A and X_A about A will be zero.
(**Note** that, we have chosen the point A because the lines of action of two of the three unknown forces pass through this point, thereby producing zero moments.)

For rotational equilibrium, the sum of the moments of the remaining forces should be zero. The remaining forces are forces due to the weights of the ladder and the person. This can be considered to be applied at the centre of gravity of the ladder and the person. We can take the weight as a force acting downwards from the centre of gravity, i.e., the middle of the ladder. The distance of the force from the point where moments are to be calculated is half the length of AB. The weight can be resolved into two forces — one along AB, and the other perpendicular to the ladder. There is no moment of the force along AB at A since the fixed point lies along the direction of the force. The remaining moment, due to **R** and the component of weight opposite to **R**, must balance. This gives us the following equation:

$$W\left(\frac{AB}{2}\right)\cos 60^\circ - R.AC = 0 \Rightarrow R = W/3.$$

So, if R = W/3, the combined body will be in rotational equilibrium.

Now, what about translational equilibrium? Since the combined body is at rest along any direction, the net force in any direction is zero. So, the combined body is in translational equilibrium.

———×———

Why don't you try some exercises now?

E21) i) A man can exert a force of 1000N. He pulls a rope fastened to the top of a post, the rope being twice the length of the post. A horizontal force **P** is applied to the middle of the post to keep it from falling, i.e., to keep it in equilibrium. Draw the figure and mark the forces, including reactions, if any. Find the horizontal force **P**.

ii) What happens if a horizontal force **P′** is applied at a point 3/4th of the post from the bottom, and the pole is to be kept from falling? Which horizontal force is larger?

With this we come to the end of our discussion on moments. Let us take a brief look at what we have covered in this unit.

12.6 SUMMARY

In this unit we have looked at ways of helping students understand the following points.

1. What a rigid body is, and why rotation and translation are rigid body transformations.

2. When rotation is about a fixed axis, angular displacement, angular velocity and angular acceleration are vectors with direction parallel to that of the axis of rotation.

3. The 'moment of a force' is the rotational analogue of 'force' in the context of translation.

4. The moment of a force **F** about a point O is the vector **OP** × **F**, where P is the point of application of **F**. On the other hand, **the moment of F about a line is a scalar**.

5. The physical and geometrical interpretation of moment of a force about a point.

6. A couple consists of two equal forces, opposite in direction, and with distinct lines of action.

7. When couples come into play, and how to calculate their moments.

8. The moment of a couple is not dependent upon the point about which it is calculated.

9. Several forces can be reduced to a couple only if there is no translation in any direction under their net effect.

10. A body is in equilibrium if the forces acting on it have no translational or rotational effect.

12.7 COMMENTS ON EXERCISES

E1) For instance, some people may call a length of telephone wire a rigid body. However, it is not a rigid body because a translation need not shift all parts of the wire through the same distance.

E2) Here you could look at the principles for encouraging learning that are discussed in Block 1 of this course. Accordingly decide on a series of activities and discussions.

E3) For instance, what about the rotation of the earth about its axis, but relative to the sun? In this case, the axis is not fixed with respect to the sun as the origin. You can ask your students to think of examples of situations around them.

E4) One way is, of course, the opposite of the currently used convention, namely, the clockwise direction is positive, and the positive direction of W is its vector in the upward direction. Are there any other possibilities?

E5) Suppose a body has an angular displacement θ_1 about L_1 and θ_2 about the axis L_2. Then, is $\theta_1+\theta_2 = \theta_2+\theta_1$ always? Check this by rotating some objects. You

would then see that this doesn't usually happen. Therefore, the laws of addition of vectors wouldn't hold.
However, you should try and prove that angular velocity is a vector even when the rotation is about a moving axis.

E6) a) i) The moment of the force 100N about O = $100 \times 1.2 \sin 30°$ Nm = 60 Nm.

ii) If F is the magnitude of the horizontal force, then
$F \times (1.2) \sin 60°\ \text{m} = 60\ \text{Nm} \Rightarrow F = \frac{100}{\sqrt{3}}$ N.

iii) The force applied will be least when it is at right angles to OA and p is greatest, i.e., p = 1.2. If F_1 is the magnitude of the least force, then $F_1(1.2)\ \text{m} = 60\ \text{Nm} \Rightarrow F_1 = 50$ N.

b) Let the point of application of the force 240 N be at a distance r metres from O. Then

$240\text{N} \times r \sin 30°\ \text{m} = 60\text{Nm} \Rightarrow r = 0.5.$

E7) A centrifuge is an implement used for separating suspended particles from the liquid they are in. The centrifuge can be manually rotated by 2 forces on opposite arms applied in opposite directions. In the case of a screw being opened with fingers or with a spanner, the force on two opposite sides also needs to be applied in opposite directions.

Moments are the turning effect produced due to forces. In the case of the screw, the axis of rotation is through the centre of the screw and passing through the tip. The moment is the cross product of the applied force and the position vector of the point of application with respect to the axis of rotation. Notice that the application of the force in the opposite direction would move the screw in the opposite direction. When forces are applied in one direction the screw moves down, and it moves up when the force is in the opposite direction.

A similar analysis can be done for a centrifuge.

E8) See the matter following these exercises.

E9) See the matter following these exercises.

E10) In the first case, both **p** and **q** tend to turn the body in the same direction. So their moments are in the same direction. In the second case, **p** and **q** tend to turn the body in opposite directions. So the turning effect is the difference of the turning effects of each.

E11) Did they apply the definition of **M** as a vector product?

E12) If $\mathbf{P}_1$ and $\mathbf{P}_2$ are two forces acting at a point A and **P** is their resultant, then $\mathbf{P} = \mathbf{P}_1 + \mathbf{P}_2$.
The vector moments of $\mathbf{P}_1$, $\mathbf{P}_2$ are given by $\mathbf{AO} \times \mathbf{P}_1$ and $\mathbf{AO} \times \mathbf{P}_2$, respectively. Their sum is $(\mathbf{AO} \times \mathbf{P}_1) + (\mathbf{AO} \times \mathbf{P}_2) = \mathbf{AO} \times \mathbf{P}$, the moment of their resultant **P**.
This proves the theorem.

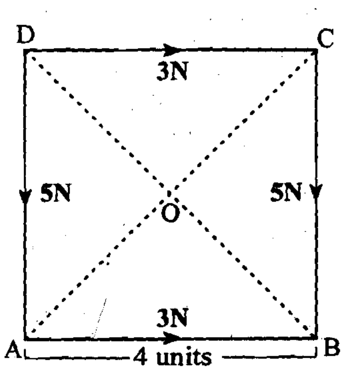


Fig. 21

E13) The situation is shown in Fig. 21. The forces along CB and DC tend to rotate the body in a clockwise direction, while the other two forces tend to rotate the body in an anti-clockwise direction. Therefore, the sum of the moments of the four forces about B

$= 3N \times 0 - 5N \times 0 - 3N \times 4m + 5N \times 4m = 8$ Nm

Similarly, the total moment about C = 32 Nm.

The centre O of the square is at a distance of 2m from each side of the square.

$\therefore$, the sum of the moments about O

$= -3N \times 2m + 5N \times 2m + 3N \times 2m - 5N \times 2m = \mathbf{0}$.

In each case, the sign tells us that the body will rotate about B in the anti-clockwise direction, about C in the anti-clockwise direction, and no rotation will take place about O.

E14) Let the line of action of the force **P** make an angle θ with the side AB of the square. Then the magnitude of the moment of the force **P** about B = P 10 sinθ.

$\therefore$ P 10 sinθ = 20 (1)

Similarly, the moment of **P** about D will be = P 10 cosθ.

$\therefore$ P 10 cosθ = 30 (2)

From (1) and (2), we get $\tan\theta = \frac{2}{3}$.

So, $\sin\theta = \frac{2}{\sqrt{13}}, \cos\theta = \frac{3}{\sqrt{13}}$

$\therefore P = \frac{2\sqrt{13}}{2} = \sqrt{13}.$

To find the moment of **P** about C, one way would be to consider the components of **P** along AB and AD. These are $P\cos\theta = 3N$ and $P\sin\theta = 2N$, respectively. Then the moment of **P** about C is the algebraic sum of the moments of the components 3N and 2N about C = $-3N \times 10m + 2N \times 10m = -10Nm$.

Some student might have used another route for solving the problem — taking the components of **P** as Pcosθ and Psinθ along AB and AD, respectively, and then finding the moments of **P** about B and about D, to get Equations (1) and (2).

E15) In each problem, the first step is to draw a diagram of the square, and indicate the appropriate forces. While there may not be an error in drawing the square, the students usually err in depicting the forces to be taken into account for calculating the moments. Students also tend to err in recognizing the axis of rotation and in understanding the direction of the force.

Computational errors are not significant, but the **conceptual errors** open a window into the mind of the child. Divide the errors into these two broad areas, and then further analyse and specify the errors made.

E16) This interpretation is similar to that of the moment of a force. So, the moment of a couple is twice the area of a triangle with base being the magnitude of the force and height being the length of the arm.

What kinds of examples, activities and exercises would you expose your students to for clarifying their geometric understanding of this concept? How does this understanding help the students in dealing with the concept?

E17) Find out the different ways they tried to solve this and the errors, if any. Also find out why they made those errors — a misunderstanding, or just a miscalculation? Did they use the equation $0.18 = 0.04 \times F$? Or, did they write $F = 0.18 \times 4$ N? Or, $0.18 = 4 \times F$? Each equation used has a different understanding behind it, which you need to find out by talking to the student.

E18) The moment of the couple is 8 Nm. The distance between the hands is 60 cm, or 0.6 m. So, the force, **P**, that he must exert with each hand, is given by

$$P \times 0.6\text{m} = 8\text{Nm} \Rightarrow P = \frac{40}{3}\text{N}.$$

E19) i) The couple moment produced = 30 N × 10 m = 300 Nm.
ii) The resulting couple moment = 30 N × 20 m = 600 Nm.
iii) The moment about the merry-go-round axis in this case = 60 N × 20 m = 1200 Nm.

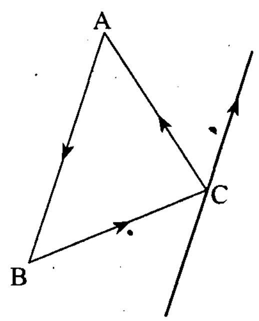
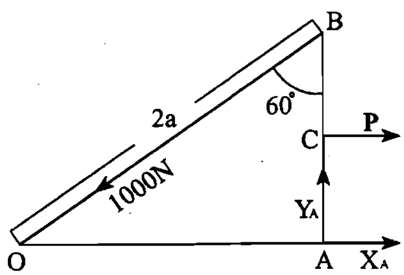


Fig. 22

E20) The resultant of the forces **BC** and **CA** is the force acting through C and parallel to AB in the opposite direction, as shown in Fig. 22. Along with the force **AB**, it constitutes a couple whose moment is twice the area of the triangle.

There is no contradiction, as the forces are not acting on the same point. Since the net force, when translated to one point, is zero, there is no translation because of the three forces. The moments in this situation are not balanced, so the parallel forces would produce a rotation.

E21) (i) $\angle$ ABO = 60°, because AB = a, OB = 2a. So, $\cos \angle OBA = \frac{a}{2a} = \frac{1}{2}$.



AB = a

Fig. 23

The reaction at A has components X_A and Y_A, as shown in Fig.23. When all the forces are in equilibrium, the algebraic sum of the moments about any point must be zero. It is convenient to choose this point as A so that the moment of the force of reaction at A is zero.

$$\therefore 1000\text{N} \times \frac{\sqrt{3}}{2} a = \mathbf{P} \frac{1}{2} a \Rightarrow \mathbf{P} = \sqrt{3}\, 1000\text{N}.$$

(ii) $\mathbf{P'}$ is applied at $\frac{3}{4}$a from the bottom of the post. For equilibrium we must have

$$1000\text{N} \times \frac{\sqrt{3}}{2} a = \mathbf{P'} \frac{3}{4} a \Rightarrow \mathbf{P'} = \frac{2}{3}\sqrt{3}\, 1000\text{N}.$$

So, $P' < P$.